Weak Interaction

Part 1
HT 2003

http://www-pnp.physics.ox.ac.uk/~weber/teaching

Introduction

- This lecture will give an introduction to the theory of weak interaction.
- At the end you will know the basics of
  - nuclear decays
  - weak particle decays
  - effects of weak interactions at high energies
  - …
- You already know about the radioactive decays and this will be put into the greater context.

Agenda (part 1)

- Charged current weak interaction
  - W exchange
  - Fermi theory
    (4 particle point-like interaction)
- V-A theory
- Nuclear beta decay
- Parity violation
- Test of V-A theory
  - Neutrino helicity
  - π and K decays
  - W decays
- Unitarity violation at high energies

The Standard Model

- Three generation of quarks and leptons
- interaction via g, γ, Z, $W^{\pm}$
- mass generation via Higgs

0, ½ **υ_e** **0 eV**	**-1, ½** **e** **0.511 MeV**
0, ½ **υ_μ** **0 eV**	**-1, ½** **μ** **106 MeV**
0, ½ **υ_τ** **0 eV**	**-1, ½** **τ** **1.78 GeV**

+2/3, ½ **u** **0.3 GeV**	**-1/3, ½** **d** **0.3 GeV**
+2/3, ½ **c** **1.5 GeV**	**-1/3, ½** **s** **0.5 GeV**
+2/3, ½ **t** **175 GeV**	**-1/3, ½** **b** **4.7 GeV**

g

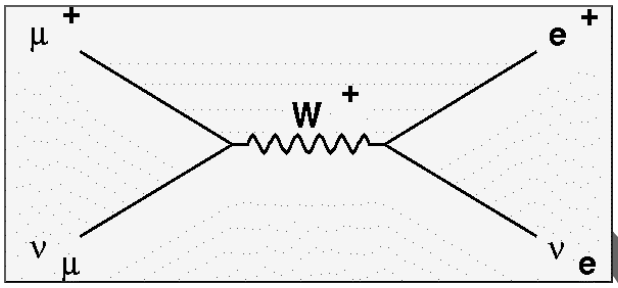
γ

Z or W

H

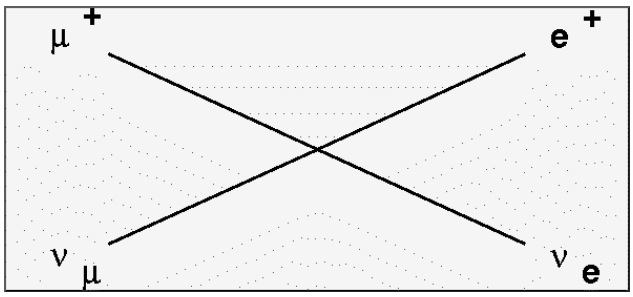
V-A Theory

- Charged Current (CC) weak interaction is due to W exchange



$$\propto \frac{1}{m_W^2 + q^2}$$

- At low energies: 4 point interaction



$$\propto \frac{1}{m_W^2} \propto \frac{G_F}{\sqrt{2}}$$

- current current interaction

$$H_{weak} = G_F \overline{j^\mu} j_\mu \text{ with } j_\mu = \overline{\Psi}\gamma_\mu(1-\gamma_5)\Psi$$

combination of vector (V) and axial-vector (A) current

$$\mathrm{V} = \overline{\Psi}\gamma_\mu\Psi \qquad \mathrm{A} = \overline{\Psi}\gamma_\mu\gamma_5\Psi$$

Non-relativistic limit

- Consider non-relativistic limit of theory, e.g. nuclear beta decay:

$$\Psi = \begin{pmatrix} \phi \\ \frac{\sigma p}{E+M}\varphi \end{pmatrix} \approx \begin{pmatrix} \phi \\ 0 \end{pmatrix}$$

  - V interaction
    - 0 component of nucleon current:

      $\overline{\Psi}_i \gamma_\mu \Psi_f = \Psi_i^+ \gamma_0 \gamma_0 \Psi_f = \phi_i^+ \phi_f$

    - 1,2,3 space components

      $$\overline{\Psi}_i \gamma_\mu \Psi_f = \overline{\Psi}_i \begin{pmatrix} 0 & \sigma_\mu \\ \sigma_\mu & 0 \end{pmatrix} \Psi_f = \begin{pmatrix} \phi_i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ \phi_f \end{pmatrix} = 0$$

    - Fermi transition ($\Delta S=0$)

  - A interaction
    - 0 component

      $\overline{\Psi}_i \gamma_5 \gamma_\mu \Psi_f = 0$

    - 1, 2, 3 space component

      $\overline{\Psi}_i \gamma_5 \gamma_\mu \Psi_f = \phi_i^+ \sigma_\mu \phi_f$

    - Gamow-Teller transition ($\Delta S=0,1$)

Rate of weak nuclear decays

- Fermi's golden rule

$$\lambda = \frac{4\pi^2}{h}\left|M_{fi}\right|^2 \frac{dN}{dE_f}$$

- Assume four point interaction (V):

$$M_{fi} = G_\beta \int \psi_e^*(r)\,\psi_\upsilon^*(r)\,\psi_p^*(r)\,\psi_n(r)\,d^3r$$

- Electrons and neutrinos are free particles leaving the nucleus:

$$\psi_{e,\nu} = L^{-3/2}\exp(i\underline{k}_{e,\nu}\underline{r})$$

$$M_{fi} = \frac{G_\beta}{L^3}\int \exp(i\underline{q}\underline{r})\,\psi_p^*(r)\,\psi_n(r)\,d^3r$$

with $\underline{q} = \underline{k}_e + \underline{k}_\nu$

- Typical beta decay
  - q = 1 MeV and r = 5 fm
  - exp(iqr) = 1
  - electron and neutrino take no orbital momentum away

$$M_{fi} = \frac{G_\beta}{L^3}\int \psi_p^*(r)\,\psi_n(r)\,d^3r$$

- Selection rules (Fermi)
  - we found $\Delta S = 0$ and $\Delta L = 0$
  - therefore: $\Delta J = 0$ and $\Delta P = (-1)^L$
  - allowed Fermi-transition

- Selection rules (Gamow-Teller)

$$M_{fi} = \frac{\lambda G_\beta}{L^3} \int \underline{\sigma} \psi_p^*(r)\, \underline{\sigma} \psi_n(r)\, d^3 r \qquad (A)$$

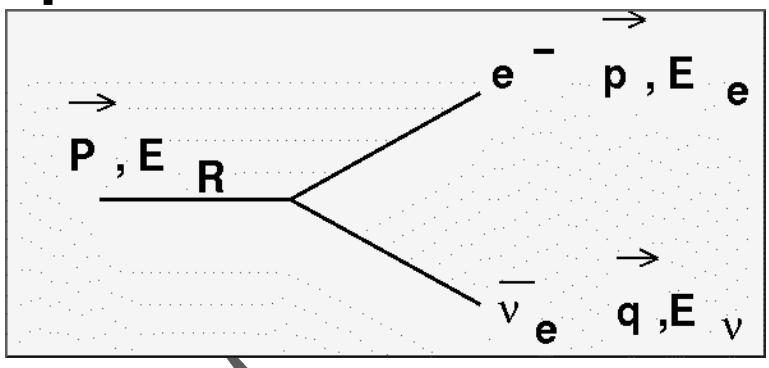
  - λ=1.24 for nuclear beta decay
  - we found $\Delta S = 0, 1$ and $\Delta L = 0$
  - therefore: $\Delta J = 0, 1$ and $\Delta P = 0$
- M_{fi} is a constant for allowed transitions!
- Spectrum depends on phase space only.

Beta Decay Spectrum

$$E_f = E_R + E_\nu + E_e$$

$$0 = \vec{P} + \vec{p} + \vec{q}$$

$$M_R \rangle\rangle m_e \Rightarrow E_f \rangle\rangle E_R \approx 0$$



Now calculate desity of states

$$\rho = \frac{dN}{dE_f}$$

$$dN_e = 4\pi p^2 dpV / h^3$$

$$dN_\nu = 4\pi q^2 dqV / h^3$$

$$d^2N = 4\pi p^2 dpV / h^3 * 4\pi q^2 dqV / h^3$$

Convert to $dpdE_f$

$$q = (E_f - E_e)/c$$

$$dq / dE = 1/c$$

$$dN^2 = \frac{16\pi^2}{h^6 c^3} p^2 (E_f - E_e)^2 dpdE_f$$

Result :

$$\lambda(p)dp = \frac{64\pi^2}{h^7 c^3} \left|M_{if}\right|^2 p^2 (E_f - E_e)^2 F(Z,p) dpdE_f$$

