High Energy neutrino cross-sections

HERA-LHC working week Oct 2007

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- Updated predictions of high energy v and v CC cross-sections
- Within conventional framework NLO DGLAP
- With systematic accounting for PDF uncertainties
- Including general mass variable flavour treatment of heavy quarks
- Relevant to neutrino telescopes: Ice-Cube, ANTARES, KM3NET
  air shower arrays: Pierre Auger array
  radio detectors: RICE, ANITA

The point is to estimate how well known conventional predictions are in order to
See when we really have unconventional behaviour at small-x
BFKL ln(1/x) resummation
non-linear gluon recombination
etc

hep-ph: arxiv.0710.5303

High energy neutrino interactions probe very low-x (as we shall see) and thus the most relevant experimental information comes from HERA, not from lower energy neutrino experiments

In this field the cross-sections were given by Gandhi et al in 1996

This used CTEQ4-DIS PDFs and is significantly out of date, since the most extensive and accurate HERA data at low-x were published well after this date.

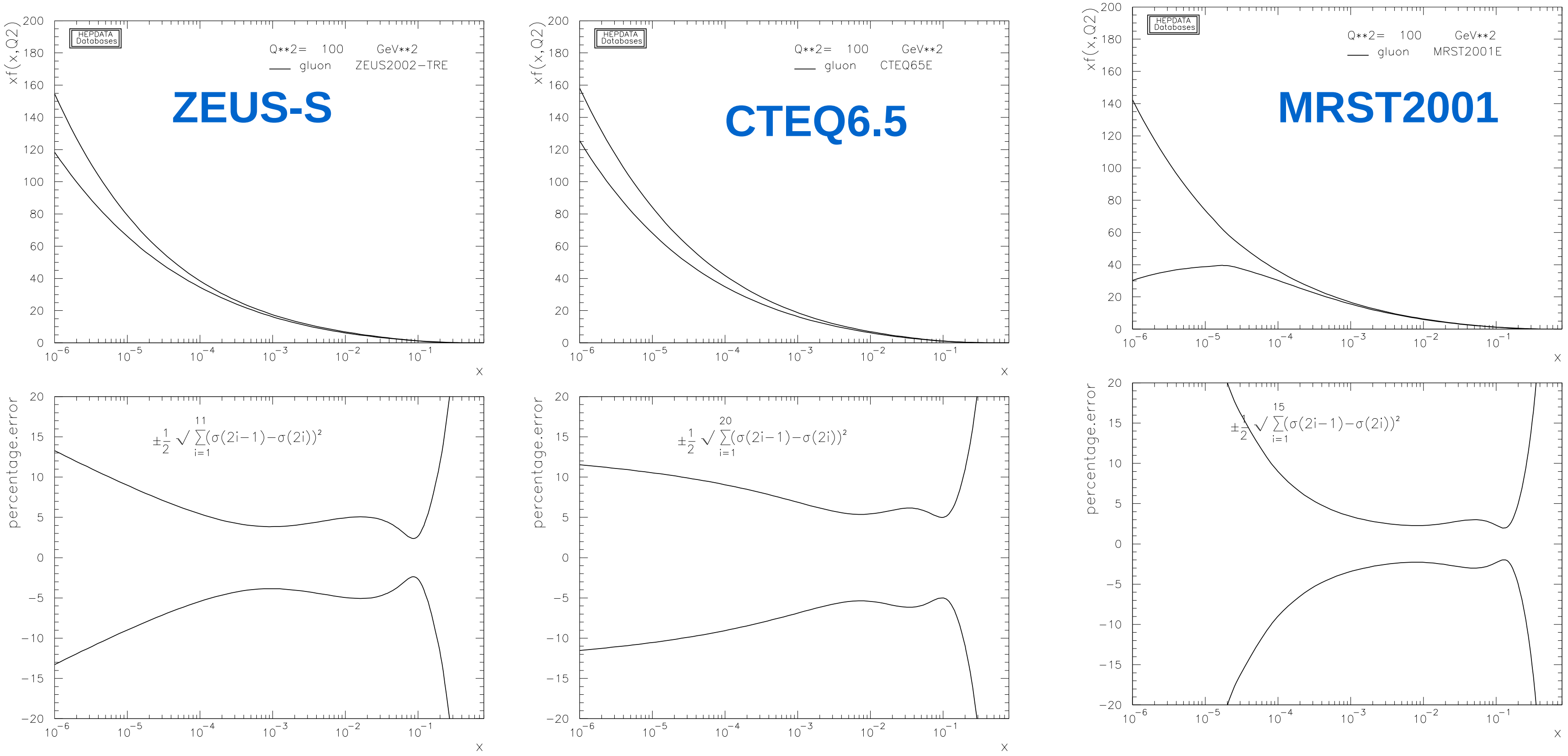
CTEQ4 PDFs no longer represent a good fit to HERA data

	Fixed-tgt	HERA	DY-W	Jets	Total
# Expt pts.	1070	484	145	123	1822
CTQ4M ~’98	1414	666	227	206	2513
MRS98 ~’98	1398	659	111	227	2396
CTQ6M 02	1239	508	159	123	2029
MRST01/2	1378	530	120	236	2264
MRST04	1315	519	129	154	2157

This is a table of $\chi 2$ values for modern and out of date PDFs produced by Wu-Ki Tung at DIS04

Need to use updated PDFs, present work updates ZEUS-S PDFs

Also modern PDFs come with uncertainty estimates from the experimental systematic and statistical errors of the input data so that one need not depend on ad hoc procedures like comparing the central value of one PDF set with another



Compare uncertainty estimates on the gluon for NLO PDFs

ZEUS-S uncertainty estimates are comparable those of CTEQ and MRST

ZEUS-S-13 fit is an updated version of the published ZEUS-S fit (Phys Rev D67, 012007) which was a global fit to DIS data. The updates are

1. Include all ZEUS HERA-I data 2. Free parameters Cg and au

Form of the parametrization at $Q^2_0 = 7\ \mathrm{GeV}^2$

- $xuv(x) = Au\ x^{au}\ (1-x)^{bu}\ (1 + c_u\ x)$
  $xdv(x) = Ad\ x^{ad}\ (1-x)^{bd}\ (1 + c_d\ x)$
  $xS(x) = As\ x^{as}\ (1-x)^{bs}\ (1 + c_s\ x)$
  $xg(x) = Ag\ x^{ag}\ (1-x)^{bg}\ (1 + c_g\ x)$
  $x\Delta(x) = x(d-u) = A\Delta\ x^{av}\ (1-x)^{bs+2}$

No χ2 advantage in more terms in the polynomial

No sensitivity to shape of $\Delta = \bar{d} - \bar{u}$
AΔ fixed consistent with Gottfried sum-rule - shape from E866

Assume $\bar{s} = (\bar{d}+\bar{u})/4$ consistent with ν dimuon data

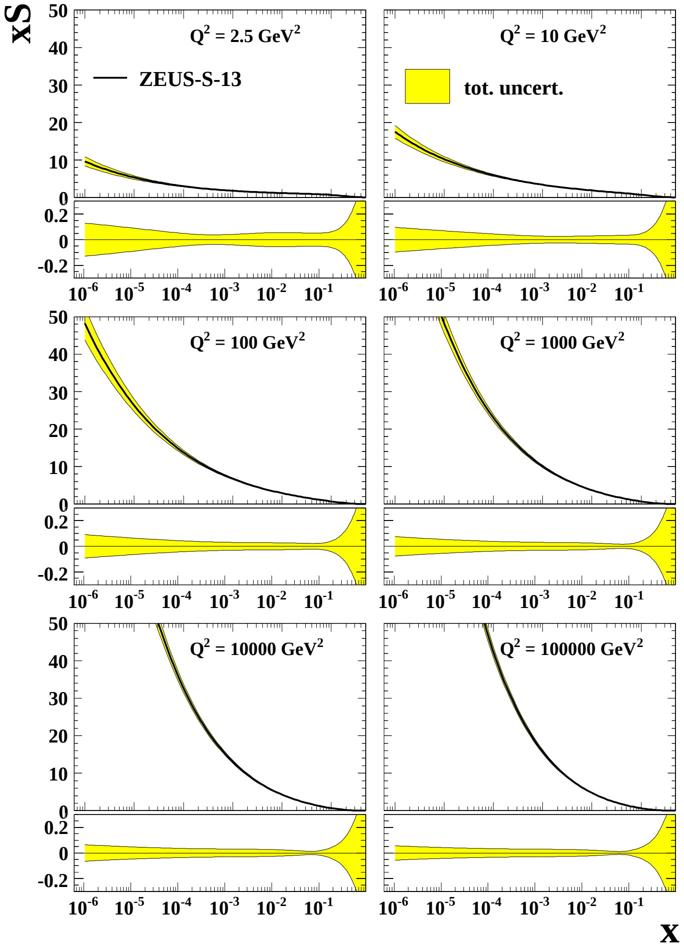
Au, Ad, Ag are fixed by the number and momentum sum-rules

