

# Accelerators and detectors

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# Accelerators and detectors

## 1 Basics

To understand the properties of an object we need to see how it interacts with other objects. The typical experiment in nuclear or particle physics involves firing a projectile (e.g. proton, neutron, electron, ...) at a target. The projectile and target undergo an interaction (possibly creating a long-lived excited intermediate state which then decays). The particles at the end may not be the same ones you started with, but nevertheless we wish to detect them.

In some cases we don't have to do the 'firing' ourselves. For example many of the excited states of interest in nuclear physics – the unstable nuclei – were created long ago by high-energy collisions within stars. We just have to wait for them to decay.

The topic of how to make, accelerate, detect and identify particles is an enormous one. If you can answer the in-line questions marked marked with arrows ( $\Rightarrow$ ) you are doing well.

## 2 Accelerators

Scattering experiments are easiest to interpret if the initial state is a beam of particles with a known momentum  $\mathbf{p}$ . To make such a beam particles must either be accelerated to large  $\mathbf{p}$ , or must come from the decay of a parent which itself had large  $\mathbf{p}$ .

To accelerate the beam of particles, they will need to have interactions with an external field. The only forces which are active over macroscopic length scales are the electromagnetic and gravitational forces. The gravitational forces on subatomic particles are very much smaller than the electromagnetic ones, so we conclude that we should use electromagnetic fields to change  $\mathbf{p}$ . The particles to be accelerated will then have to be charged. We can achieve this by (for example) pulling electrons off a hot cathode ( $e^-$  beam) or using such an electron beam to strip electrons from atoms to create positively-charged ions (e.g. protons beam from kicking the electrons off Hydrogen).

Since the magnetic field changes only the direction of  $\mathbf{p}$ , we have to use electrical fields to increase the energy of the particles. How big an  $\mathbb{E}$  field can be achieved? From knowledge of atomic energy levels, we expect that the electric strength is of order  $0.1 \text{ V}/10^{-10} \text{ m} \sim 10^8 \text{ V m}^{-1}$ . Fields stronger than this will pull electrons from the metal surface, so cannot be sustained. To get energies of order

$$\frac{d\mathbf{p}}{dt} = Q[\mathbb{E} + \mathbf{v} \times \mathbf{B}]$$

The Lorentz force law.  $Q$  is the particle's charge,  $\mathbf{p}$  its momentum,  $\mathbf{v}$  its velocity, and  $\mathbb{E}$  and  $\mathbf{B}$  the electric and magnetic fields, respectively.



Figure 1: A superconducting niobium cavity designed for a high-energy  $e^+e^-$  linear collider. The cavities operate at radio frequencies and can support fields up to  $50 \text{ MV m}^{-1}$ .

MeV can therefore be done in a small scale,<sup>1</sup> whereas to get TeV-scale energies from linear acceleration would require constant acceleration over distances of order  $10^{12} \text{ V} / (10^8 \text{ V m}^{-1}) \sim 10^4 \text{ m}$ .

### 2.0.1 Linear accelerators

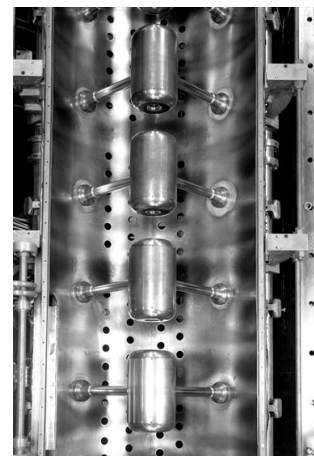
The problem we encounter with a constant electric field is that to achieve high energies we need an enormous potential difference. Large potential differences tend to break down because of electrical discharge (sparking) to nearby objects.

We can get around this by realizing that only that only the local  $\mathbb{E}$  field needs to be aligned along  $\mathbf{v}$ , and only while the particle is in that particular part of space.

We can use time-varying electric fields to achieve large energy without large static potential differences. In the margin you can see a picture of a **linear accelerator** or **linac**. In this device we have a series of cylindrical electrodes with holes through the middle of each which allow the beam to pass through. Electrodes are attached alternately to either pole of an alternating potential. The particles are accelerated in bunches. As each bunch travels along we reverse the potential while the bunch is inside electrode (where this is no field). We can then ensure that when the bunch is in the gap between the electrodes the field is always in the correct direction to keep the particle accelerating.

The oscillating potential on the electrodes may be created by connecting wires directly from the electrodes to an oscillator. For radio frequency AC oscillations we instead bathe the whole system in an electromagnetic standing wave, at the right frequency and phase to provide continuous acceleration.

<sup>1</sup>An exception will be discussed in lectures.



The linear accelerator injector to the CERN proton synchrotron.  
©CERN

## 2.1 Bending, and circular accelerators

By bending our particle beams into a circle we can reuse the same accelerating components many times over.