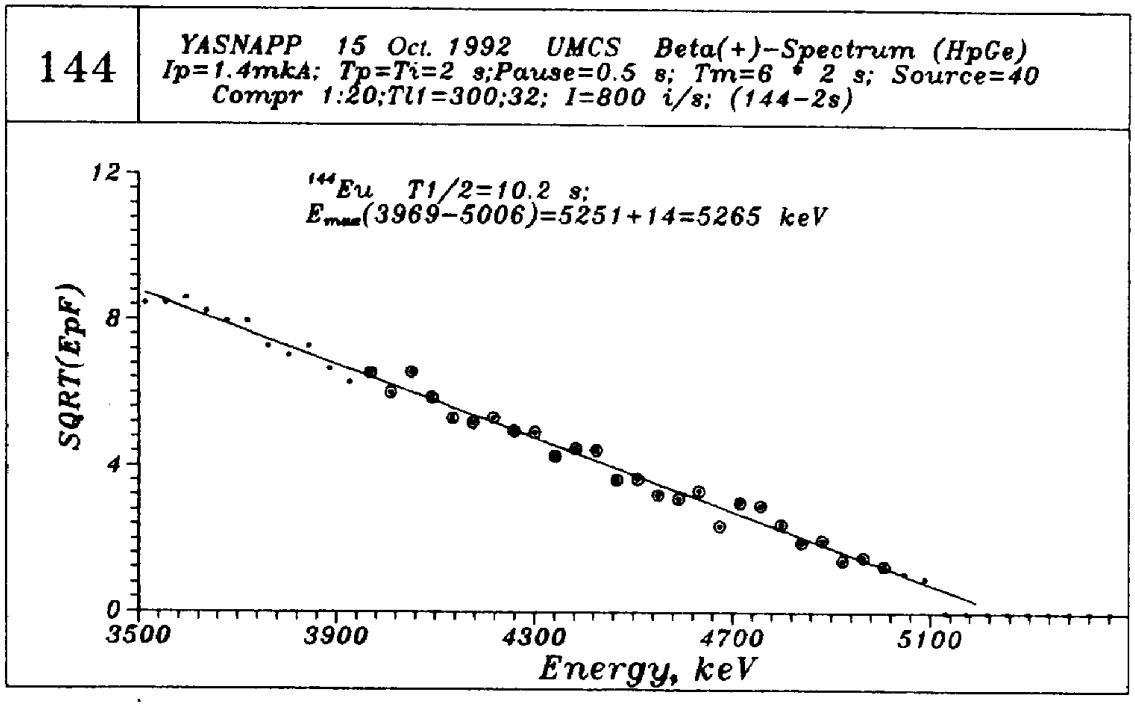
Coulomb correction : Fermi function $F(Z,p)$

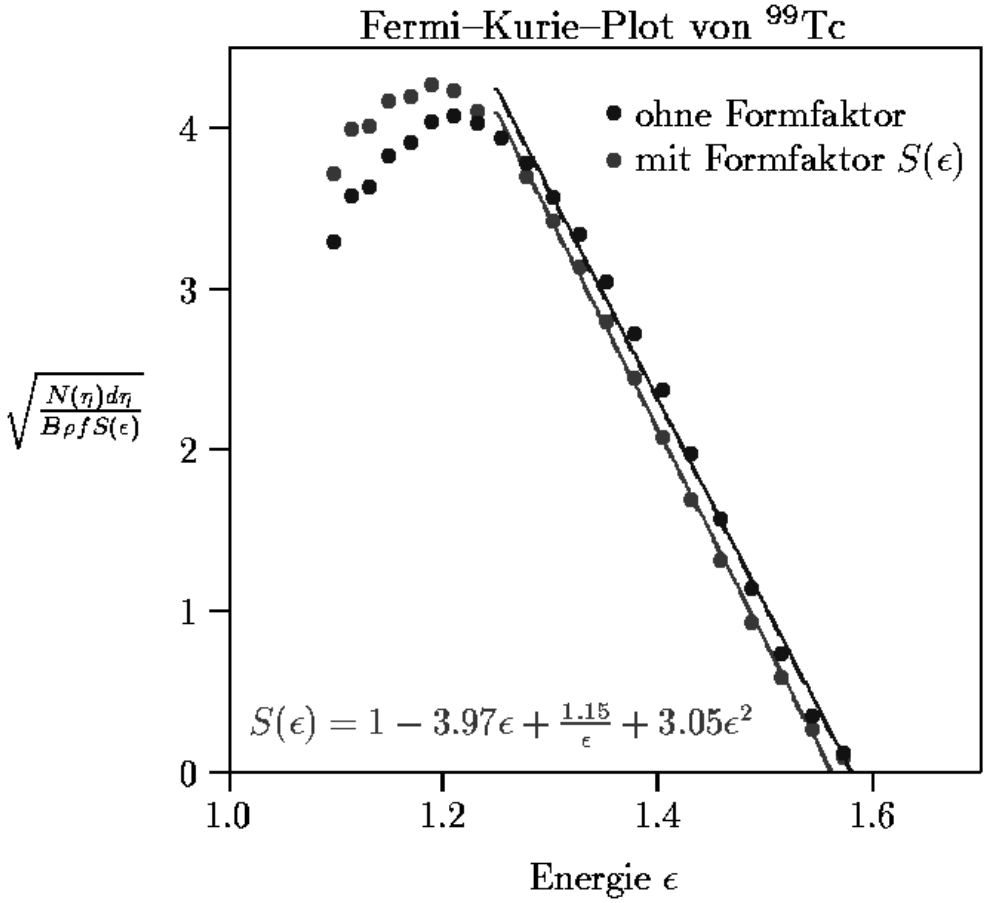
Now define Kurie - function K :

$$K(Z,p) \equiv \sqrt{\frac{\lambda(p)}{p^2 F(Z,p)}}$$

Linear function of E_e

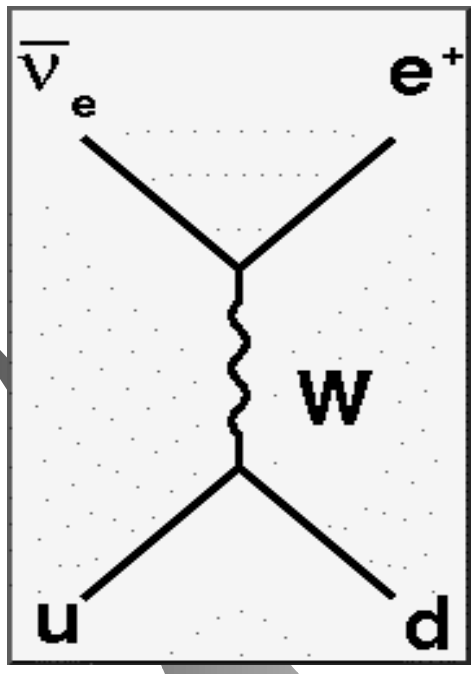
Curie-Plot


144
YASNAPP 15 Oct. 1992 UMCS Beta(+)-Spectrum (HpGe)
Ip=1.4mkA; Tp=Ti=2 s;Pause=0.5 s; Tm=6 * 2 s; Source=40
Compr 1:20;Tli=300;32; I=800 i/s; (144-2s)
144Eu T1/2=10.2 s;
Emax(3969-5006)=5251+14=5265 keV
SQRT(EpF)
12
8
4
0
3500
3900
4300
4700
5100
Energy, keV


Fermi–Kurie–Plot von 99Tc
ohne Formfaktor
mit Formfaktor $S(\epsilon)$
$\sqrt{\frac{N(\eta)d\eta}{Bpf S(\epsilon)}}$
$S(\epsilon) = 1 - 3.97\epsilon + \frac{1.15}{\epsilon} + 3.05\epsilon^2$
4
3
2
1
0
1.0
1.2
1.4
1.6
Energie ϵ

Inverse Beta Decay

- Fermi's Golden Rule:



$$W = \left(\frac{4\pi^2}{h}\right)\left|M_{if}\right|^2 \frac{dN}{dE_f}$$

$$\frac{dN}{dE_f} = \frac{dN}{dp_f}\frac{dp_f}{dE_f}$$

$$= \frac{4\pi p_f^2}{h^3}\frac{dp_f}{dE_f}$$

$$E^2 = p^2c^2 + m^2c^4$$

$$2E = 2pc^2\frac{dp}{dE}$$

$$\frac{dp}{dE} = \frac{E}{pc^2} = \frac{1}{\beta c}$$

$$\frac{dp_f}{dE_f} = \frac{1}{v_f}$$

beam flux $\Phi = \sigma n_a$

targets (unit area) = n_b

interaction rate $= \sigma\Phi = \sigma n_a v_i$

$$W = \frac{4\pi^2}{h}\left|M_{if}\right|^2 \frac{4\pi p_f^2}{h^3}\frac{dp_f}{dE_f} = \sigma v_i$$

$$\sigma = \frac{16\pi^3}{h^4}\left|M_{if}\right|^2 \frac{p_f^2}{v_i}\frac{dp_f}{dE_f}$$

$$= \frac{G_F^2}{\pi}\left|M\right|^2 p^2 \qquad (v_i = v_f \approx c = \frac{h}{2\pi} = 1)$$