ad=au for low-x valence since there is little information to distinguish

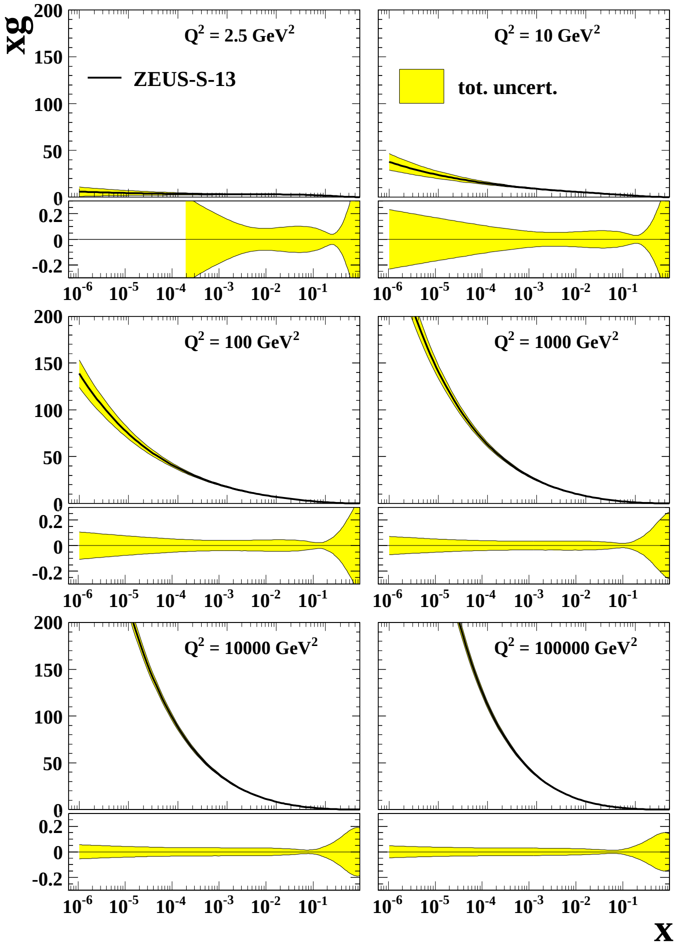
→ 13 parameters for the PDF fit

The fit uses the conservative OFFSET method to estimate the PDF uncertainty resulting from the experimental errors. Presents 68% CL uncertainties

Sea

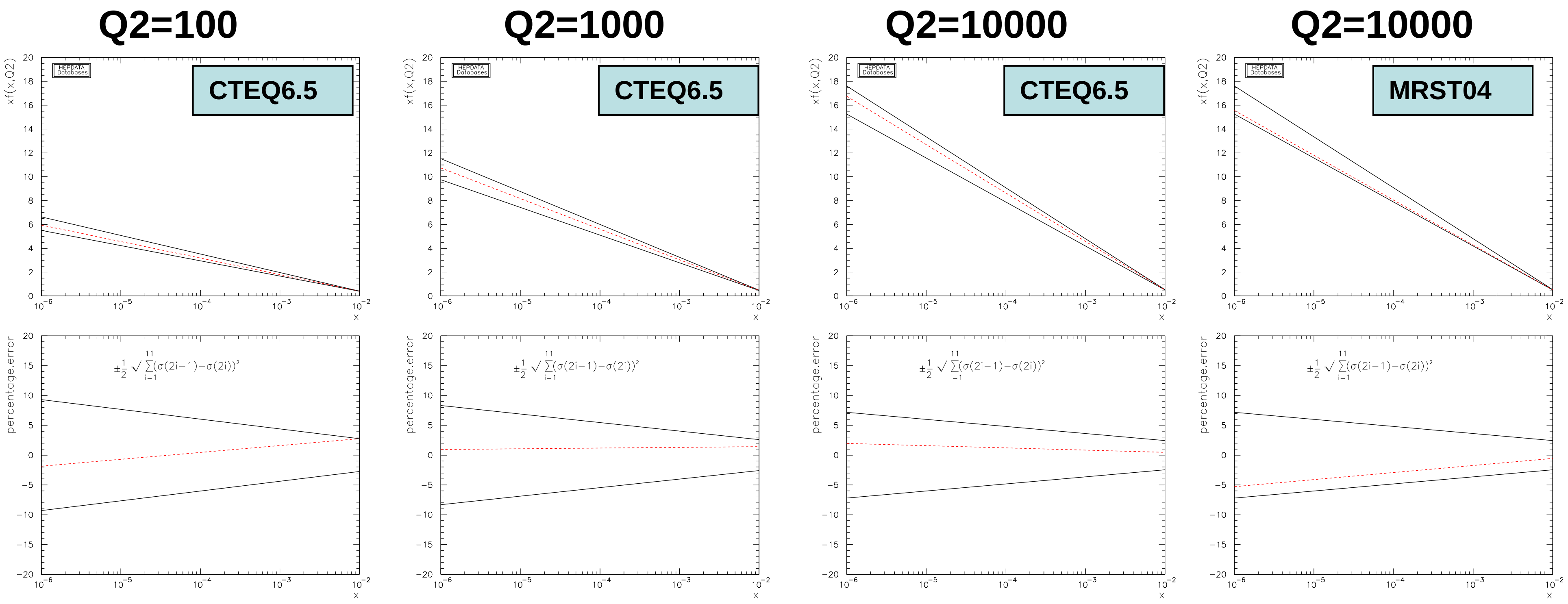


Gluon



The Low-x behaviour of the neutrino cross-sections is Sea dominated and the Sea is driven by the gluon by g → q qbar splitting. As we go to very high Q^2 the importance of low-x is greater

Uncertainties in the PDFs increase as we go to low x- although less so at larger Q^2



Comparing ZEUS-S-13 PDFs in black with CTEQ6.5/ MRST04 in red for upbar quark

for relevant x, Q2 ranges

Compatibility of modern PDFs

Why not just use off the shelf PDFs and do it yourself ?

Firstly you’ll need the corresponding coefficient functions for NLO (and correct heavy quark treatment)

Secondly, they don’t extend to large enough Q^2 or low enough x

Define the cross-section and reduced cross-section

$$\frac{d^2\sigma(\nu(\bar{\nu})N)}{dx\, dQ^2} = \frac{G_F^2 M_W^4}{2\pi(Q^2+M_W^2)^2 x}\sigma_r(\nu(\bar{\nu})N)$$

$$\sigma_r(\nu N) = \left[Y_+ F_2^{\nu}(x,Q^2) - y^2 F_L^{\nu}(x,Q^2) + Y_- xF_3^{\nu}(x,Q^2)\right],$$

$$\sigma_r(\bar{\nu} N) = \left[Y_+ F_2^{\bar{\nu}}(x,Q^2) - y^2 F_L^{\bar{\nu}}(x,Q^2) - Y_- xF_3^{\bar{\nu}}(x,Q^2)\right],$$

At LO in QCD for isoscalar nucleon targets we have

$$F_2^{\nu} = x(u+d+2s+2b+\bar{u}+\bar{d}+2\bar{c}), \quad xF_3^{\nu} = x(u+d+2s+2b-\bar{u}-\bar{d}-2\bar{c}),$$