Beams are bent by magnetic fields,  $\mathbf{B}$ , with a force given by

$$\mathbf{f} = Q\mathbf{v} \times \mathbf{B}.$$

The acceleration is perpendicular to the velocity, so the Lorentz factor  $\gamma$  is unchanged, and the Lorentz force law reduces to

$$Qv_{\perp}B = \gamma_v m a,$$

where  $a$  is the lab acceleration, and  $v_{\perp}$  is the velocity perpendicular to the magnetic field. The particle's motion will describe a circle (or a helix if we start it off with non-zero momentum component  $p_{\parallel}$  in the direction parallel to  $\mathbf{B}$ ). The acceleration in the lab frame must satisfy

$$a = \frac{v_{\perp}^2}{R},$$

where  $R$  is the radius of the circle. Combining this with the Lorentz force gives

$$Qv_{\perp}B = \frac{\gamma m v_{\perp}^2}{R}.$$

For  $p_{\parallel} = 0$  the particle describes a circle of radius

$$R = \frac{p_{\perp}}{QB},$$

where  $\mathbf{p}_{\perp} = \gamma_v m \mathbf{v}_{\perp}$  are the momentum components perpendicular to  $\mathbf{B}$ .

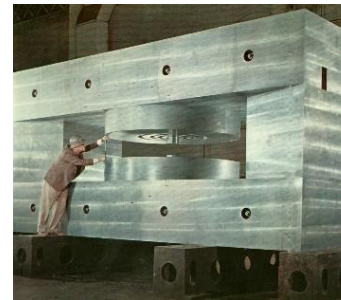
If we have a particle with charge  $|e|$  and we express  $p_{\perp}$  in GeV,  $B$  in Tesla and  $R$  in meters then we have the simple scaling law that

$$\boxed{p_{\perp} = 0.3BR} \quad [\text{GeV, Tesla, meters}].$$

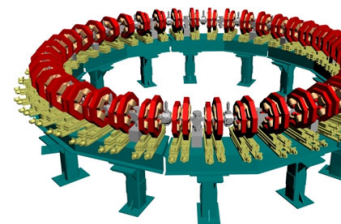
The maximum energy achievable for proton beams (currently  $E = 3.5$  TeV for protons at the LHC, CERN, Geneva) is limited by the product  $BR$ . Large scale superconducting magnets can reach fields of order of a few Tesla, so for a TeV-momentum beam we'll need  $R \sim \text{km}^2$ .

The other effect one needs to worry about in circular accelerators is **synchrotron radiation**. This is the electromagnetic radiation emitted when relativistic charges accelerate, which they must do to describe a circle. The synchrotron energy loss is proportional to  $\gamma^4$ . Electrons have a much smaller mass than e.g. protons and hence a larger  $\gamma$  for the same energy or momentum. Electron beam energies are

<sup>2</sup>Astronomical sources can have large magnetic fields over longer distances, and accelerate particles to even higher energies. Cosmic rays impinging on the earth's atmosphere from space have been observed with energies above  $10^{20}$  eV! [Takeda et al.(1998)]



To make a high-momentum beam you might want one big magnet...



... or perhaps lots of small ones.

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are usually limited by synchrotron losses <sup>3</sup>. Electrons in circular colliders have been accelerated to energies of up to about 105 GeV at LEP (CERN, Geneva).

⇒ Find out what is meant by a **cyclotron** and a **synchrotron**. Explain the difference to a friend.

### 3 Introduction to detectors

We want to study the properties of particles. For us to infer their presence they have to undergo *interactions* with some material that then leads to a detectable signal.

Historically a variety of techniques have been used to detect the presence of particles. Geiger and Marsden worked with  $\alpha$  particles. They bashed them off nuclei and then into a **phosphor screen**. When the  $\alpha$  hits the screen it partially ionized the atoms there. When those atoms de-excite they can create visible light [Geiger and Marsden(1909)]. (Old cathode ray tube televisions and monitors work on similar principles.) **Cloud chambers** contain a super-saturated gas solutions. The gas is ready to form droplets, but there is an energy barrier caused by surface tension which prevents the formation of small droplets (and hence large ones). Charged particles depositing energy via ionization allow droplets to form along their path, forming a visible track.

**Bubble chambers** are not so different. They use a super-heated liquid (e.g. liquid Hydrogen), just ready to boil. When the particle passes through, it generates enough energy for gas bubbles form, creating a trail of bubbles along the track.<sup>4</sup>

Both droplets and bubbles will scatter light, so can be photographed, producing snazzy pictures that show the particle trajectories.

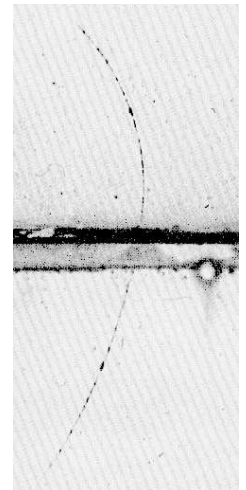
Most modern particle detectors are rather different. They are almost all designed to produce **electrical** signals, rather than photographic ones. They need some method of liberating charge — charge which is subsequently amplified and digitized. By feeding the signals into a computer, very large numbers of interactions can be analyzed. Computer analysis is easier on the eyes than scanning thousands of photos.

### 4 Particle interactions with matter

If we are going to detect a particle then it had better **interact** with something.

<sup>3</sup>These losses are a pain if you want to get the beam to high energy. But they have their uses. The intense x-rays emitted are just what you want if you are a materials scientist, crystallographer or biologist.

<sup>4</sup>One of the few instances of a great physics idea being inspired by someone staring at a pint of beer.



Cloud chamber photograph showing a positively charged particle with the same mass as the electron – an anti-electron or **positron**. This is how anti-matter was discovered. The lead plate in the middle of the picture is used to slow the positron so that the direction of motion (and hence the charge) can be inferred. From [Anderson(1933)].