- Fermi transitions
  $\Delta J = 0$ fl $M^2=1$
- G-T transitions
  $\Delta J = 1$ fl $M^2=3$ (Why? Spin!)
- Total cross section
  (order of magnitude)

$$\sigma = \frac{4G_F^2}{\pi} p^2$$

- Electron extreme relativistic

$$p \approx \frac{(E-Q)}{c} \approx 1\,\mathrm{MeV}$$

$$\mathrm{G_F} = 10^{-5}\ \mathrm{GeV}^{-2}$$

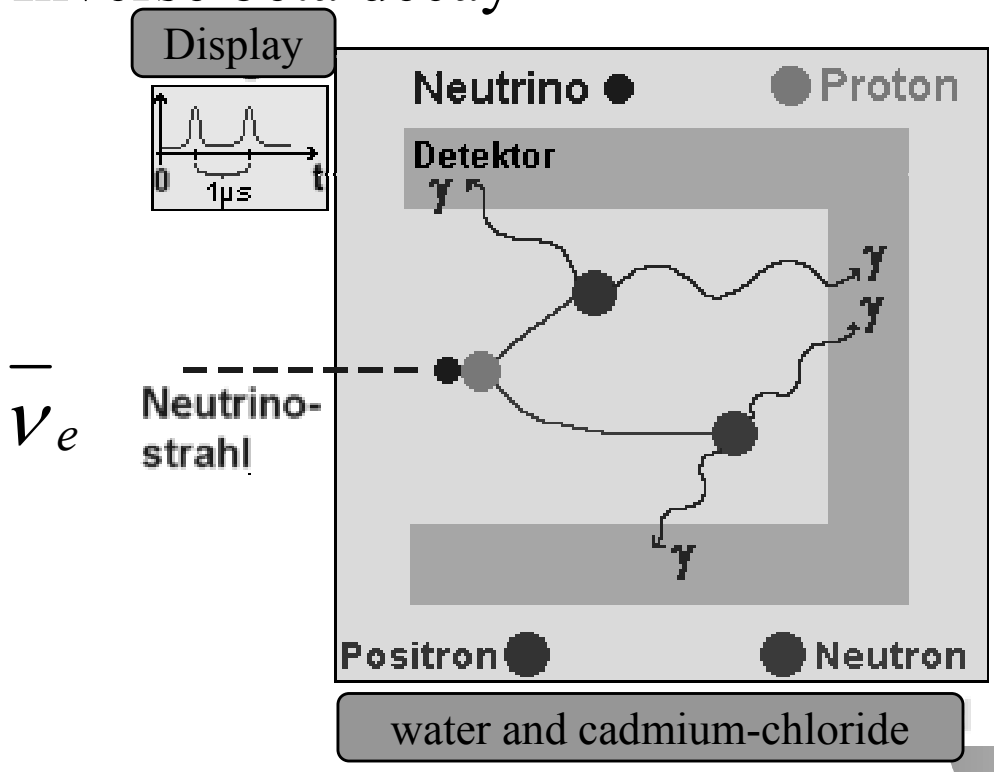
- Total cross section:

$$\sigma = O(4*10^{-18}\ fm^2)$$

(tiny! tiny! tiny! tiny! tiny! tiny!)

Discovery of the Neutrino

- Reines & Cowan (1956)
  - Inverse beta decay



  - Positron annihilation (prompt)

$$e^+ + e^- \rightarrow \gamma + \gamma$$

  - Neutron capture (delayed)
    after neutron became thermal

$$n + Cd \rightarrow Cd^* \rightarrow Cd + n\gamma$$

  - Where do you get anti-neutrinos from?

What have we learned today?

- Standard Model (know before)
- V-A Theory
- Charged current interactions
- Types of nuclear beta decays
  - Fermi
  - Gamow-Teller
- Kinematics of allowed decays
- Inverse beta decay
- Discovery of the neutrino

- Next Lecture:
  Experimental tests of V-A theory
  1. Parity violation
  2. W decay
  3. Pion decay
  4. Helicity of neutrino

Experimental Tests of V-A Theory

- We constructed a V-A theory for charged current weak interaction with build in Parity violation

- Now test the V-A theory

1. Parity violation in nuclear beta decay (Maximum violation! Why?)

2. W decay angular distribution

   $W^+ \to l^+ \nu_l$

3. Pion decays to electron and muons

   $\pi \to \mu\nu$ and $\pi \to e\nu$

4. Helicity of neutrino

Parity Violation in W.I.

- What is parity

$$\hat{P}\,\vec{r} = -\vec{r} \quad \text{i.e.} \quad (x,y,z) \rightarrow (-x,-y,-z)$$

$$\hat{P}\Psi(\vec{r}) = \Psi(-\vec{r})$$

- Eigenvalues

$\hat{P}^2 = \hat{1} \quad \Rightarrow \quad$ Eigenvalues $p = \pm 1$

- Parity conservation:
  - QM tells us:

$$\text{If } \left[H,\hat{P}\right] = 0, \text{ then } \frac{d\left\langle \hat{P} \right\rangle}{dt} = 0$$

  - Therefore
    1. Observed states will have definite parity. Why?
    2. Parity is conserved in interactions

- Examples of operators

$\hat{P}s = s$ (scalar : a number)

$\hat{P}p = -p$ (psuedoscalar : spin * momentum)

$\hat{P}V = -V$ (vector : momentum)

$\hat{P}A = A$ (axial vector : angular momentum = postion * velocity)

Example

- Parity conservation and helicity

$$\hat{H} = \frac{\vec{s}\vec{p}}{|\vec{s}||\vec{p}|}$$

This is the helicity operator!

- If parity is conserved, expectation value of pseudo-scalar = 0

$$\langle p \rangle = 0$$

- Proof:

$$\begin{aligned}
\langle p \rangle &= \langle \Psi | p | \Psi \rangle \\
&= \langle \Psi | p\hat{P}^2 | \Psi \rangle \\
&= -\langle \Psi | \hat{P}p\hat{P} | \Psi \rangle \quad \text{with} \quad p\hat{P} = -\hat{P}p \\
&= -\langle \Psi | p | \Psi \rangle \\
&= -\langle p \rangle
\end{aligned}$$

Structure of Weak Interaction

- Weak interaction is due to vector current V and axial-vector current A:

$$V = \overline{\Psi}\gamma_\mu \Psi \qquad A = \overline{\Psi}\gamma_\mu \gamma_5 \Psi$$

- The interaction is V-A

$$V\text{-}A = \overline{\Psi}\gamma_\mu (1-\gamma_5)\Psi$$

- It is equivalent to say:

Interaction is with left-handed particles only!

- Because:

$$\Psi_{LH} = \frac{1}{2}(1-\gamma_5)\Psi$$

This is a chirality-projector!

Parity and V-A Theory

- W couples to left handed particles! Weyl representation for gamma matrices:

$$\frac{1}{2}(1-\gamma_5)=\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

Projects left handed states!

- Massless limit (or high energies):

$$\Psi_- = N\begin{pmatrix}0\\1\\0\\K\end{pmatrix} \qquad \Psi_+ = N\begin{pmatrix}K\\0\\1\\0\end{pmatrix} \qquad \text{with } K=\frac{m}{E+p}$$

Helicity and chirality are the same!

$$\frac{1}{2}(1-\gamma_5)\Psi_- = \Psi_- \qquad \text{and} \qquad \frac{1}{2}(1-\gamma_5)\Psi_+ = 0$$

- Weak interaction generates net helicity! fl Parity violation!

Parity Violation

- A V-A current current interaction is violating parity:

$$P\ V = -V$$

$$P\ A = A$$

$$(V\text{-}A)(V\text{-}A) = VV + AA - 2AV$$

$$P\ (V\text{-}A)(V\text{-}A) = VV + AA + 2AV$$

- Was originally build into theory but not understood!

- Now is understood as a consequence of W interaction to left handed particles! (Not understood?)

Is parity conserved?

- Yes:
  - Strong interaction
  - Electromagnetic interaction
  - Gravity?
- Everybody expected it to be conserved in weak interaction!
- First hint was the θ-τ puzzle!

$$\tau \to 2\pi \quad l = 0 \quad P = P_\pi^2 = +1$$

$$\theta \to 3\pi \quad l = 0 \quad P = P_\pi^3 = -1$$

- But both particle have same mass and lifetime, i.e. must be the same particle

- Parity is violated !!!!!
  (direct test by Wu!)

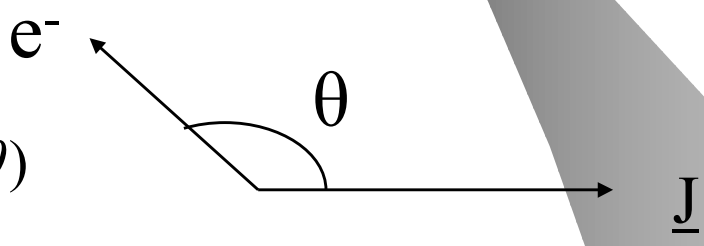
Experimental test of P-violation

- Measure decay spectrum of Cobalt beta decay

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni} + e^{-} + \bar{\nu}_e$$

  - ^{60}Co at T=0.01 K
  - all spins are parallel in external field
- Measure electron angular distribution