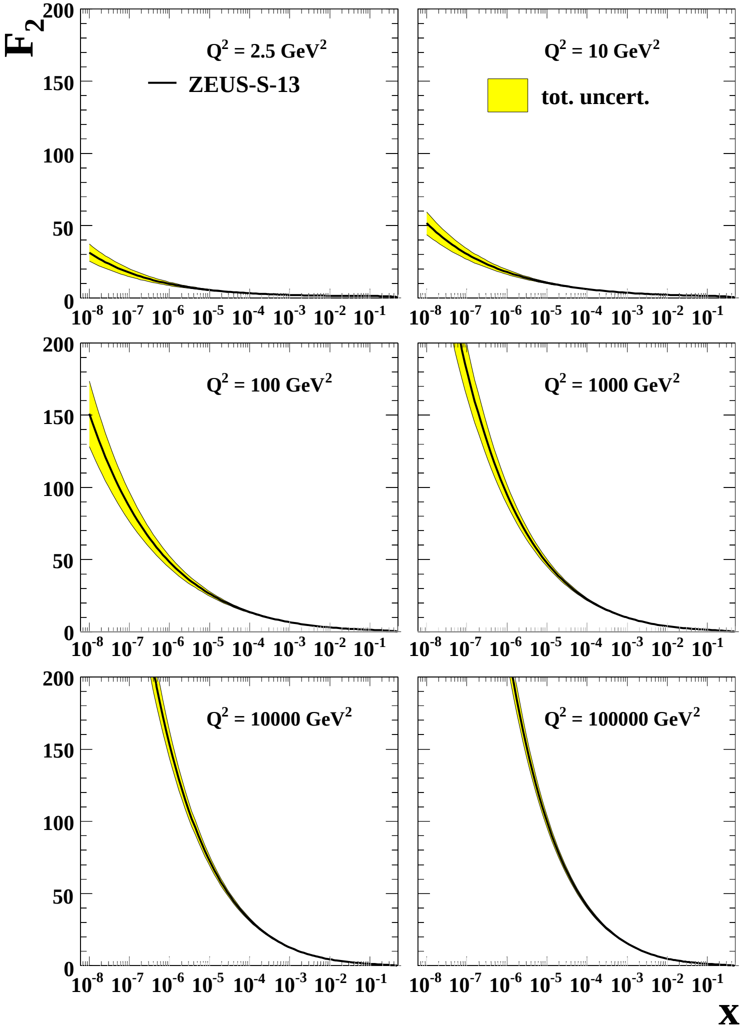
$$F_2^{\bar{\nu}} = x(u+d+2c+\bar{u}+\bar{d}+2\bar{s}+2\bar{b}), \quad xF_3^{\bar{\nu}} = x(u+d+2c-\bar{u}-\bar{d}-2\bar{s}-2\bar{b}).$$

Assuming, $s=\bar{s}$, $c=\bar{c}$, $b=\bar{b}$, we obtain $F_2^{\nu}=F_2^{\bar{\nu}}$, whereas $xF_3^{\nu}-xF_3^{\bar{\nu}} = 2(s+\bar{s}+b+\bar{b}-c-\bar{c}) = 4(s+b-c)$.

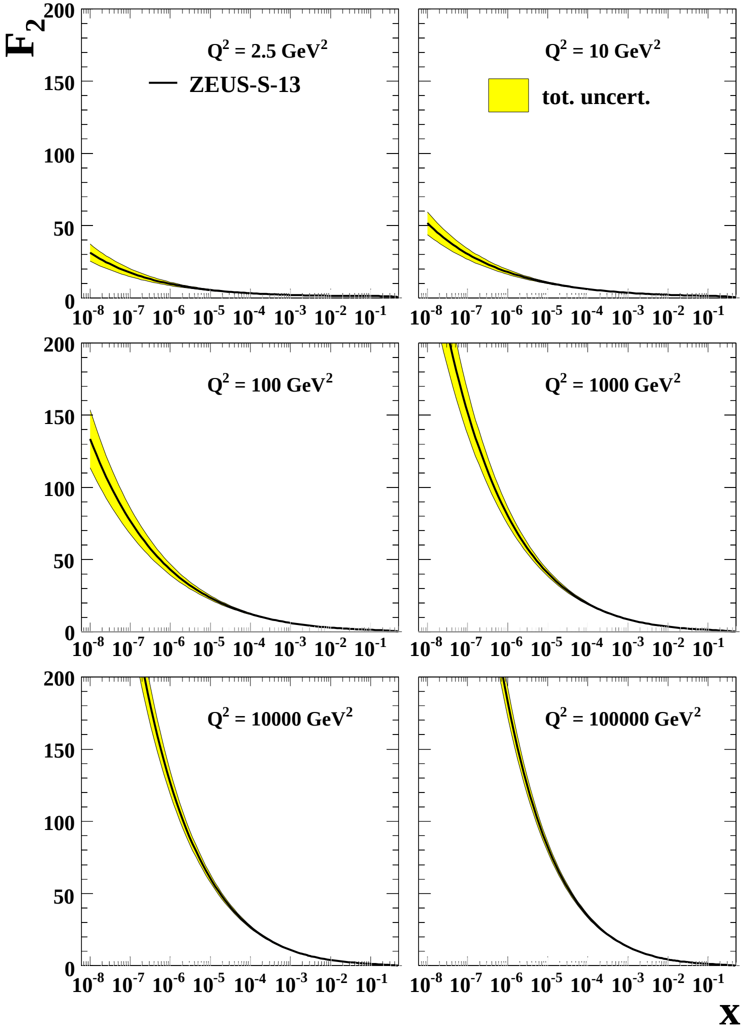
At NLO we convolute with coefficient functions, but these expression still give us a good idea of dominant contributions

Note the b will be very suppressed until $Q^2 >> m_{top}^2 \sim 30000$ GeV^2, because we are consider charged current cross-sections and the b to t coupling is dominant

The b contribution to F2 and FL is never more than 20%, but the effect on xF3 is much more dramatic