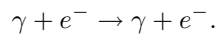
The interactions of particles depend on their properties – particularly their masses, charges, and couplings to other particles. We don't need to know about them all, but let's consider some examples.

### 4.1 Photon interactions

When a photon strikes some material object the result depends on the energy of that photon.

The dominant interactions of photons in some example materials are shown in Figure 2 for a range of photon energies. At low energy the photon is usually coherently absorbed by the atom, leading to the ejection of the electron in a process known as **photoelectric absorption**. In the figure you can see the large cross-section, with sharp spikes when the photon has enough energy to knock electrons out of more tightly-bound shells.

**Compton Scattering** At higher energy the photon acts as if scattering elastically from stationary 'free' electrons. This is known as **Compton scattering**:



Let's investigate the kinematics. Let the photon four-momentum be P and the electron four-momentum be Q. We'll use the same symbols but with primes after the scatter. Then energy-momentum conservation is given by

$$P + Q = P' + Q'.$$

To eliminate the components of Q', recognize that we get rid of the energy and momentum components of the electron by taking

$$Q'^2 = (P + Q - P')^2. \tag{1}$$

The electron can be assumed to be at rest. Without loss of generality the four-vectors can be expressed as

$$P = \begin{pmatrix} E \\ E \\ 0 \\ 0 \end{pmatrix} \quad Q = \begin{pmatrix} m_e \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad P' = \begin{pmatrix} E' \\ E' \cos \theta \\ E' \sin \theta \\ 0 \end{pmatrix}.$$

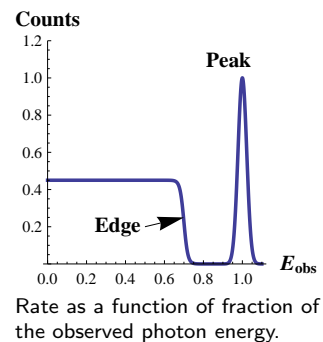
Expanding (1) we find that

$$EE'(1 - \cos \theta) = m_e(E - E').$$

which can be rewritten

$$E' = \frac{m_e E}{E(1 - \cos \theta) + m_e}.$$

But what do we observe? If the photon is scattered through sufficiently small



## 4.1 Photon interactions

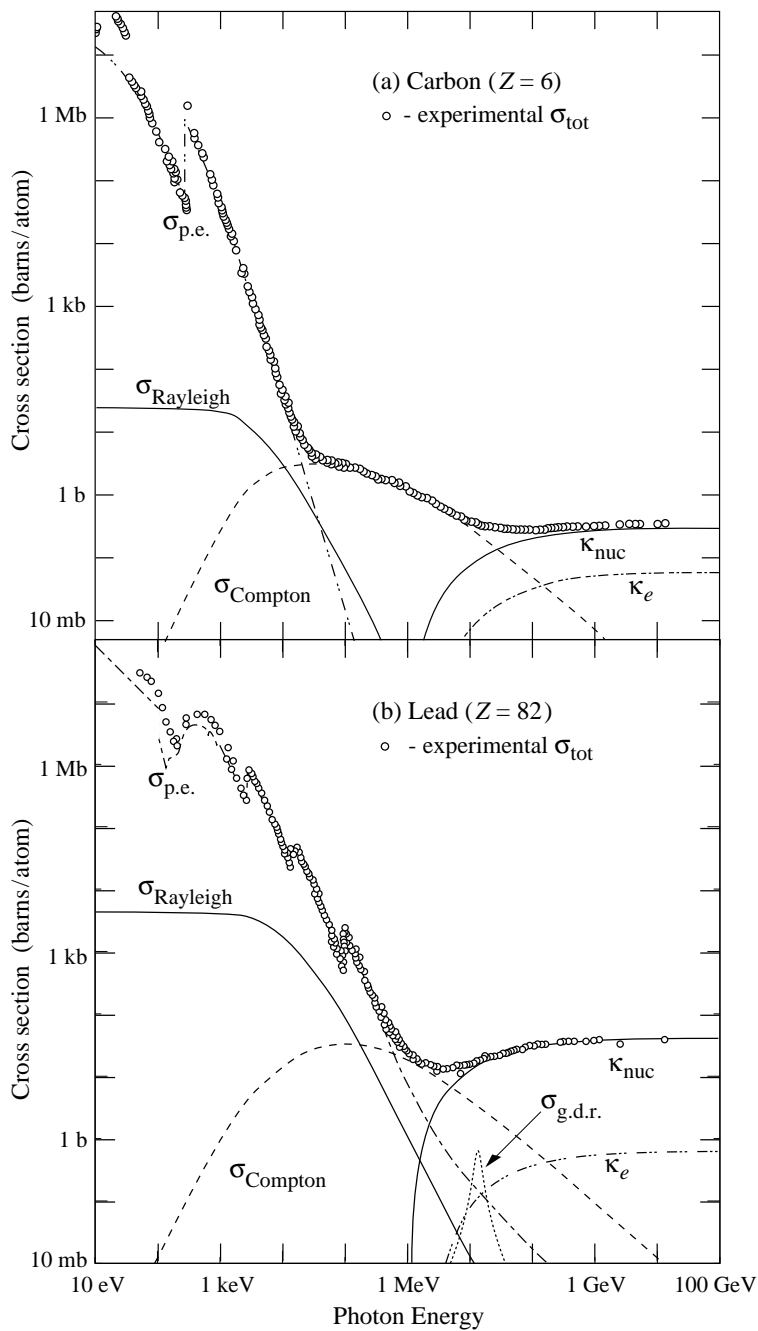


Figure 2: Photon total cross sections as a function of energy in carbon and lead, showing the contributions of different processes. Most important are:  
**(p.e.)** Atomic photoelectric effect (electron ejection, photon absorption)  
**(Compton)** Compton scattering from an electron:  $\gamma + e^- \rightarrow \gamma + e^-$   
**(nuc)**  $\gamma \rightarrow e^+e^-$  pair production in the nuclear electric field.  
 From [PDG(2008)].