$$I(\theta) = 1 - \frac{v}{c}\cos(\theta)$$

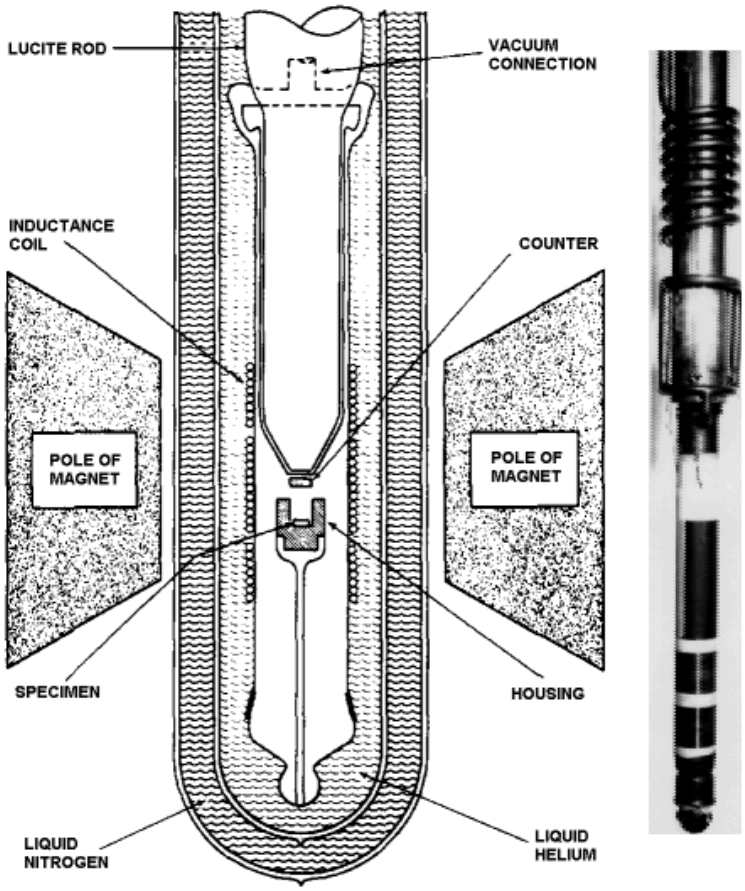


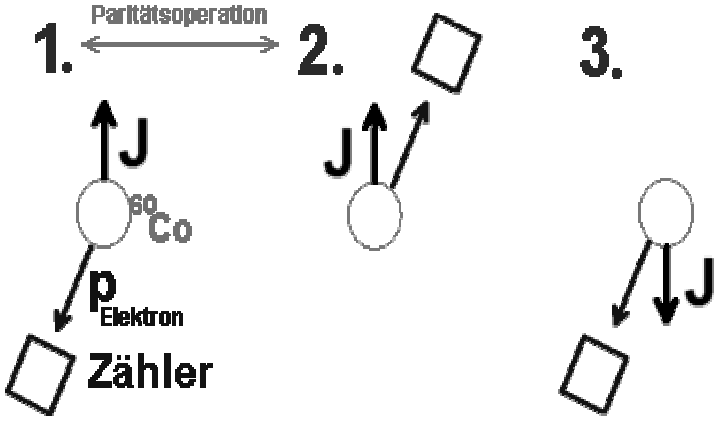
- Now calculate:

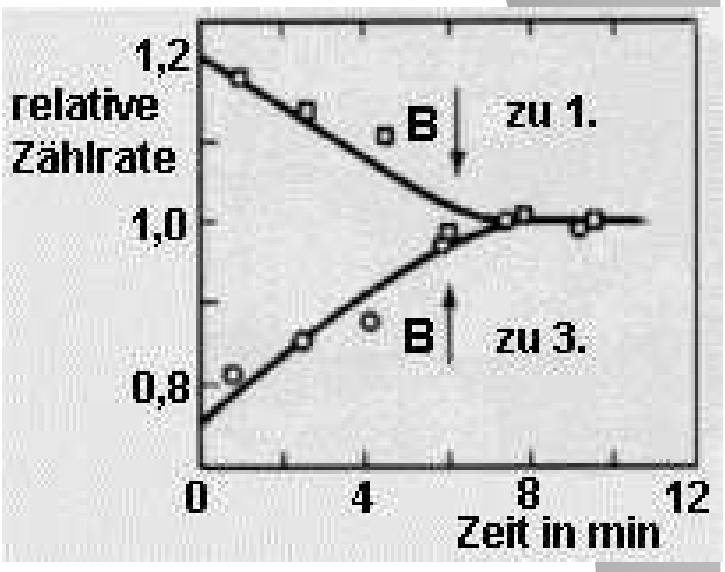
$$\left\langle \vec{\sigma}\vec{p} \right\rangle \propto \left\langle \cos\theta \right\rangle \neq 0$$

But, this is a pseudo-scalar and has to be 0, if parity is conserved!

Wu's experiment







Parity and Nuclear states

- If parity is violated in CC weak interaction, how can we have parity selection rules in nuclear beta decay?
- Initial an final nuclear states are eigenstates of the strong interaction! Eigenstates of parity:

$$\Psi(-\vec{r}) = \eta\Psi(\vec{r})$$

- Consider allowed decays:

$$\begin{aligned} I &= \int \Psi_f^*(\vec{r})\Psi_i(\vec{r})d^3\vec{r} \\ &= \int \Psi_f^*(-\vec{r})\Psi_i(-\vec{r})-d^3\vec{r} \\ &= \eta_f\eta_i\int \Psi_f^*(\vec{r})\Psi_i(\vec{r})d^3\vec{r} \\ &= \eta_f\eta_i I \end{aligned}$$

- I=0, unless $\eta_i = \eta_f = \pm 1$

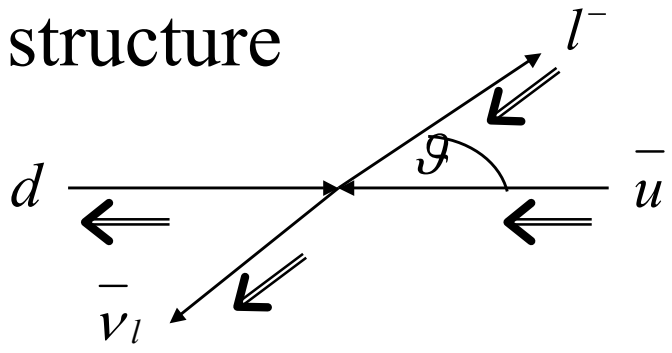
- No change in parity of nuclear wave function!

W Decay

- Charged current weak interaction
  - couples to LH particles
  - couples to RH anti-particles
- Extreme relativistic approach (valid for W decay)
  - LH = helicity minus (-)
  - RH = helicity plus (+)
- W production and decay
  - valence quarks dominate

$$\bar{u} + d \rightarrow W^- \rightarrow l^- \bar{\nu}_l$$

- Spin structure

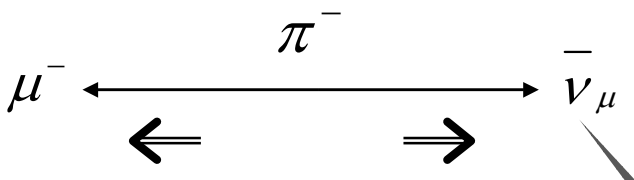


$$A(\vartheta) = d^1_{1,1}(\cos\vartheta) = \frac{1}{2}(1+\cos\vartheta)$$

$$\frac{dN}{d\cos\vartheta} = |A(\vartheta)|^2 = K(1+\cos\vartheta)^2$$

Pion and Kaon Decay

- Angular momentum conservation



- Implications:
  - muon is RH, but CC WI couples to left handed particles
- In relativistic limit
  - left handed = helicity –
  - decay suppressed
- We therefore expect:
  - pion decays mostly to muons and
  - rarely to electrons
- Now:
  Let's calculate the decay rate

Decay Kinematics

- Momentum conservation in CMS

$$p_\pi = p_\mu + p_\nu \quad \text{and} \quad \left|\vec{p}_\mu\right| = \left|-\vec{p}_\nu\right| = p$$

$$m_\pi = \left(p^2 + m_\mu^2\right)^{\frac{1}{2}} + p$$

$$p = \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \quad \text{and similar} \quad E_\mu = \frac{m_\pi^2 + m_\mu^2}{2m_\pi}$$

- Relativistic calculation of Lorenz invariant phase space (Lips)

$$dLips = \frac{Vd^3\vec{p}_1}{(2\pi)^3 2E_1}\frac{Vd^3\vec{p}_2}{(2\pi)^3 2E_2}(2\pi)^4\delta^4(p_0 - p_1 - p_2)$$

$$= \frac{V}{8\pi^2}\frac{\left|\vec{p}\right|}{m_0}$$

Decay Dynamics

- W couples to left handed particles, but we have a helicity (+) lepton. Remember:

$$\Psi_+ = N\begin{pmatrix} K \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \text{with} \quad K = \frac{m}{E+p}$$

Lorenz invariant normalisation

$$\Psi^+\Psi = 2E$$

$$N^2(1+K^2) = 2E$$

$$N_l^2 = \frac{2E_l}{1+K^2}$$

$$N_\nu^2 = 2E_\nu$$

- Use Weyl representation

$$\frac{1}{2}(1-\gamma_5) = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$ to project left handed states

- LH state

$$\Psi_{LH} = \frac{1}{2}(1-\gamma_5)\Psi_+ = N\begin{pmatrix} K \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

- Matrix element:

$$M = \Psi^{\nu}_{RH}\Psi^{l}_{LH}$$

$$|M|^2 = N_l^2 N_\nu^2 K^2 \qquad \text{with} \qquad K = \frac{m_l}{E_l + p_l}$$

$$= 4E_\nu E_l \frac{K^2}{1+K^2}$$

$$= Cp_l m_l^2$$

- Decay rate

$$\Gamma = |M|^2 \, Lips = Cp_l^2 m_l^2$$

- Decay ratios

$$R = \frac{\Gamma(\pi \to e\nu)}{\Gamma(\pi \to \mu\nu)} = \frac{m_e^2}{m_\mu^2}\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}$$

$$= 1.283 * 10^{-4} \qquad \text{(theory)}$$

$$= (1.230 \pm 0.004) * 10^{-4} \qquad \text{(experiment)}$$

(similar for K decays)

- Striking evidence for V-A form of CC weak interaction

Helicity of the Neutrino

- Can we measure the helicity of the neutrino?
- Consider the following decay:

$$^{152}\mathrm{Eu} \xrightarrow{e^-} {}^{152}\mathrm{Sm}^* + \nu_e \rightarrow {}^{152}\mathrm{Sm} + \gamma(960\ \mathrm{KeV})$$