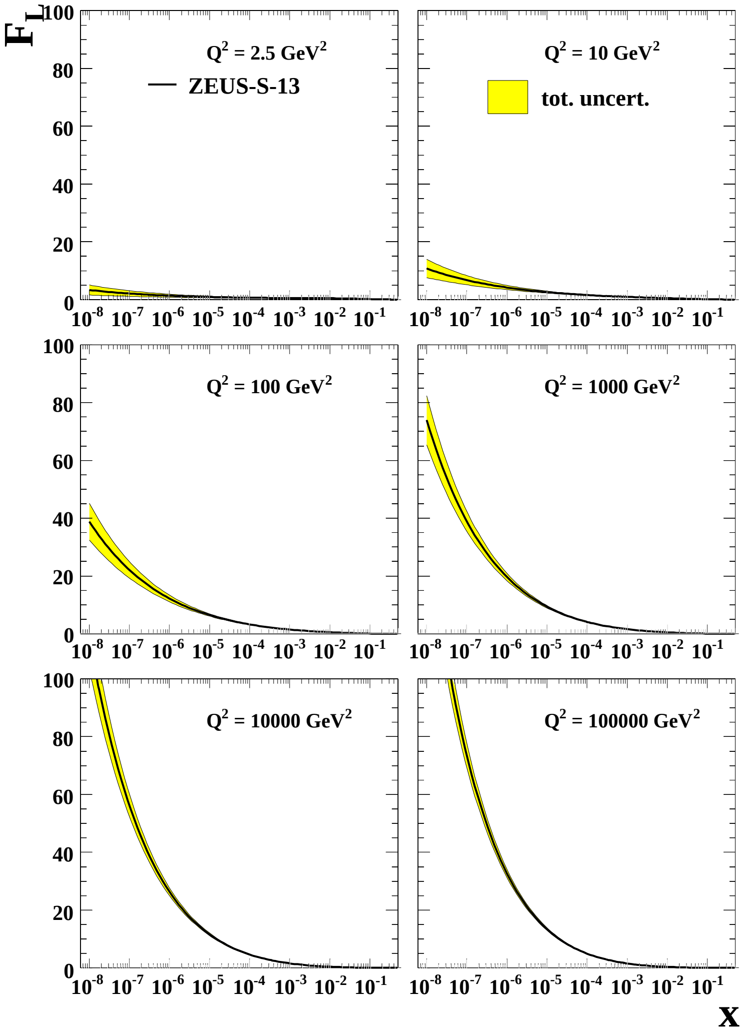


With b

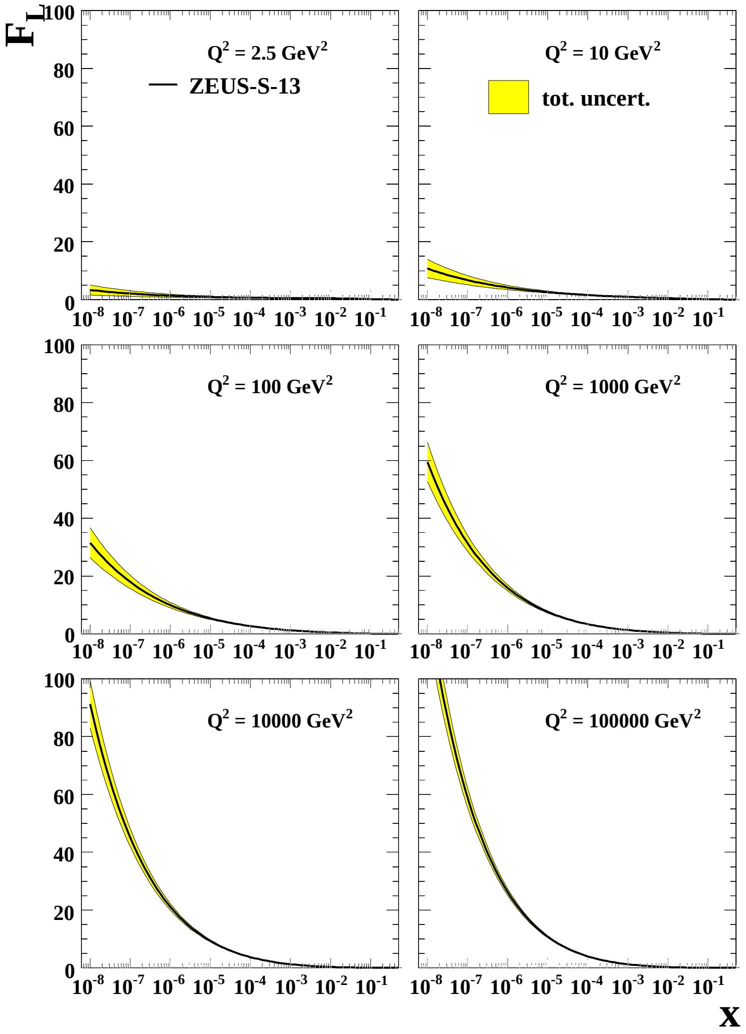


Without b

These plots show F2 from an NLO calculation using a zero-mass variable flavour scheme for the heavy quarks, with and without including the b quark

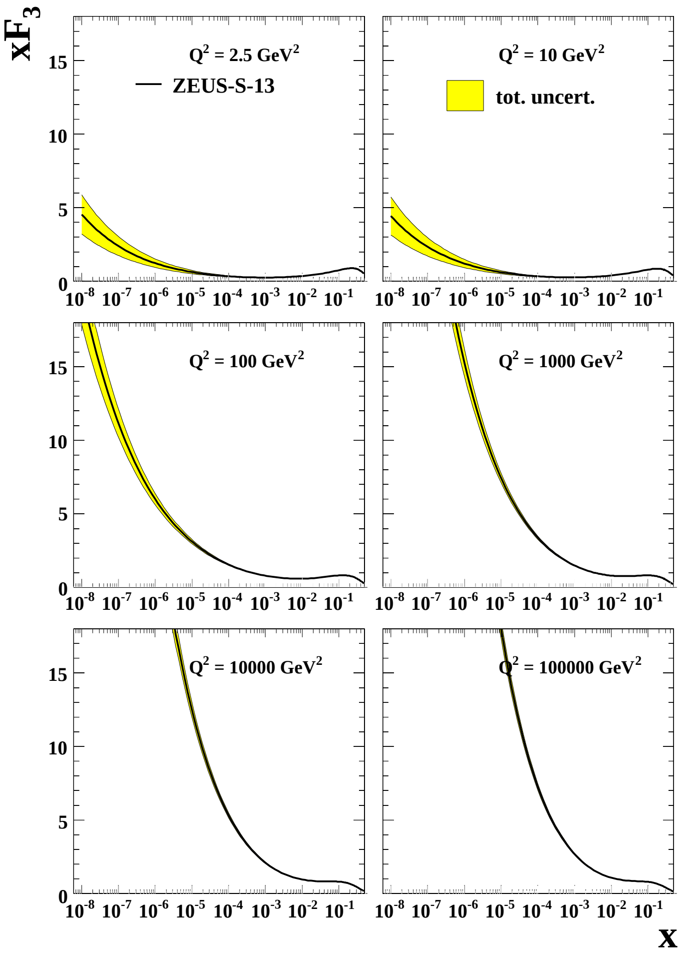


With b



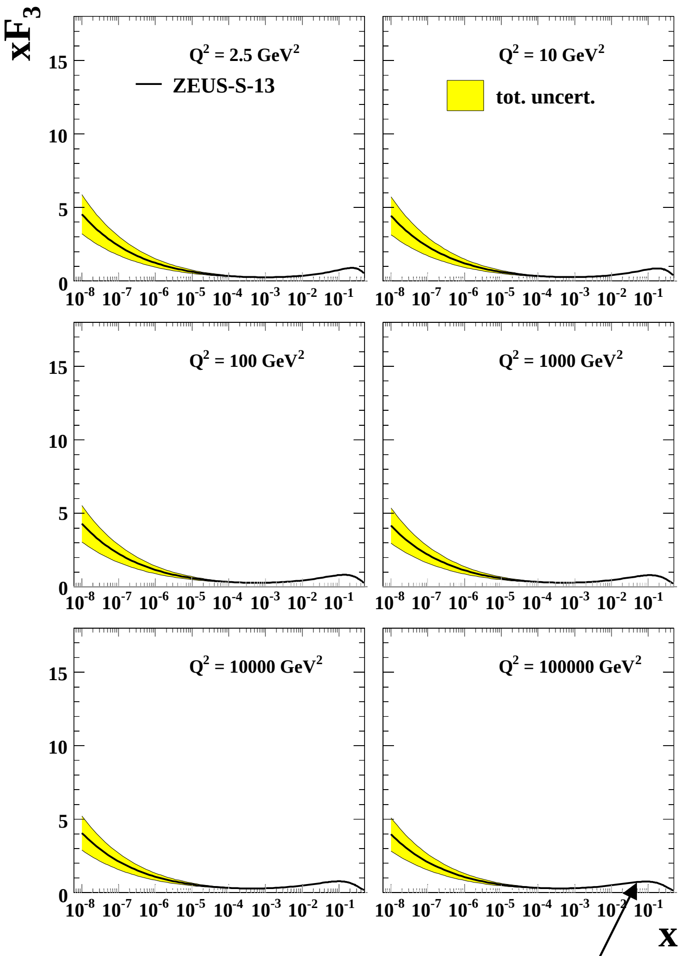
Without b

These plots show FL from an NLO calculation using a zero-mass variable flavour scheme for the heavy quarks, with and without including the b quark