## 4.2 Very high-energy electrons and photons

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angles that all of its energy *and all of the energy of the scattered electron* end up being absorbed in the detector then one gets a *peak* corresponding to  $E$ .

What if the photon scatters through a sufficiently large angle that it is not subsequently absorbed by the material? Then some of the energy will not be observed, and the visible signal will be smaller. The energy lost depends on the angle through which the photon is scattered.

Assuming the photon is lost, the *minimum* energy is lost when  $E'$  is smallest, which is when the photon back-scatters such that  $\cos\theta$  is close to  $-1$ . When an ensemble of scattering events are observed (e.g. when detecting many individual gamma rays one-by-one from an isotopic decay) the distribution of *observed* energies shows a sharp drop corresponding to scattered photon energy

$$E' = \frac{m_e E}{2E + m_e}.$$

The sharp drop at visible energy  $E - E'$  is known as the **Compton edge**.

**Pair creation** At still higher energies, photons with  $E > 2m_e$  can create  $e^+e^-$  pairs from interactions in the vicinity of an atomic nucleus,

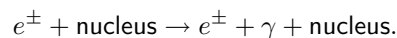


The nucleus absorbs some of the momentum from the photon. The energy of the incoming photon clearly has to be greater than twice the rest-mass-energy of the electron for the reaction to proceed.

⇒ Why can the above reaction *not* happen in a vacuum?

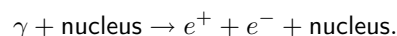
## 4.2 Very high-energy electrons and photons

Very high energy electrons in the presence of a charged nucleus will accelerate (electrons more so than e.g. protons). Accelerated charges emit electromagnetic radiation – photons:



The process above is known as **Bremsstrahlung** (from the German 'braking radiation'.)

As described above in §4.1, the photons produced — if they have enough energy — can lead to **pair-creation** of further electrons and positrons in the nuclear electric field



Those electrons and positrons can themselves undergo further Bremsstrahlung. More electrons, positrons and photons are created through repeated cycles of Bremsstrahlung and pair-creation until the energy of the photons is too small to generate electron-positron pairs.



### 4.3 Very high-energy, strongly interacting particles

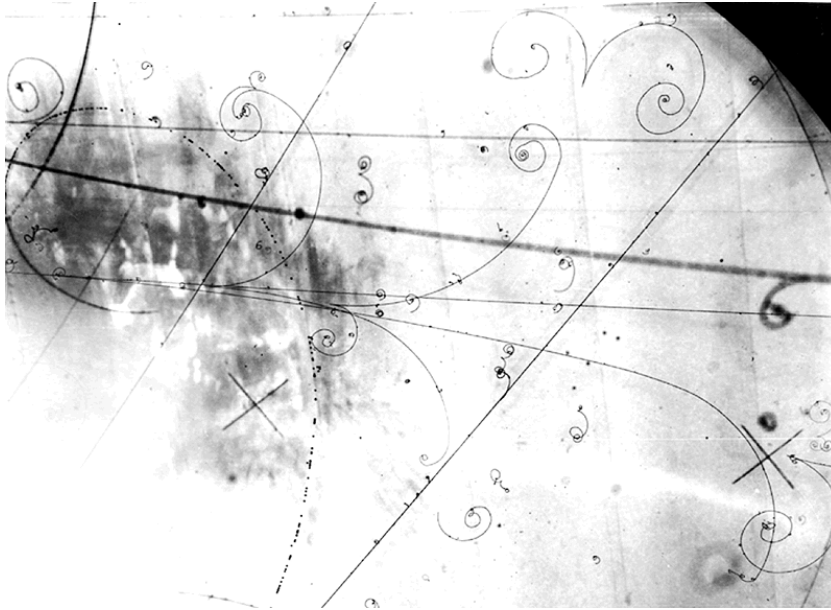
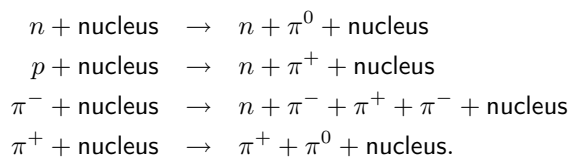


Figure 3: Bubble chamber photograph showing the creation of electron-positron pairs  $\gamma \rightarrow e^+ + e^-$ , and photon creation from Bremsstrahlung.

The net effect is to create a *cascade* or shower of electrons, photons, and positrons. As these come to rest they create a lot of subsequent ionization. The average amount of ionization will be proportional to the energy of the incoming electron or photon. A particle sensitive to this ionization — for example a crystal which produces light proportional to the energy deposited in it — then acts as an **electromagnetic calorimeter**.<sup>5</sup>

### 4.3 Very high-energy, strongly interacting particles

Particles which couple to the *strong nuclear force* — such as neutrons, protons, kaons and pions can undergo strong-force reactions with atomic nuclei. For very high-energy projectiles these mostly result in the creation of new strongly-interacting particles, e.g.