J=0 1 1/2 0 1

- Conservation of angular momentum
  - Neutrino spin is opposite to direction of J in $^{152}Sm^*$
  - Spin of γ is parallel to J

$$^{152}\mathrm{Sm}^* \rightarrow {}^{152}\mathrm{Sm} + \gamma$$

J= 1 0 1

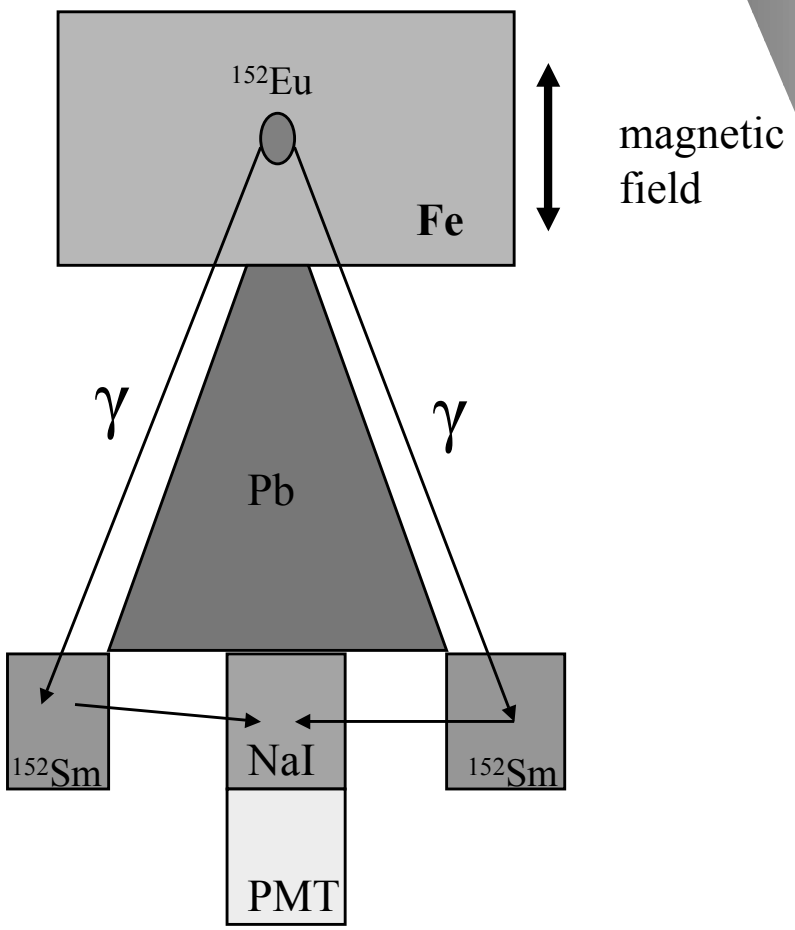
- Therefore:
  - γ emitted forward has same polarisation as $^{152}Sm^*$
  - γ emitted forward has same helicity as ν_e
- Forward γ measures neutrino helicity

Neutrino Helicity (exp.)

- Goldhaber et al.
- Tricky bit: identify forward γ
- Use resonant scattering!

$$\gamma + {}^{152}\mathrm{Sm} \rightarrow {}^{152}\mathrm{Sm}^{*} \rightarrow {}^{152}\mathrm{Sm} + \gamma$$

- Measure γ polarisation with different B-field orientations

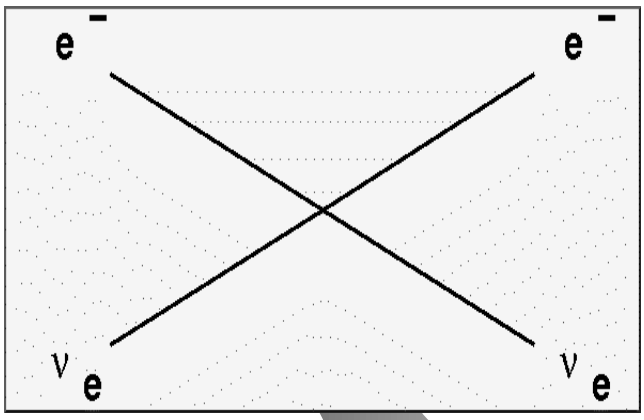


Problems at High Energy

- Fermi theory is base on 4 point contact interaction.
- Consider:

$$\nu_e + e^- \rightarrow \nu_e + e^-$$

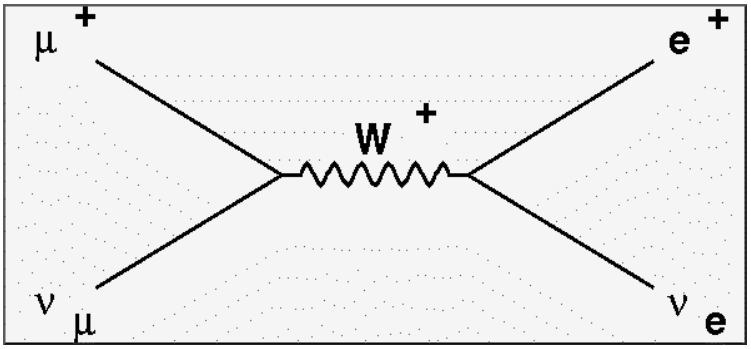
$$\sigma_{tot} = \frac{G_F^2 s}{\pi}$$



- Unitarity limit:
  scattering probability > 1

$$\sigma_{u.l.} = \frac{\lambda^2}{8\pi} \quad \text{with} \quad \lambda = \frac{h}{p_{cms}}$$

- At p=300 GeV CC WI violates the unitarity limit!
- Solution: The W-Boson



$$\propto \frac{1}{m_W^2 + q^2}$$

Summary (Part 1)

- We constructed a V-A theory for charged current weak interaction with build in Parity violation

- Different applications:

1. Nuclear Beta decay
2. Parity violation in nuclear beta decay
3. W decay angular distribution
4. Pion decays to electron and muons
5. Helicity of Neutrino
6. Unitarity violation at high energies

Weak Interaction (3 Families)

Part 2
HT 2003

Content

- So fare we have only considered weak interaction involving u and d quarks and electrons and neutrinos.

- Now we will learn about:

1. 3 generation of leptons
2. Universal coupling strength
3. LEP data & number of generations. Why are there 3 generations???
4. The s quark and Cabibbo's theory
5. FCNC and the need for the c quark
6. b and t quark
7. Generalised theory of quark mixing

Leptons

- Muon is heavier version of electron
  - $m_e = 0.511$ KeV
  - $m_\mu = 106$ MeV
- Rabbi's unanswered question: Who ordered the muon?
- Experimental facts:
  - Not seen: $\mu \to e\gamma$
  - Normal decay: $\mu \to e\nu_\mu\nu_e$
  - electron neutrino ≠ muon neutrino
  - neutrino ≠ anti-neutrino
- One more lepton neutrino pair was discovered (SLAC)

$$e^+ + e^- \to \tau^+ + \tau^-$$

$$\tau \to e\nu_\tau\nu_e \quad \text{or} \quad \mu\nu_\tau\nu_\mu$$

  Signature: electron and muon in one event
- Tau neutrino discovered in 2001!

Lepton Universality

- Leptons are all the same, just heavier and unstable!
- Experimental test:
  - Measure W boson decay ratios!

    $Br(W \to e\nu_e) = Br(W \to \mu\nu_\mu) = Br(W \to \tau\nu_\tau)$

    Experimental data (Jan 2002)

    $$Br(W \to e\nu_e) = 10.72 \pm 0.16\%$$

    $$Br(W \to \mu\nu_\mu) = 10.57 \pm 0.22\%$$

    $$Br(W \to \tau\nu_\tau) = 10.74 \pm 0.27\%$$

  - Measure tau decay ratio!

    $Br(\tau \to e\nu_e\nu_\tau) = Br(\tau \to \mu\nu_\mu\nu_\tau)$

    Experimental data

    $$Br(\tau \to e\nu_e\nu_\tau) = 17.84 \pm 0.06\%$$

    $$Br(\tau \to \mu\nu_\mu\nu_\tau) = 17.37 \pm 0.06\%$$

  - Compare

    $\tau \to e\nu_\tau\nu_e$ and $\mu \to e\nu_\mu\nu_e$

Leptonic lepton decay

- Decay of tau/muon into electron + neutrinos is 3 body decay (like nuclear beta decay)

$$\frac{d\Gamma}{dp} = Cp^2(E_0 - E)^2$$

- Extreme relativistic approx.: p=E

$$\Gamma = \int dp \frac{d\Gamma}{dp} = \int_0^{E_0} dp\ Cp^2(E_0 - p)^2 = \frac{C}{30} E_0^5$$

$$\Gamma(\tau \to e\nu\nu) = \left(\frac{m_\tau}{m_\mu}\right)^5 \Gamma(\mu \to e\nu\nu)$$

$$Br(\tau \to e\nu\nu) = \frac{\Gamma(\tau \to e\nu\nu)}{\Gamma_{total}}$$

- Requires precise determination of Tau mass!!! (Threshold scan at BES)
- Results:

$$\tau_\tau = 291.3 \text{ fs} \quad \text{(predicted)}$$

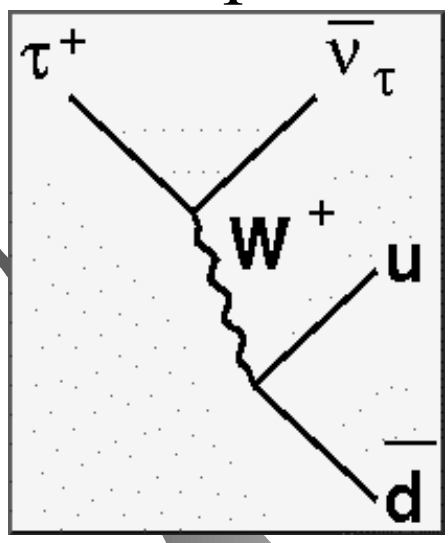
$$\tau_\tau = 290.6 \text{ fs} \quad \text{(measured)}$$