With b

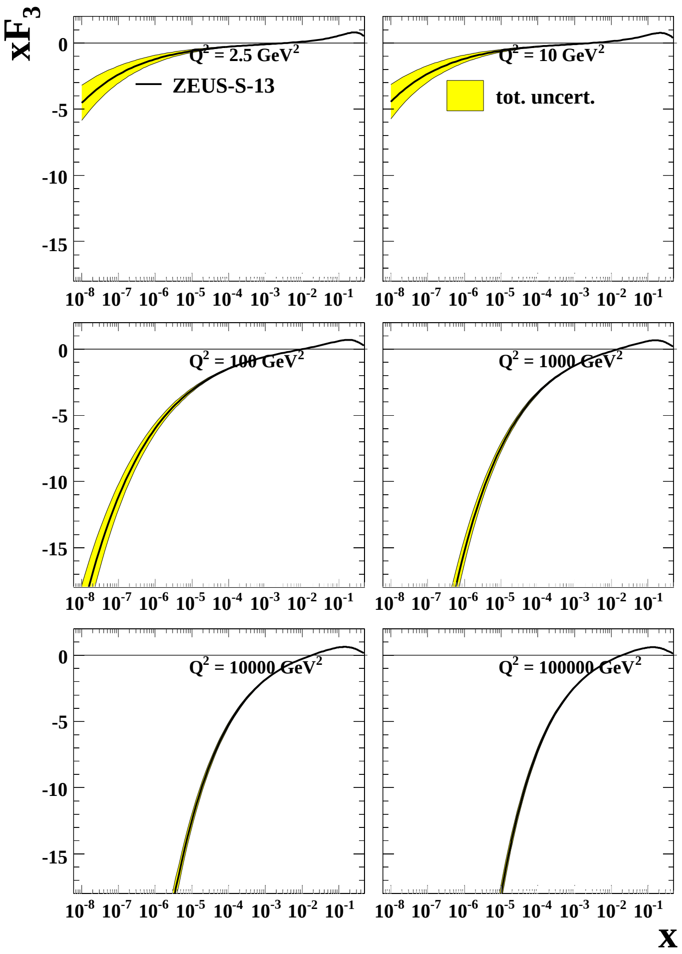
These plots show xF3$^{\nu}$ from an NLO calculation using a zero-mass variable flavour scheme for the heavy quarks, with and without including the b quark



Without b

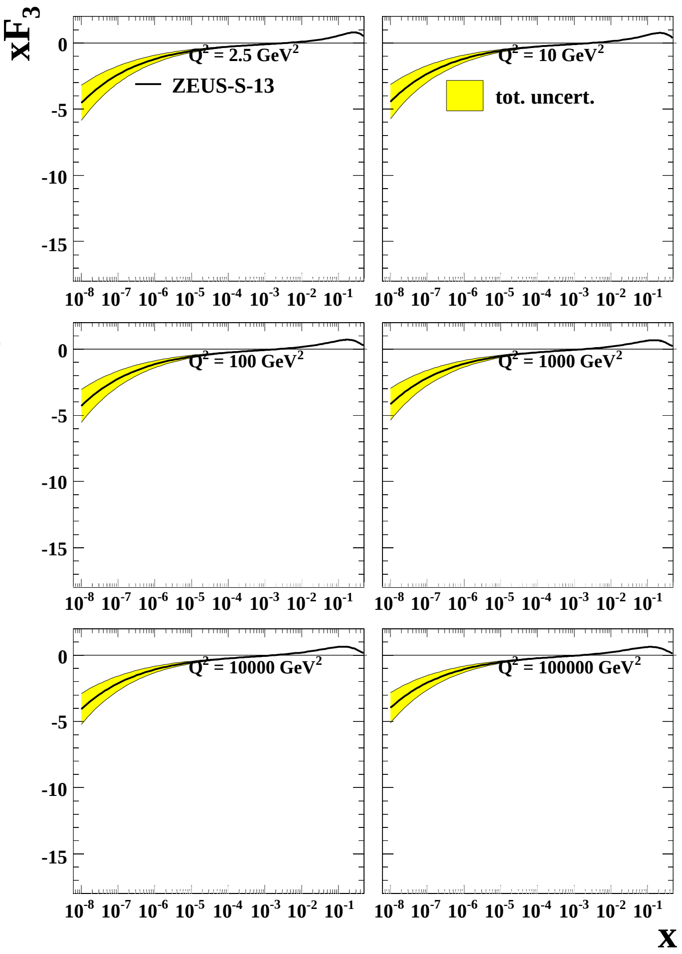
$$xF_3^{\nu} = x\left(u_v + d_v + 2(s - \bar{c}) + 2b\right)$$

The dramatic effect of b on xF3 can be understood from looking at the LO expressions and realising that the valence quarks have a small contribution confined to high-x, whereas at low x the s and cbar parts of the sea are close to cancelling- leaving xF3 as b dominated



With b

These plots show xF3$^{\bar{\nu}}$ from an NLO calculation using a zero-mass variable flavour scheme for the heavy quarks, with and without including the b quark



Without b

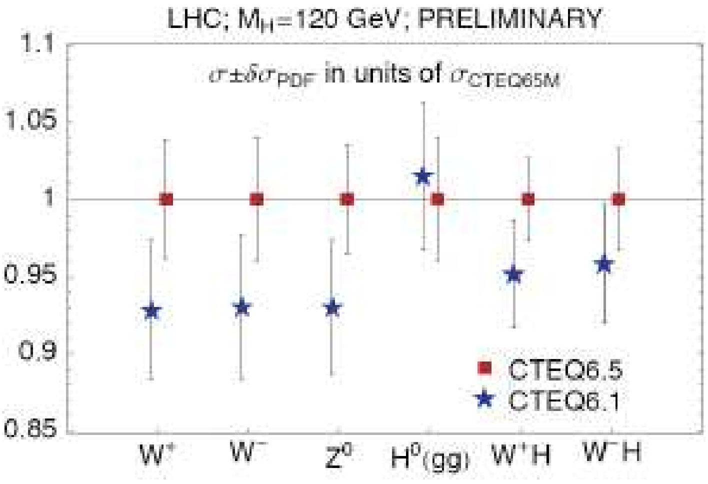
$$xF_3^{\bar{\nu}} = x(u_v + d_v + 2(c - \bar{s}) - 2\bar{b}).$$

There is also a dramatic effect of b on xF3 for antineutrinos such that xF3 at small x is negative and bbar dominated $xF_3^{\nu} \sim -xF_3^{\bar{\nu}}.$

However, ultimately the high energy cross-sections are not very sensitive to the treatment of b

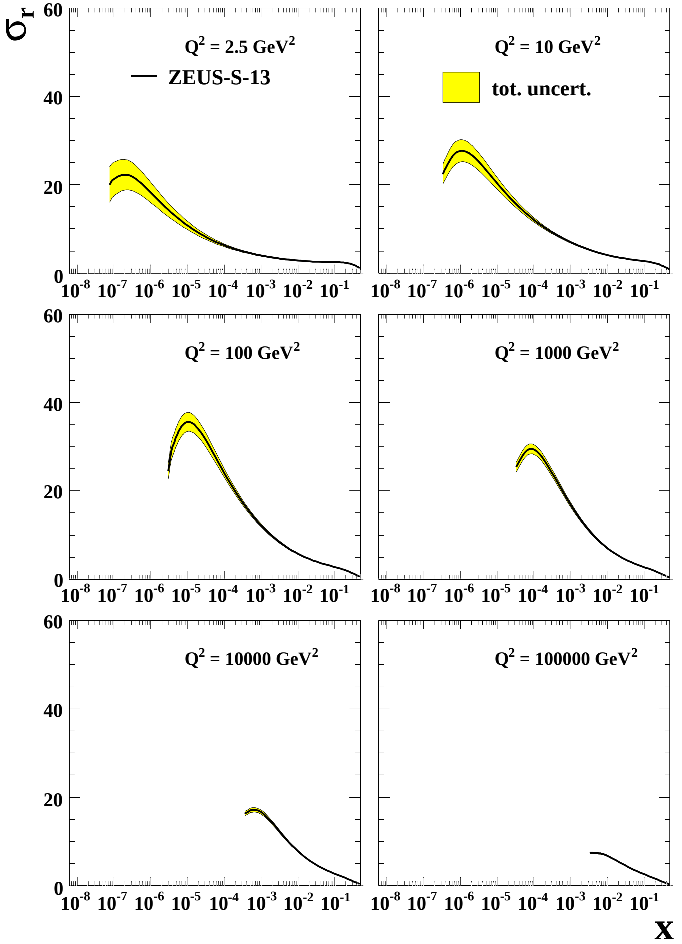
Firstly because xF3 is not a large contribution compared to F2, $xF_3 \lesssim F_2/5$,and it is suppressed by the y dependence such that the b contribution to the reduced cross-section is less than ~25%, even at very high Q^2