Rather like in the electromagnetic case, cascades of such interactions create large numbers of charged pions — the lightest strongly interacting particles — and photons from the subsequent decay  $\pi^0 \rightarrow \gamma\gamma$ . This is known as a 'hadronic shower', and

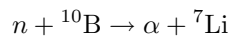
<sup>5</sup>Calorimeter = device for measuring energy.

devices which exploit it to determine the energy of the original strongly-interacting particle are imaginatively referred to as **hadronic calorimeters**.

#### 4.4 Detecting neutrons

High energy neutrons will create hadronic showers (described above). If the neutron energy is less than or even close to  $m_\pi \approx 140$  MeV they won't be able to play that game.

Low energy neutrons can be detected by inducing them to undergo nuclear reactions which lead to ionizing particles. For example neutrons impinging on a gas of  $BF_3$  can undergo the reaction



which has a large cross-section. The  $\alpha$  particle ionizes the gas which, in the presence of an electric field, leads to a current.

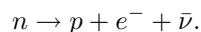
#### 4.5 Detecting neutrinos

Neutrinos have extremely small cross sections, so one needs very large fluxes – and preferably very large detectors – to stand a chance of detecting them.

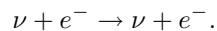
Though the cross-section is very small (see problem set 1), rare collision reactions can be observed. For example when a neutrino strikes a neutron the following reaction can occur



The interaction which allows this – the weak nuclear interaction – is the same one that is responsible for nuclear beta decay:



Neutrinos can undergo other interactions, for example elastic scattering from electrons



The moving electron from this reaction, or from the nucleon-changing scattering reaction (2), can then be detected. We'll talk more about the interactions of neutrinos after we discuss the only force they couple to – the weak nuclear force.

#### 4.6 Measuring properties

**Momentum** can be measured using the bending radius of the particle in an applied magnetic field,  $p_\perp = QBR$ .

**Speed** can be measured from:

- **Time of flight** (to of order ps timing resolution).
- Energy deposited through **ionization**. Energy loss  $-\frac{dE}{dx}$  depends on  $v$  (see the first problem sheet), so if you measure the amount of ionization you will learn about the speed.
- The electromagnetic equivalent to the sonic boom – the **Čerenkov radiation** created when a particle passes through a medium faster than the local speed of light in that medium.

⇒ Look up the meaning of Čerenkov radiation. Convince yourself that this sort of radiation should be emitted at angle  $\theta = \cos^{-1} \frac{1}{\beta\eta}$  from the particle's trajectory, where  $\beta = v/c$  and  $\eta$  is the refractive index of the medium.

**Energy** can be measured by total absorption of the particle within some active medium (a **calorimeter**).

**Lifetime** can be measured with a good clock for reasonably long-lived species. For short-lived species we can infer the lifetime from the distance travelled before decaying — provided that the lifetime and speed are such that the mean distance travelled  $\beta\gamma ct_{\text{decay}}$  is a measurable length. If the particle is very short-lived, we can measure its Breit-Wigner width,  $\Gamma$ , and infer the lifetime  $\tau = \hbar/\Gamma$ .

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## Key ideas

- Charged particles can be accelerated according to the Lorentz force law

$$\frac{d\mathbf{p}}{dt} = Q[\mathbf{E} + \mathbf{v} \times \mathbf{B}]$$

- Linear accelerators are limited by the strength of the electric field achievable.
- Circular accelerators are limited by the strength of the magnetic field for protons, and by synchrotron radiation for electrons.
- The momentum of particles can be measured from their bending in magnetic fields
- A useful formula relating bending radius, momentum, and magnetic field is

$$p_{\perp} = 0.3BR \quad [\text{GeV, Tesla, meters}].$$

- For a particle to be detected it has to interact with some sensitive material.
- The energy of particles can be measured by absorbing them in a calorimeter.
- Modern particle detectors are designed to produce electrical signals that can be amplified and digitised.

## .A Details of a linear acceleration

*Non-examinable*

### .A.0.1 Constant electric field

Let's calculate how the speed and position of an particle will vary when it is accelerated from rest in a constant electrical field.

For a constant electric field  $\mathbf{E} \parallel \mathbf{v}$ , and starting at rest we can work in one dimension. Since the parallel component of the field stays the same under the Lorentz transformation

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel},$$

and since the Lorentz force law reduces to

$$f = \mathbf{E}Q$$

the proper acceleration (i.e. the acceleration in the frame in which the particle is instantaneously at rest) is

$$a_0 = \frac{\mathbf{E}Q}{m}. \quad (3)$$

---

If the motion is relativistic it's convenient to work in terms of the rapidity  $\rho$  which is defined by

$$\boxed{\tanh \rho = v/c} \quad (4)$$

The reason that rapidity is useful is that it is easy to add successive Lorentz transformations if their relative rapidity is known. To show this, we rewrite the usual 1D Lorentz transformation in terms of the rapidity,

$$\Lambda_a = \begin{pmatrix} \gamma_a & -\beta_a \gamma_a \\ -\beta_a \gamma_a & \gamma_a \end{pmatrix} = \begin{pmatrix} \cosh \rho_a & -\sinh \rho_a \\ -\sinh \rho_a & \cosh \rho_a \end{pmatrix}$$

showing only the  $ct$  and  $x$  components. Here  $\Lambda_a$  is the L.T. corresponding to a boost by rapidity  $\rho_a$  or equivalently by a velocity  $v_a = c \tanh \rho_a$ .