Hadronic Tau Decay

- If CC WI is universal, can we predict hadronic decay ratio?

$$\tau \to \nu_\tau + \text{ hadrons}$$

$$(\pi, \rho, \eta, a, ...)$$



- Count number of final states:

$$\Gamma(\tau \to \nu_\tau + \text{hadrons}) = N_c \times \Gamma(\tau \to e\nu_e\nu_\tau)$$

$$Br(\tau \to \nu_\tau + \text{hadrons}) = \frac{N_C}{N_C + 2}$$

$$= 0.60 \quad \text{(theory)}$$

$$= 0.64 \quad \text{(experiment)}$$

- Question:
  Why is there a 4% difference?
- Answer:
  QCD radiative correction!
  (Can be used to measure $\alpha_s(m_\tau)$

Neutrinos and Lepton Number

- Questions:
  - Are muon neutrinos and electron neutrino the same?
  - Are neutrino and anti-neutrino the same?
- Facts:
  - In SM

    $\nu_e + n \rightarrow p + e^-$ (possible)

    $\bar{\nu}_e + p \rightarrow n + e^+$ (possible)

    $\bar{\nu}_e + n \rightarrow p + e^-$ (NOT possible)
  - Experimental search

    $\bar{\nu}_e + {}^{37}\mathrm{Cl} \rightarrow {}^{37}\mathrm{Ar} + e^-$

    radioactive Argon isotope was not seen!

  - Neutrinoless double beta decay

    ${}^{76}_{32}\mathrm{Ge} \rightarrow {}^{76}_{34}\mathrm{Se} + 2e^- + (2\bar{\nu})$

Neutrinos and Lepton Number

- Neutrinoless double beta decay

$$^{76}_{32}\mathrm{Ge} \rightarrow {}^{76}_{34}\mathrm{Se} + 2e^{-}$$

  - Only possible, if neutrinos have a Majorana component

$$\nu_e = \overline{\nu_e}$$

  - The 2 anti-neutrinos could annihilate!

- Question:
  - How can we distinguish SM and exotic reaction?

- No evidence for Majorana neutrinos yet!

- First evidence for lepton number violation comes from neutrino oscillations (later in course)!

Lepton Number Conservation

Q	L_e=1	L_μ =1	L_τ =1
0	ν_e	ν_μ	ν_τ
1	e-	μ-	τ-

- Anti-particles have opposite lepton numbers!
- Example: $\mu^- \rightarrow e^- + \nu_\mu + \overline{\nu_e}$

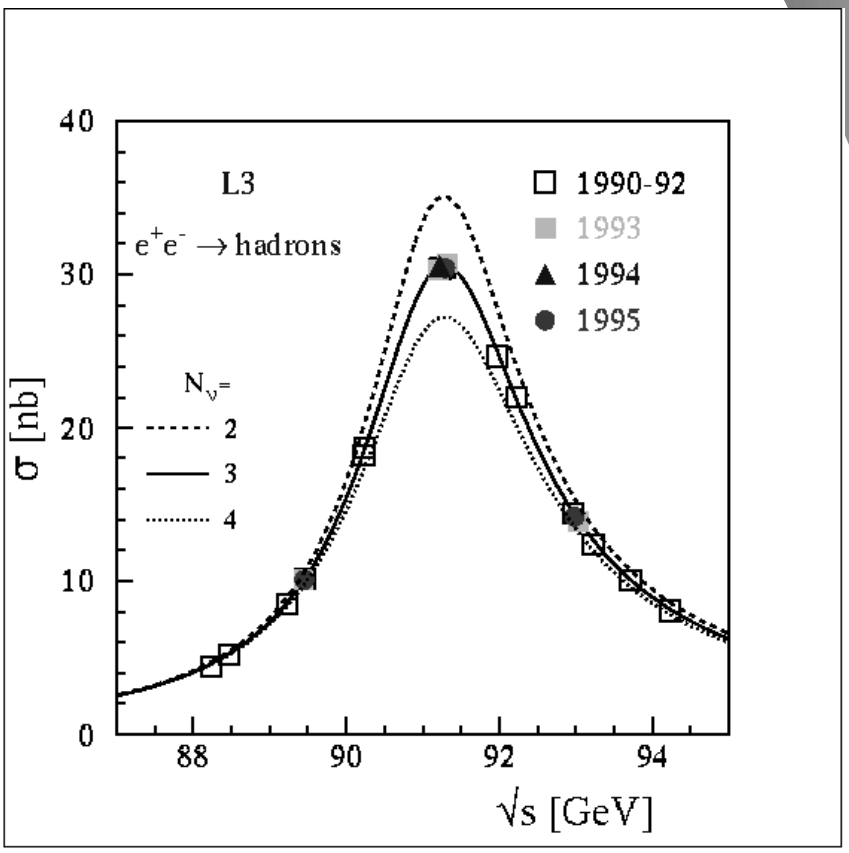
L_μ =	1	0	1	0
L_e =	0	1	0	-1
L =	1	1	1	-1

- Universal strength for all CC WI vertices.
- All vertex factors g for the lνW vertex are the same!

Number of Families

- Are there any more generations of particles? Maybe just too heavy to be produced at colliders yet?
- Neutrino is always light = massless!
- Look for neutrinos!
- Studies at LEP:

$$e^+ + e^- \rightarrow Z \rightarrow f + \overline{f} \qquad f = (e^-, \nu_e, u, d, \mu^-, ...)$$



- There are only 3 generations!
  $N_\nu = 2.9841 \pm 0.0083$

Summary

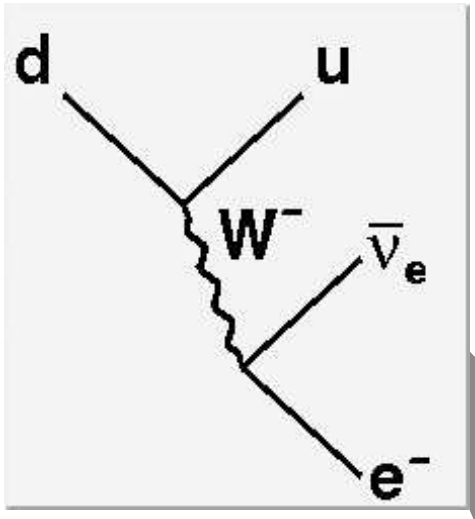
- There are three generations of fermions.
- They have a universal coupling strength to the W
  - W boson decay ratio
  - Tau lepton decay ratios
  - Tau/muon relative lifetime
- Lepton number is a conserved quantum number.
  Why?
- Neutrinos and anti-neutrinos are different.

- **Last Lecture**:
  - WI and quarks
  - Cabibbo's theory
  - FCNC
  - CKM matrix

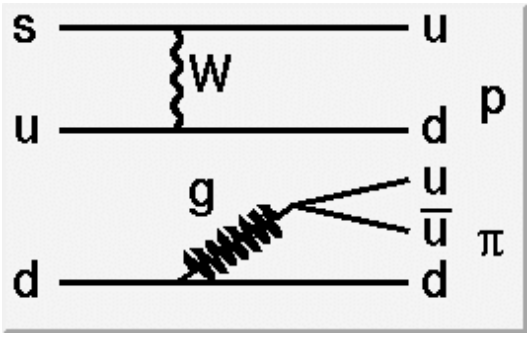
Weak Interaction and Quarks

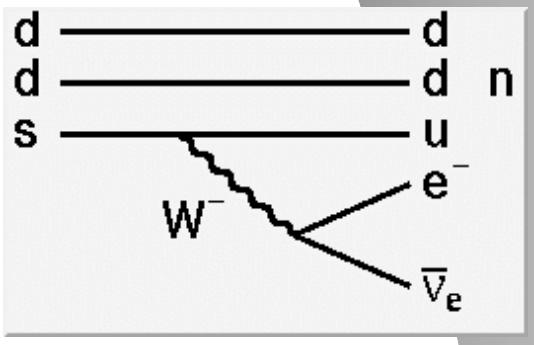
- Compare interaction strength of non-strange and strange decays:

1. Beta decay



2. Strange quark decays





in quark model s becomes u quark explains selection rules:

- $\Delta Q = \Delta s$
- $\Delta I = 1/2$

Cabibbo Theory

- Measure strength of weak interaction for different processes:

$$\mu^- \to \nu_\mu + e^- + \bar{\nu}_e \Rightarrow G_F$$

$$^{14}\mathrm{O} \to {}^{14}\mathrm{N} + e^+ + \nu_e \Rightarrow G_{ud} = 0.95 G_F$$

$$K^- \to \pi^0 + e^- + \bar{\nu}_e \Rightarrow G_{us} = 0.05 G_F$$

- Cabibbo theory:
  - quark mass eigenstates are eigenstates of strong interaction but **NOT** of weak interaction
  - CC WI couple with universal strength to rotated quark states.