More significantly because b is suppressed until $Q^2 > m_{top}^2$~30000, but in the full (non-reduced) cross-section the W propagator suppresses contributions for which $Q^2 >> M_W^2$~6400 GeV^2



However even though the treatment of the b to t threshold is not crucial it is important to evaluate the PDFs using a general mass variable flavour number scheme at low Q2, because this treatment affects the shape of the PDFs at low Q2 and thus by evolution at high Q2, as we have recently seen in the differences between the recent CTEQ6.5 PDF predictions for LHC cross-sections as compared to the previous CTEQ6.1 predictions

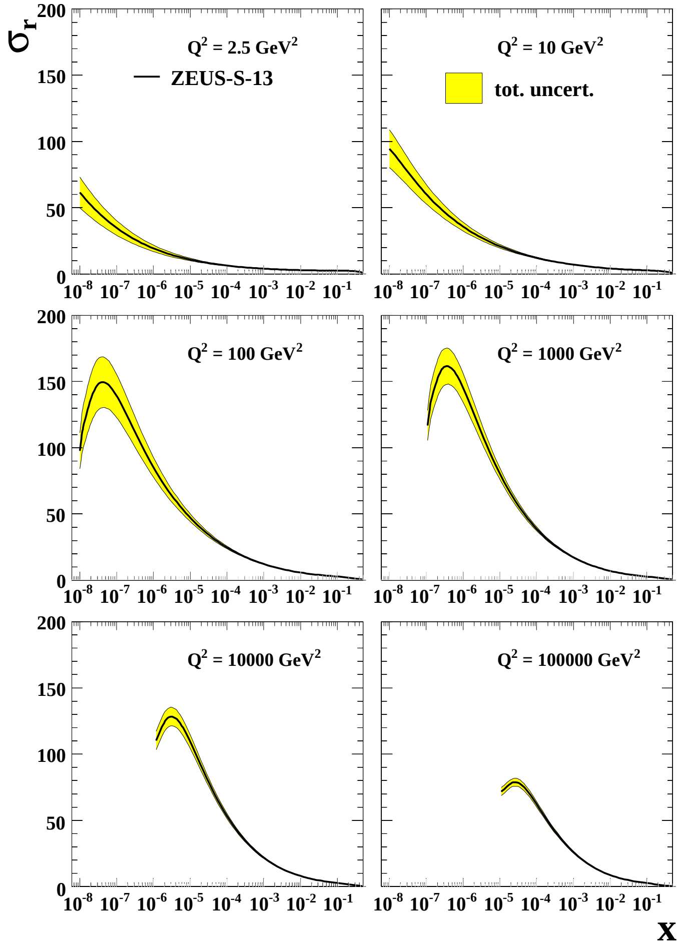
Neutrino reduced cross-section vs x for different Q^2 bins evaluated with general mass variable flavour number scheme



S = 3.6 10^7

Dominant contributions to the reduced cross-section from middling Q^2 and low-x (but perhaps not as low as we might have thought)

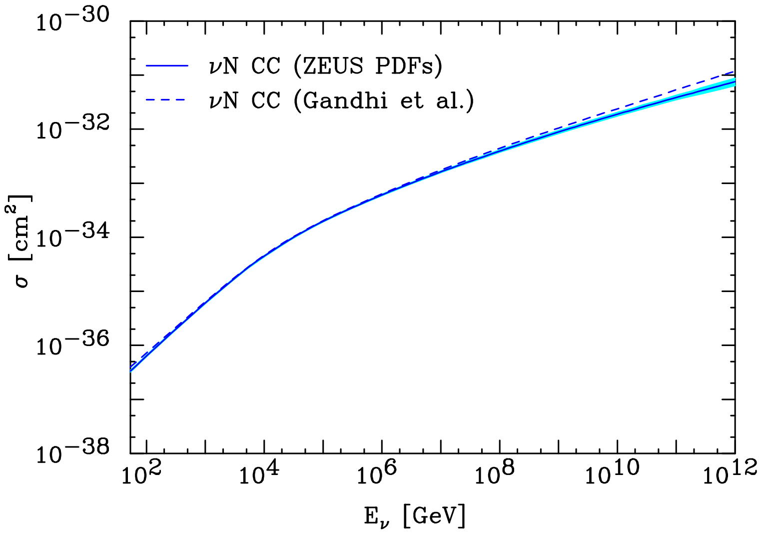
Hence PDF uncertainties on these cross-sections are not so very large

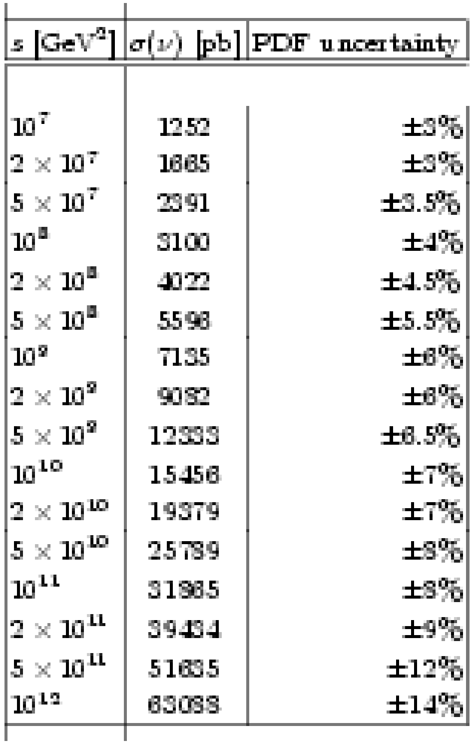


S =10^{10}

Note position of kinematic cut-off x> Q^2/s for y<1. This ensures that very low x is not probed at high Q2, until neutrino energies are very large indeed

The total neutrino cross-section is obtained by integrating





s [GeV^2]	$\sigma(\nu)$ [pb]	PDF uncertainty
10^7	1252	±3%
2×10^7	1665	±3%
5×10^7	2391	±3.5%
10^8	3100	±4%
2×10^8	4022	±4.5%
5×10^8	5598	±5.5%
10^9	7135	±6%
2×10^9	9082	±6%
5×10^9	12333	±6.5%
10^{10}	15456	±7%
2×10^{10}	19379	±7%
5×10^{10}	25789	±8%
10^{11}	31985	±8%
2×10^{11}	39434	±9%
5×10^{11}	51635	±12%
10^{12}	63088	±14%

The PDF uncertainties increase slowly with neutrino energy as lower and lower x is probed

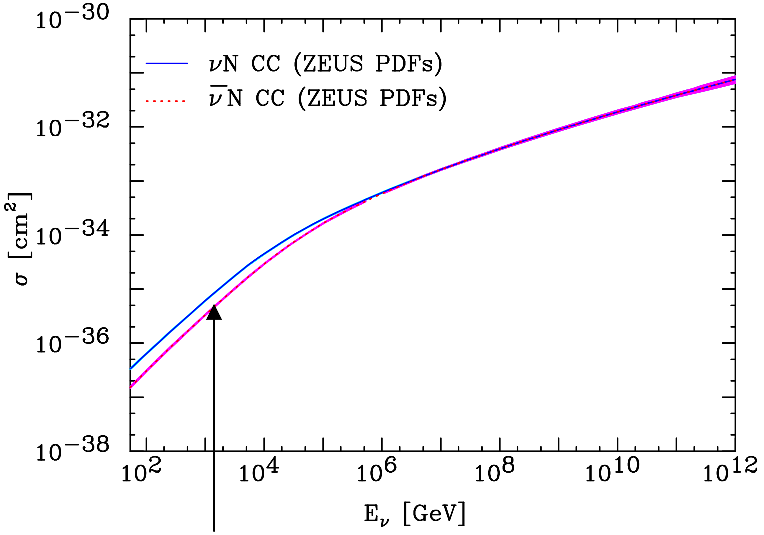
At higher neutrino energies the new estimates of the cross-section differ from those of the 1996 estimate of Gandhi et al reflecting the fact that the more recent HERA data showed a somewhat less steep rise of F2 at small-x.