By multiplying a pair of such matrices together with different rapidities we can see that the combined operation of two Lorentz transformations along the same axis is given by

$$\Lambda_b \Lambda_a = \Lambda_{a+b}.$$

This is a neat result. The Lorentz transformation for a combined boost has rapidity given by *the straight sum* of the rapidities of the two individual transforms:

$$\rho_{a+b} = \rho_a + \rho_b. \quad (5)$$

Remember that the same can *not* be said for the velocity. The resultant velocity is *not* the direct sum of the two individual velocities (unless  $v \ll c$ ):

$$v_{a+b} = \frac{v_a + v_b}{1 + v_a v_b / c^2} \neq v_a + v_b.$$

By taking derivatives of (4) we can see that, close to the instantaneous rest frame (IRF) of the particle,

$$\left. \frac{d\beta}{d\rho} \right|_{\text{IRF}} = 1.$$

This means that we can use the proper acceleration (in the series of instantaneous rest frames) to calculate the rate of change of rapidity with respect to proper time

$$a_0 = \left. \frac{dv}{dt} \right|_{\text{IRF}} = c \left. \frac{d\beta}{d\tau} \right|_{\text{IRF}} = c \frac{d\rho}{d\tau}.$$

The IRF subscript shows quantities calculated very close to the instantaneous rest frame.

In the last expression all of the quantities ( $c$ ,  $d\tau$ ,  $d\rho$ ) are unchanged by any boost along the  $x$ -axis.<sup>6</sup> So the expression  $a_0 = c \frac{d\rho}{d\tau}$  is valid in *any*  $x$ -boosted frame, not just the IRF.

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<sup>6</sup>This is clear for  $a_0$ ,  $c$ , and  $d\tau$  all of which are invariants by definition. To see that  $d\rho$  is not changed by the boost let us show that *differences* in rapidity must be unmodified by the L.T. along the  $x$ -axis. Consider a general difference of rapidity ( $\rho_a - \rho_b$ ). If both  $a$  and  $b$  are boosted by rapidity  $\rho_c$ , then from (5) the difference becomes  $(\rho_a + \rho_c) - (\rho_b + \rho_c) = \rho_a - \rho_b$ , i.e. the rapidity difference is unchanged. This is true for *any* difference in rapidities (along the  $x$ -axis) so it must be true for  $d\rho$ .

---

Since (5) implies that rapidities are additive, we can calculate the rapidity at any proper time  $\tau$  as the sum of lots of little boosts, each of rapidity  $d\rho$ :

$$\rho = \int d\rho = \int \frac{d\rho}{d\tau} d\tau = \frac{a_0\tau}{c} = \frac{\mathbb{E}Q\tau}{mc} \quad (6)$$

(substituting for  $a_0 = c d\rho/d\tau$  from above).

It's now easy to find a parametric equation for the motion. We just Lorentz transform using the matrix  $\Lambda_\rho$  to boost from the lab frame to the IRF, which we can do for any proper time. For a particle initially at rest at  $X_0 = (0, x_0, 0, 0)$  (with  $x_0$  to be determined later),

$$X = \Lambda_\rho X_0.$$

The components of the boosted vector can then be parametrized in terms of the proper time,

$$\begin{aligned} x &= x_0 \cosh(a_0\tau/c) \\ ct &= x_0 \sinh(a_0\tau/c), \end{aligned} \quad (7)$$

since  $\rho = a_0\tau/c$ .

We can also find the speed in terms of the proper time using the definition of the rapidity (4)

$$v = c \tanh \rho = c \tanh(a_0\tau/c).$$

We haven't yet worked out the constant  $x_0$ . Let's do so. First we find the small velocity at very early times when  $t$  is small and (since  $v$  is small)  $t \approx \tau$ . Differentiating (7) at early times gives

$$v|_{v \ll c} = \left. \frac{dx}{dt} \right|_{v \ll c} = \left. \frac{dx}{d\tau} \right|_{v \ll c} = \frac{x_0 a_0}{c} \sinh\left(\frac{a_0\tau}{c}\right) \Big|_{v \ll c} \approx \frac{x_0 a_0^2}{c^2} \tau.$$

Comparing with this with  $v = a_0 t$  we see that our conveniently-chosen coordinates were such that our previously undetermined initial position must have been  $x_0 = c^2/a_0$ .

To get the energy and momentum we recognize that the components  $P = (E, pc)$  can be obtained from the energy-momentum four-vector of the initially stationary particle  $P_0 = (m, 0)$  by the same overall Lorentz transform as above,

$$P = \Lambda_\rho P_0.$$

Therefore the energy at any proper time is given by  $E = m \cosh \rho$  and the momentum is  $p = m \sinh \rho$  where the rapidity  $\rho$  at proper time  $\tau$  can again be found from (6).