$$u = u$$

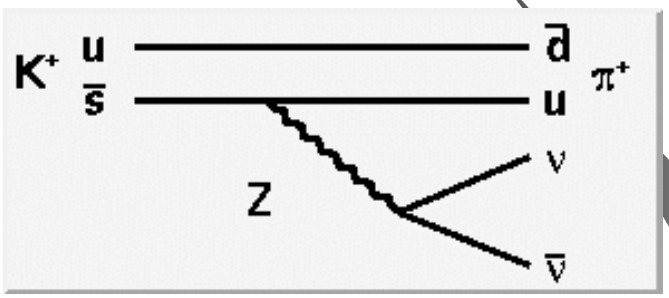
$$d = d\cos\theta_c + s\sin\theta_c$$

  - Ratio of $G_{us}/G_F = \sin^2\theta_c$
  - Fit to many different reactions

$$\theta_c \approx 0.25$$

Flavour Changing Neutral Currents

- Why don't we see FCNC?
- Naively one would expect to see FCNC, if NC couples to uu or $d_c d_c$!



$$d_c = d\cos\theta_c + s\sin\theta_c$$

$$\begin{aligned} J^0_{NC} &= u\bar{u} + d_c\bar{d}_c + s_c\bar{s}_c \\ &= u\bar{u} + d\bar{d}\cos^2\theta_c + s\bar{s}\sin^2\theta_c \\ &\quad +(s\bar{d} + \bar{s}d)\cos\theta_c\sin\theta_c \end{aligned}$$

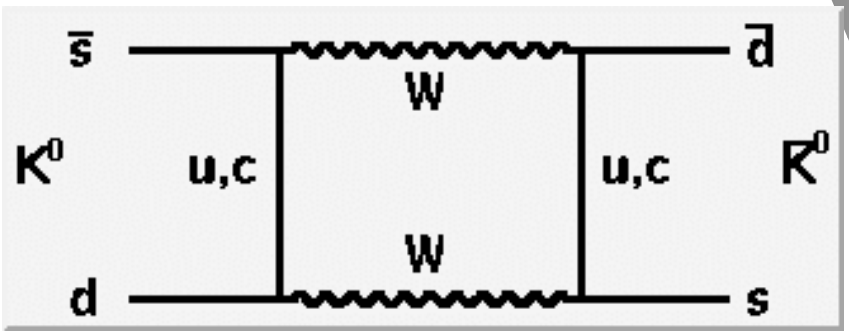
- GIM mechanism kills unwanted FCNC (1970), but one has to introduce a new quark doublet:

$$\begin{pmatrix} u \\ d_c \end{pmatrix}, \begin{pmatrix} c \\ s_c \end{pmatrix} \qquad \begin{aligned} d_c &= d\cos\theta_c + s\sin\theta_c \\ s_c &= -d\sin\theta_c + s\cos\theta_c \end{aligned}$$

FCNC

$$J^0_{NC} = u\bar{u} + d_c\bar{d}_c + s_c\bar{s}_c + c\bar{c}$$
$$= u\bar{u} + c\bar{c} + (d\bar{d} + s\bar{s})(\cos^2\theta_c + \sin^2\theta_c)$$
$$+(s\bar{d} + \bar{s}d - s\bar{d} - \bar{s}d)\cos\theta_c \sin\theta_c$$
$$= u\bar{u} + c\bar{c} + d\bar{d} + s\bar{s}$$

- Δs=0:
  No FCNC for lowest order weak interactions, but possible as higher order corrections!



  vanishes, if $m_u = m_c$
- Measured rate of transition allowed prediction of m_c!
- Discovery of the J/ψ in 1974 was triumph for quark model and GIM!

GIM

- Other consequences
  - In charm quark decays:
    c→s and c→d are possible, because
    $\tan^2 \theta_c \approx 0.05$
    Find Kaons in decay of charmed particles!
  - Charm production in neutrino beams

1.) $\nu_\mu + d \rightarrow \mu^- + u$

2.) $\nu_\mu + d \rightarrow \mu^- + c$ followed by $c \rightarrow \mu^+ + s + \nu_\mu$

Signature for 2.) is a muon pair!

$$\sigma_1 \propto \cos^2 \theta_c$$

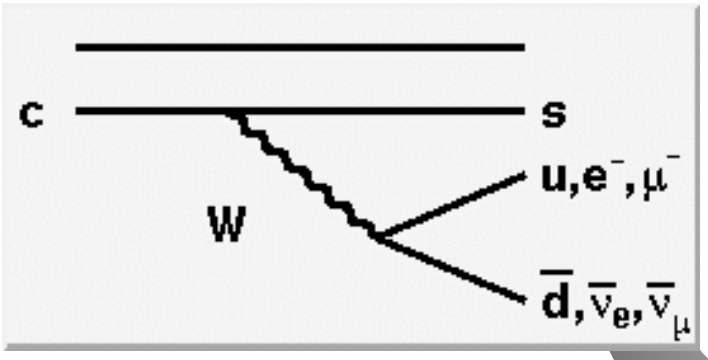
$$\sigma_2 \propto \sin^2 \theta_c$$

$$\Rightarrow \frac{\sigma_2}{\sigma_1} \propto \tan^2 \theta_c$$

(plots)

Charm Decays

- Simple spectator model assumes c quark decays as if it was a free quark. (Neglecting strong interaction effects.)



- Expect:
  - Lifetime of D^0 and D^+ are the same
    Experiment: $\tau(D^0) \simeq 2.5 \times \tau(D^+)$
  - Hadronic decay width:

$$\Gamma(D \to hadrons) = N_c \times \Gamma(D \to e\nu + hadrons)$$

$$\Rightarrow Br(D \to e\nu + hadrons) = \frac{1}{2 + N_c} = 0.2$$

    Experimental values:

$$Br(D^0 \to e\nu X) = 6.8 \pm 0.3\%$$

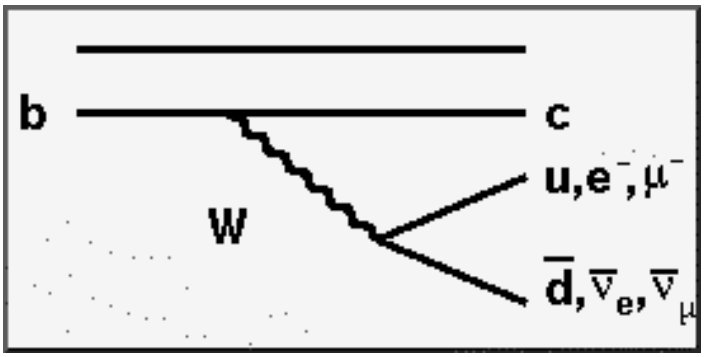
$$Br(D^+ \to e\nu X) = 17.2 \pm 1.9\%$$

- Simple spectator model works for D^+ but not for D^0!
  - charm mass to low for reliable perturbative predictions
  - D^0 has extra annihilation diagrams

B Decays

- One more quark was discovered very soon
  - Discovery in $p + N \to \mu^+ + \mu^- + X$
  - Studied in $e^+ + e^- \to J/\Psi \to \mu^+ + \mu^-$

- Similar story for B decays!



- Simple spectator model works better:
  - $m_b > m_c$
  - $\alpha(m_b) < \alpha(m_c)$
  - perturbation theory works better

$$Br(B \to l\nu + hadrons) = \frac{1}{3 + 2N_c} \cong 0.11$$

- Naïve expectation from universality of CC WI
  - $Br(B \to l\nu X) = 11\%$ (expected)

    $Br(B \to l\nu X) = 10.2 \pm 0.9\%$ (measured)
  - Expect some phase space suppression in charm and tau decays

- Discrepancy can be understood
  - QCD radiative corrections
  - bound state effects

- Lifetime

$$\tau(B^+) = (1.653 \pm 0.028) \times 10^{-12} \text{ sec}$$

$$\tau(B^0) = (1.548 \pm 0.032) \times 10^{-12} \text{ sec}$$

$$\frac{\tau(B^+)}{\tau(B^0)} = 1.062 \pm 0.029$$

The 6 Quark Model

- After 5th quark was discovered:
  - FCNC in theory again!
  - expect 6th quark (bottom $\Leftrightarrow$ top)
  - GIM like mechanism cancels FCNC
  - Top quark was discovered at FNAL
  - m_t=174.3±5.1 GeV
- Generalise GIM mechanism to 3 generations:
  - CC WI couples with universal strength to rotated quark states!

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

V is unitary CKM matrix

$V^+V=1$

$$\sum_j V_{jk} V_{ji}^* = 0 \qquad (i \neq k)$$

$$\sum_j \left|V_{jk}\right|^2 = \sum_j \left|V_{kj}\right|^2 = 1 \quad \Rightarrow \text{if real, orthogonal}$$

- real n x n Matrix
  - ½ n(n-1) independent parameters
    - n=2: 1 rotation angle
    - n=3: 3 rotation angles
- But V is unitary matrix
  - ½ n(n-1) mixing angles
  - ½ n(n+1) complex phases
  - absorb 2n-1 phases in definition of q- and q'-fields
- n=3 case
  - 3 mixing angles
  - 1 complex phase (CP violation)
- No prediction. Obtain from experiment

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & s_2 \\ 0 & -s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & s_1 & 0 \\ -s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta} \end{pmatrix} \begin{pmatrix} 0 & c_3 & s_3 \\ 1 & 0 & 0 \\ 0 & -s_3 & c_3 \end{pmatrix}$$

with $s_i = \sin\theta_i$, $c_i = \sin\theta_i$

- How can one obtain Cabibbo's Theory?