Thus estimates of these cross-sections from calculations going beyond NLO DGLAP (BFKL, gluon screening, recombination etc) should be compared to these new updated estimates

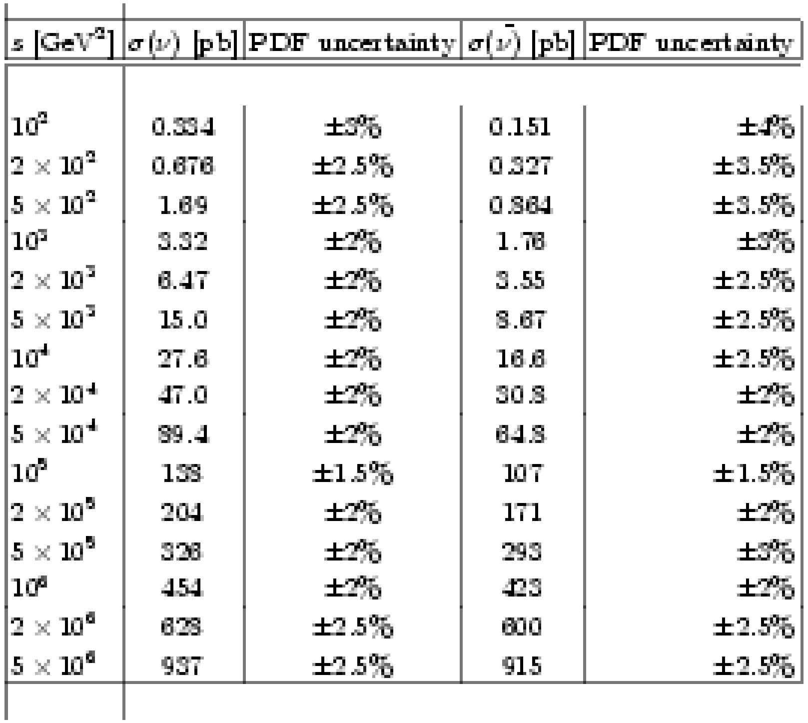
Antineutrino cross-sections are closely similar at high energies because $xF_3^{\nu} \sim -xF_3^{\bar{\nu}}$.

and xF3 contributes with opposite sign in neutrino and antineutrino cross-sections

However there are differences at low energies as we access high-x and the valence quark contribution become important



Low energy regime б ~ E



s [GeV^2]	$\sigma(\nu)$ [pb]	PDF uncertainty	$\sigma(\bar{\nu})$ [pb]	PDF uncertainty
10^2	0.334	±3%	0.151	±4%
2×10^2	0.676	±2.5%	0.327	±3.5%
5×10^2	1.69	±2.5%	0.864	±3.5%
10^3	3.32	±2%	1.78	±3%
2×10^3	6.47	±2%	3.55	±2.5%
5×10^3	15.0	±2%	8.67	±2.5%
10^4	27.6	±2%	16.6	±2.5%
2×10^4	47.0	±2%	30.8	±2%
5×10^4	89.4	±2%	64.8	±2%
10^5	138	±1.5%	107	±1.5%
2×10^5	204	±2%	171	±2%
5×10^5	328	±2%	293	±3%
10^6	454	±2%	423	±2%
2×10^6	628	±2.5%	600	±2.5%
5×10^6	937	±2.5%	915	±2.5%

As neutrino energy decrease the PDF uncertainties decrease since very low-x values are no longer probed.

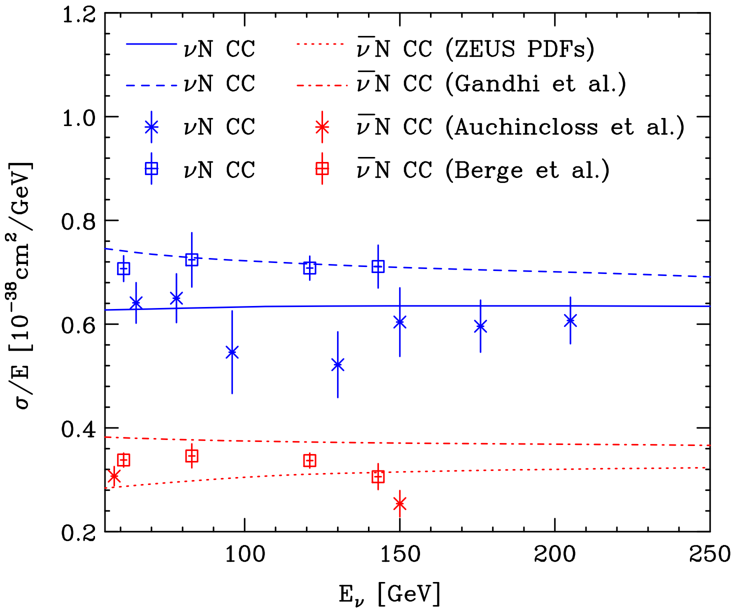
PDF uncertainties are smallest at s~10^5 corresponding to middling x, $10^{-2} < x < 10^{-1}$

PDF uncertainties increase again at lower neutrino energies as we move into the region of large **x**

And just for completeness sake lets show the new and the old predictions at very low energy compared to data

Note the perturbative predictions of the present work cannot be use for Q2 < 1 GeV^2 and hence we are missing a fraction of the lowest energy cross-sections. This is most significant in the smaller antineutrino cross-section. Hence no predictions are given for

s < 100 GeV^2 (Eν < 53.3 GeV)



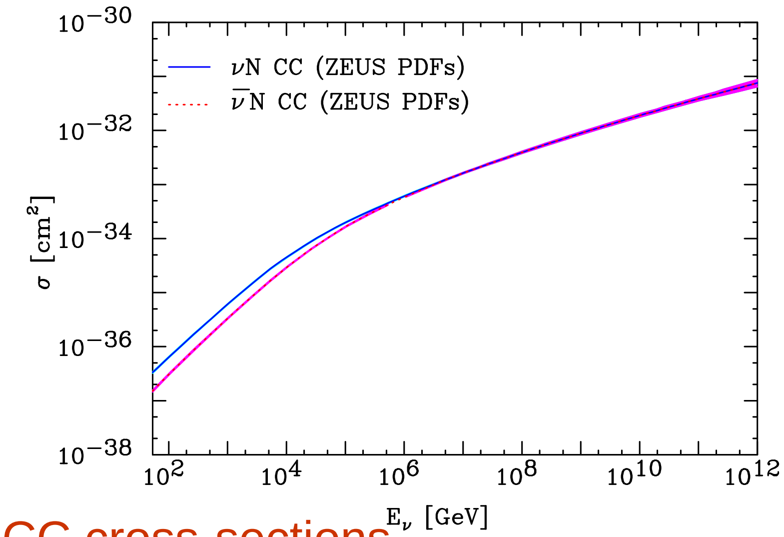
2008: Additional tables of NC cross-sections (zero-mass heavy quark treatment

s	Nu NC(pb)	Nubar NC(pb)
10^2	0.101	0.0533
10^3	1.035	0.611
10^4	8.88	5.86
10^5	50.1	40.4
10^6	184.	173.
10^7	548.	541

s	Nu/Nubar(NC)pb
10^7	547/541
10^8	1433/1430
10^9	3424/3422
10^10	7613/7613
10^11	16004/16004
10^12	32132/32132