Where should the electrodes on a linac be placed? The bunches are accelerating so will travel progressively longer distances during each period of the oscillator. Let's make the approximation that constant acceleration is very similar to lots of little accelerations, and use the results of the previous section. Eliminating the proper

time from (7) we find the following relationship between lab-frame position and time,

$$x^2 - (ct)^2 = x_0^2 \quad (8)$$

If the frequency of the applied AC voltage is  $f$  we want to change the field every half cycle, i.e. at times  $t_n = n/(2f)$ . We should therefore place the  $n$ th gap at the position the particles will be at that time, which is

$$x_n = \sqrt{x_0^2 + \left(\frac{nc}{2f}\right)^2}.$$

Let's check this answer for the non-relativistic case. For  $v \ll c$  we can Taylor expand

$$x_n = x_0 \sqrt{1 + \left(\frac{nc}{2fx_0}\right)^2} \approx x_0 + \frac{1}{2} \frac{n^2 c^2}{4x_0 f^2}.$$

Since we found above that  $x_0 = c^2/a_0$  we have  $x_n = \text{const} + \frac{1}{2} a_0 \frac{n^2}{4f^2}$  as expected from the non-relativistic formula  $x = \text{const} + \frac{1}{2} at^2$ .

## .B Charged particle ionization

*Non-examinable*

Charged particles e.g. protons will kick atomic electrons out of their ground states as they pass through the material. In the first problem set we calculated the rate at which energy is lost, assuming a 'free electron gas', and found that (in natural units)

$$-\left\langle \frac{dE}{dx} \right\rangle = \frac{4\pi n_e \alpha^2}{m_e v^2} \int_{z_{\min}}^{z_{\max}} \frac{z dz}{1+z^2}$$

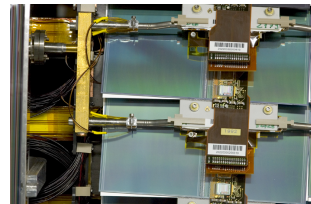
where  $z = bm_e v^2/\alpha$ ,  $b$  is the impact parameter,  $\alpha$  is the electromagnetic fine structure constant, and  $n_e$  is the number density of electrons in the material. A relativistic variation of this result is known as the **Bethe-Bloch** formula.<sup>7</sup>

The limits of the integration are set by the energies at which the approximation breaks down – which at the low-energy end is where  $E_e$  approaches the ionization energy of the material.

## .C Detector technologies

Below are a few techniques we can exploit to turn an interaction of a particle with a material into a recordable signal.

<sup>7</sup>See [PDG(2008)] for the full relativistic version.



Detail of the ATLAS Semiconductor tracker barrel during its assembly in Oxford. The silicon detector elements are approximately 60 mm wide [Abdesslam et al.(2006)].

## .C.1 Semiconductor detectors

When a charged particle passes through a semiconductor it creates electron–hole pairs – charge carriers – which can be accelerated by an applied electric field to create currents.

To work well, the semiconductor must previously have been depleted of charge carriers. This can be achieved by applying a voltage (a so-called *reverse bias*) to a junction between what are known as ‘p-type’ doped and ‘n-type’ doped types of silicon.

Low energy photons or electrons striking a semiconductor can deposit all of their energy within the material, resulting in a signal peak that corresponds to the total energy of the particle. The electrical signal is then a measure of the energy of the incoming photon or electron.

Very thin layers of semiconductor (often less than 1 mm thick), suitably instrumented, are used to detect the passage of a charged particle while only very slightly reducing that particle’s energy.

## .C.2 Gas and liquid ionization detectors

Charged particles traversing a gas or a noble liquid will create electron–ion pairs. In the presence of an electric field, those ions will create an electrical current which can be amplified (in the gas and/or electronically) and digitized.

⇒ How can we get amplification *in the gas*?

## .C.3 Scintillator detectors

Scintillators are materials which emit visible light when atomic electrons, excited by the passage of an ionizing particle, fall back to their ground states. The visible photons can be then be picked up by photomultiplier tubes, which converts that light to an electrical signal proportional to the energy deposited in the scintillator.

⇒ How does a photomultiplier work?

## Further Reading

Many of the books on nuclear and particle physics include chapters on experimental techniques. Examples in some general texts include:

- “*Introductory Nuclear Physics*”, P.E. Hodgson, E. Gadioli and E. Gadioli Erba, OUP, 2003. Chapters 4 and 5 are good introductions to accelerators



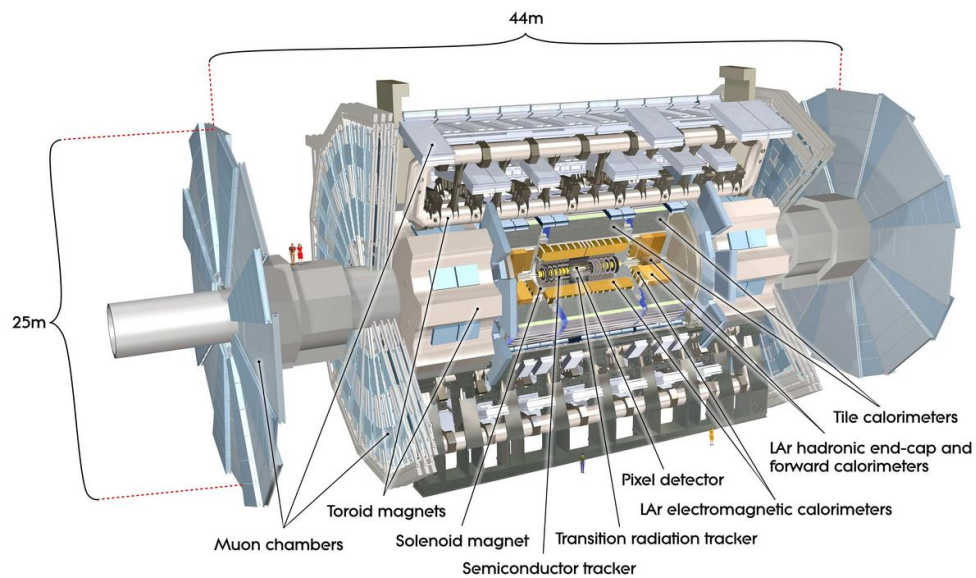


Figure 4: Diagram showing the major detector components of the ATLAS detector for the CERN Large Hadron Collider. The detector is built in concentric layers surrounding the beam-beam interaction point. LAr means liquid argon. The 'tile' hadronic calorimeter is made of alternating layers of iron and scintillator. From [Aad et al.(2008)].