CKM Matrix

- Cabibbo-Kobayashi-Maskawa quark mixing matrix
- Different parameterisations

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$\cong \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

$$\approx \begin{pmatrix} 0.9742-0.9757 & 0.219-0.226 & 0.002-0.005 \\ 0.219-0.225 & 0.9734-0.9749 & 0.037-0.043 \\ 0.004-0.014 & 0.035-0.043 & 0.9990-0.9993 \end{pmatrix}$$

$$s_1 = 0.219 - 0.226$$

$$s_2 = 0.037 - 0.043$$

$$s_3 = 0.002 - 0.005$$

Measurement of CKM angles

- V_{ud}: Compare $\pi^+ \to \pi^0 e^+ \nu_e$ and $\mu^+ \to e^+ \nu_e \overline{\nu_\mu}$
- V_{us}: Compare $\pi^+ \to \pi^0 e^+ \nu_e$ and $K^+ \to \pi^0 \nu_e \overline{\nu_\mu}$
- V_{cd}: Measure di-muon production in muon neutrino beams (see above).
- $(V_{cb})^2+(V_{ub})^2$: from lifetime of b quarks
- V_{cb}/V_{ub}: from muon spectrum in b decays $b \to u + W \to u + \mu + \nu$

  $b \to c + W \to c + \mu + \nu$
  - muon spectrum from b-u decay has higher end point as $m_c >> m_u$

    $V_{ub} = 0.002 - 0.007$

    $V_{cb} = 0.032 - 0.045$

    $\Rightarrow$long b lifetime
- Off diagonal elements are small!
  - Why????
  - Prefered decay chain: b→c→s
  - t quark: t→b+W→b+l+υ (E_{mis})

Unitarity Triangle (I)

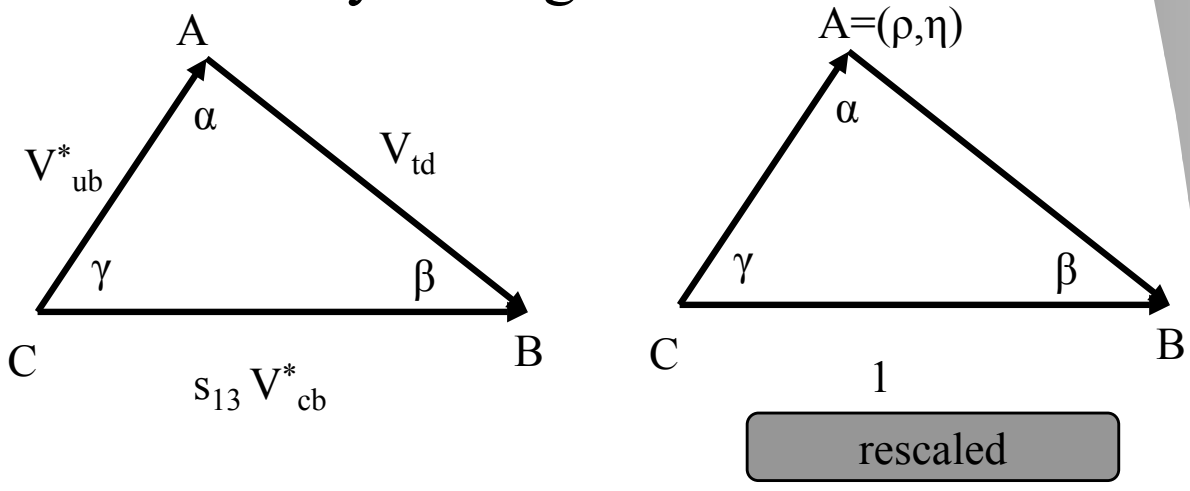
- The CKM matrix V is a unitary matrix! $VV^{+} = 1$
  - Neatly summarize information in terms of "the unitarity triangle"
  - Unitarity of 3x3 CKM matrix applied to the first and third columns yields

$$V_{ud}V_{ub}^{*} + V_{cd}V_{cb}^{*} + V_{td}V_{tb}^{*} = 0$$

  - choose $V_{cd}V^{*}_{cb}$
    real = horizontal in complex plane
  - Set cosines of small angels to unity

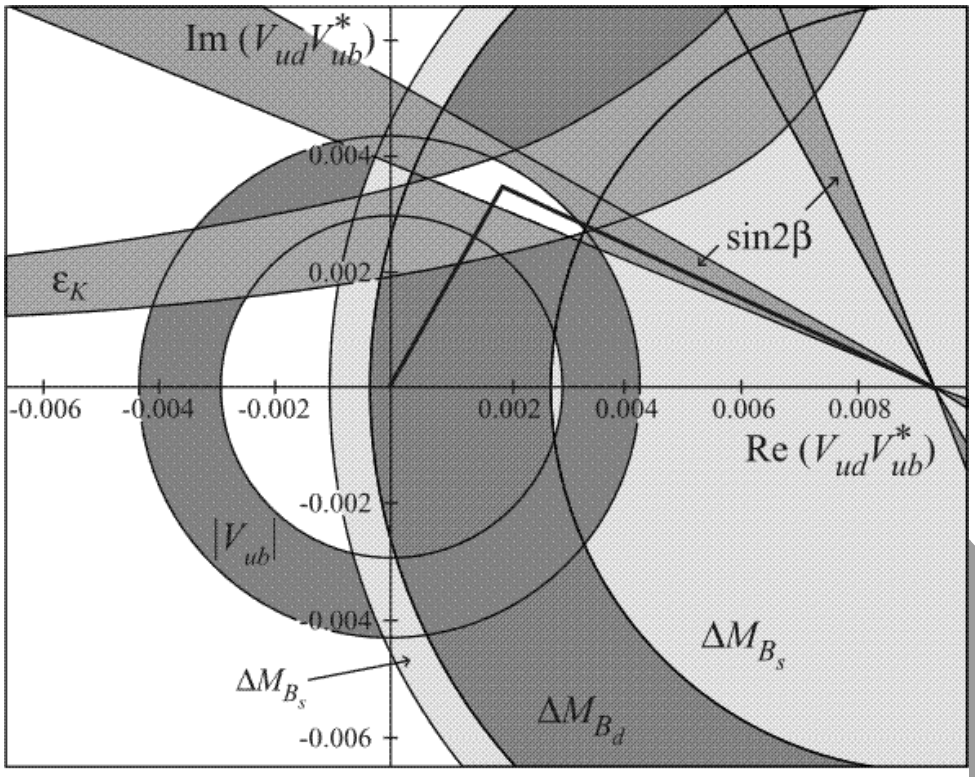
$$V_{ub}^{*} + V_{td} = \sin\Theta_{13}V_{cb}^{*}$$

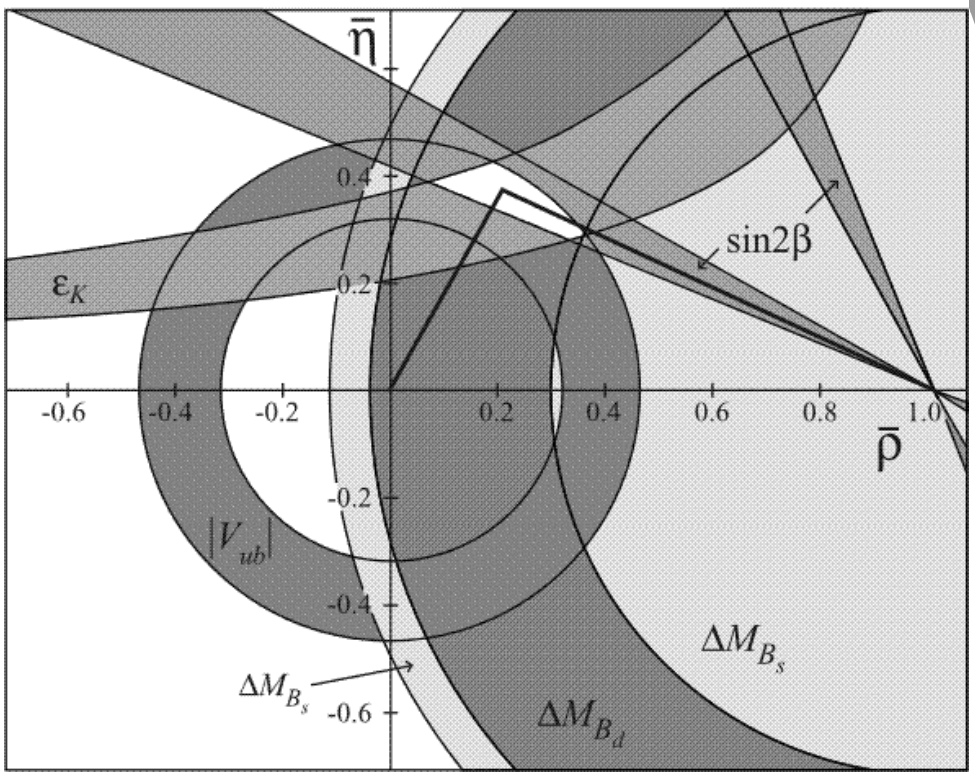
  - Unitarity Triangle



Unitarity Triangle (II)

- Why all the effort?





Summary

- Part I
  - Weak interaction and nuclear decay
    - selection rules
    - decay spectra (Curie-plot)
  - V-A theory
    - non-relativistic limit
    - ultra relativistic limit
    - particle decays
  - Experiments:
    - discovery of neutrino
    - parity violation
    - Helicity of neutrino
- Part II
  - 3 generations
  - WI and leptons
    - Lepton number conservation
  - WI and quarks
    - quark mixing
    - CKM matrix
  - Universality of WI