Summary

hep-ph: arxiv.0710.5303



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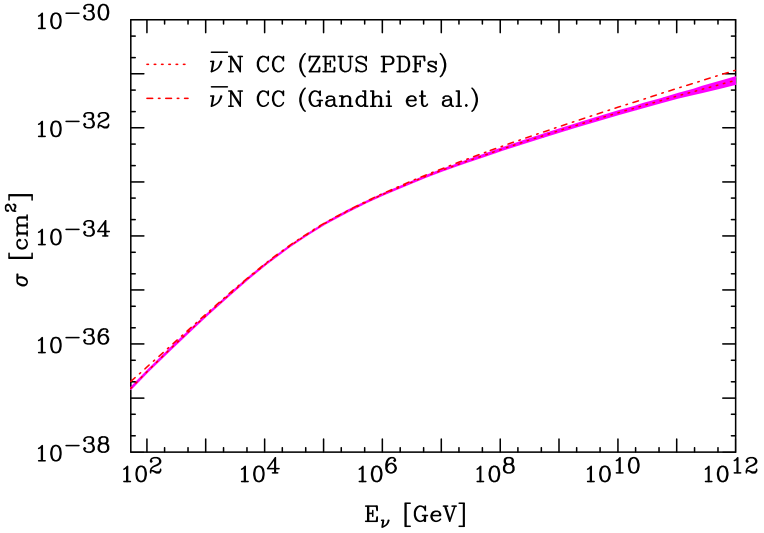
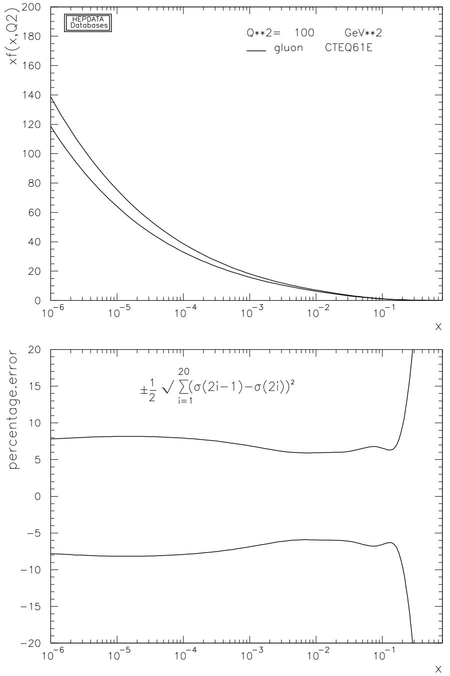
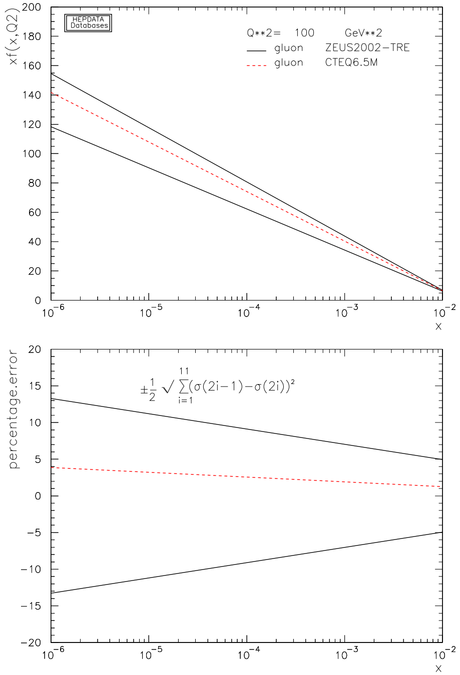
extras



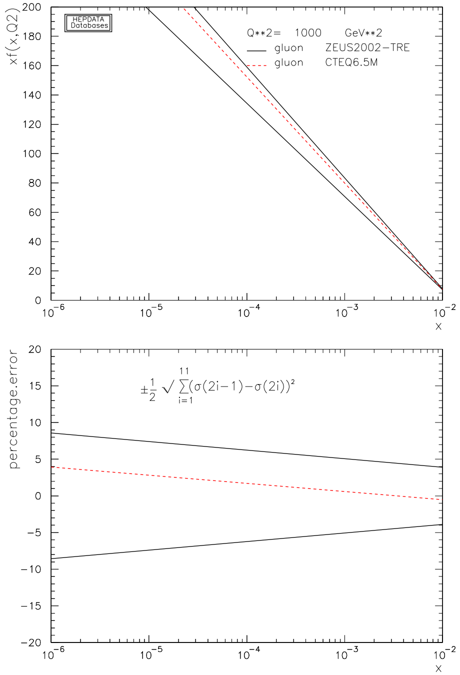
$\overline{\nu}$N CC (ZEUS PDFs)
$\overline{\nu}$N CC (Gandhi et al.)
σ [cm^2]
10^{-30}
10^{-32}
10^{-34}
10^{-36}
10^{-38}
10^2
10^4
10^6
10^8
10^{10}
10^{12}
E$_\nu$ [GeV]



HEPDATA Databases
Q**2= 100 GeV**2
gluon CTEQ61E
xf(x,Q2)
percentage.error
$\pm\frac{1}{2}\sqrt{\sum_{i=1}^{20}(\sigma(2i-1)-\sigma(2i))^2}$
x



HEPDATA Databases
Q**2= 100 GeV**2
gluon ZEUS2002-TRE
gluon CTEQ6.5M
xf(x,Q2)
percentage.error
$\pm\frac{1}{2}\sqrt{\sum_{i=1}^{11}(\sigma(2i-1)-\sigma(2i))^2}$
x



HEPDATA Databases
Q**2= 1000 GeV**2
gluon ZEUS2002-TRE
gluon CTEQ6.5M
xf(x,Q2)
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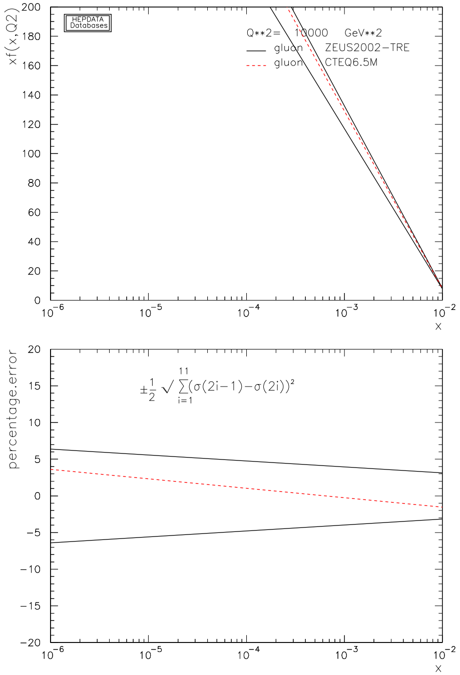
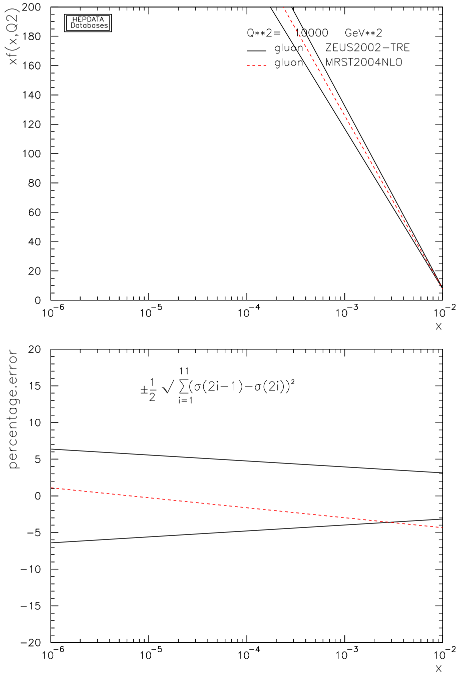
$\pm\frac{1}{2}\sqrt{\sum_{i=1}^{11}(\sigma(2i-1)-\sigma(2i))^2}$
x



HEPDATA Databases
Q**2= 10000 GeV**2
gluon ZEUS2002-TRE
gluon CTEQ6.5M
xf(x,Q2)
percentage.error
$\pm\frac{1}{2}\sqrt{\sum_{i=1}^{11}(\sigma(2i-1)-\sigma(2i))^2}$
x



HEPDATA Databases
Q**2= 10000 GeV**2
gluon ZEUS2002-TRE
gluon MRST2004NLO
xf(x,Q2)
percentage.error
$\pm\frac{1}{2}\sqrt{\sum_{i=1}^{11}(\sigma(2i-1)-\sigma(2i))^2}$
x