⇒ Muons have the same interactions as electrons but are about 200 times heavier. Why are the muon detection chambers on the outer-most layers?

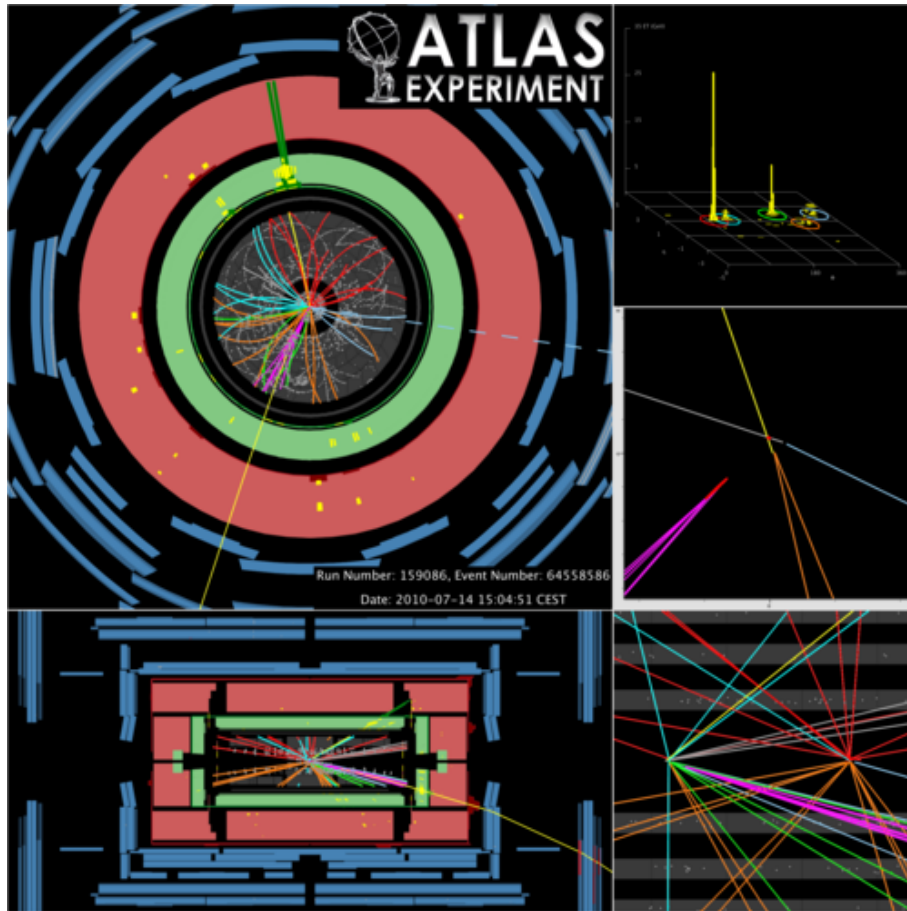


Figure 5: Event display of one of the first  $E_{\text{cm}} = 7$  TeV proton-proton collisions detected by the ATLAS experiment. Lots of particles and anti-particles have been created in the collision. They stream away from the interaction point and are detected by the various concentric layers of the detector. In the central region the charged particle tracks, which curve in the solenoidal field, are constructed from hits in layers of silicon detectors and gaseous ionization detectors. The lines represent tracks found by pattern-recognition algorithms.

**Top left** (scale  $\sim 10$  m) Projection of the detectors perpendicular to the beam.

**Bottom left** (scale  $\sim 10$  m) Projection along the line of the beam. The beam pipe would pass horizontally through the middle of this view.

**Top right** Energy deposited in the calorimeters, as a function of angle relative to the beam direction and azimuthal angle.

**Mid right** Detail (scale  $\sim 0.1$  mm) showing tracks pointing towards a **secondary vertex** caused by the decay of a 'long-lived' particle, probably one containing a  $b$ -quark.

**Bottom right** Detail (scale  $\sim$  cm) showing that two independent proton-proton collisions have coincided in time.

From <http://atlas.web.cern.ch/>

## REFERENCES

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and detectors respectively.

- “*Particle Physics*” B.R. Martin and G. Shaw. Chapter 3 gives a brief introduction at about the right level.
- “*Nuclear and Particle Physics*” W.E. Burcham and M. Jobes. Chapter 2 has lots of good stuff in it

More specialized books include . . .

- *An Introduction to Particle Accelerators*, E.J.N. Wilson, OUP (2001)
- *The Physics of Particle Detectors*, Dan Green, CUP (2000)

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- [PDG(2008)] **Particle Data Group** Collaboration, PDG, “Review of particle physics”, *Phys. Lett.* **B667** (2008) 1.
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