Reactors, energy & stars

Contents

Reactors, energy & stars

> "Anyone who expects a source of power from the transformation of these atoms is talking moonshine."
>
> *Ernest Rutherford, 1933*

Rutherford was wrong. The energy changes during nuclear reactions are of order 10^6 times larger than those during chemical reactions. They are responsible for the energy emitted by stars, including our own sun, the geothermal heat that keeps the centre of the earth. Nuclear fuels represent the overwhelming majority of the available energy resources on earth.

We have previously seen that the binding energy per nucleon B/A is typically 8 MeV. The different contributions to the binding energy, as represented in the Semi-Empirical Mass Formula, lead to a maximum B/A close to the common isotope $^{56}\mathrm{Fe}$ which has $B/A = 8.79$ MeV.

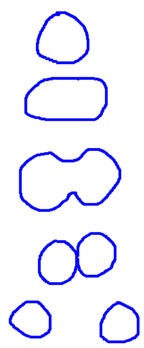
Isotope	$^{2}\mathrm{H}$	$^{4}\mathrm{He}$	$^{6}\mathrm{Li}$	$^{16}\mathrm{O}$	$^{44}\mathrm{Ca}$	$^{56}\mathrm{Fe}$	$^{107}\mathrm{Ag}$	$^{238}\mathrm{U}$
B/A [MeV]	1.112	7.074	5.332	7.976	8.658	8.790	8.554	7.570

Table 1: Examples of binding energy per nucleon.

1 Fission

1.1 Energy and barriers

From the Table 1 we see that it would be energetically favourable for nuclei in the region of $A \sim 100$ to split into two lighter nuclei, each with larger values of B/A. It is then reasonable to ask why it is that nuclei with values of $A \sim 100$ do not split into two parts, given that it is energetically possible. The reaction would be exothermic, and so must be suppressed by some mechanism.



Fission of a nucleus into two smaller nuclei

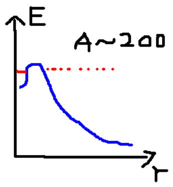
Consider the splitting of a large sphere into two smaller, equally sized, spheres. The intermediate steps must involve the first sphere elongating, then becoming ellipsoid, pinching in the middle, and finally separating. During the elongation and separation, the charges move further apart, decreasing the size of the Coulomb term. However the surface area, and hence the surface energy must increase.

At the point where the daughter nuclei have only just separated, the electrostatic energy from their proximity will be of order the Coulomb potential of

$$E_{\text{barrier}} = \frac{\alpha\,(Z/2)^2}{2r_0(A/2)^{1/3}} \tag{1}$$

where $Z/2$ and $A/2$ are the atomic number and mass number respectively of the daughter nuclei, and a symmetric split has been assumed. For $Z = 40$ and $A = 100$ the energy barrier is of height 65 MeV, which is large enough to prevent fission.

For larger values of A, the barrier (1) continues to increase relative to the final state of two well-separated daughters. However the heavier parent nucleus also has less binding energy, and so moves closer to the top of the Coulomb barrier. For A as large as about 200, the barrier becomes sufficiently small (relative to the initial parent's energy) that it becomes possible to tunnel through that barrier. Fission proceeds, at a rate determined by the tunnelling probability. Since the barrier is small, it may also be possible to push the nucleus over the barrier, if a relatively small amount of energy can be added, for example from a projectile.

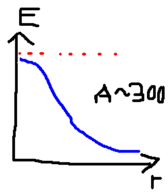


The potential barrier is small for $A \sim 200$.

If one considers values of A as large as 300, there is no barrier at all from the parent's side, and so the nucleus will immediately fall apart. Such nuclei are not observed in nature.

Let us consider how energy might be added to induce fission for two example $A \sim 200$ nuclei. For the uranium isotop $^{238}_{92}\mathrm{U}$ the energy barrier is of order 6 MeV. We can induce fission in this nucleus by bombarding it with sufficiently high-energy neutrons:

$$^{238}\mathrm{U} + n(E > 6\,\mathrm{MeV}) \longrightarrow \text{fission products.}$$



The potential barrier disappears for $A \sim 300$.

Low-energy neutrons impacting on $^{238}\mathrm{U}$ have a relatively high capture cross section through the (n, γ) process

$$^{238}\mathrm{U} + n \rightarrow {}^{239}\mathrm{U}^* \rightarrow {}^{239}\mathrm{U} + \gamma \tag{2}$$

leading to neutron capture without fission.

Fission is easier to induce in the other naturally occuring isotope of uranium $^{235}\mathrm{U}$. The arrival of even very low-energy (thermal) neutrons on $^{235}\mathrm{U}$ leads to fission with 84% probability. The radiative neutron capture reaction (n, γ) occurs with only 16% probability.

To understand why a very low-energy neutron can cause fission in $^{235}\mathrm{U}$, but that a much higher energy neutron is required to push $^{238}\mathrm{U}$ over the energy barrier, we need to consider the change in the Z and N numbers as the two isotopes gain a neutron.

Isotope	Z	N	Type
$^{235}\mathrm{U}$	92	143	Even-Odd
$^{238}\mathrm{U}$	92	146	Even-Even

On addition of a neutron, $^{235}\mathrm{U}$ moves from being even-odd to being being even-even, thus releasing pairing energy δ. By contrast $^{238}\mathrm{U}$ on absorbing a neutron moves from even-even to even-odd, for which one must pay the price of an additional δ of pairing energy. It is the release of the pairing energy δ that taps $^{235}\mathrm{U}$ over the energy barrier, no matter how low-energy the incident neutron may be.

Fissionable nuclei

Nuclei that are fissionable with slow-moving neutrons include: ${}^{235}_{92}\mathrm{U}$ and ${}^{239}_{94}\mathrm{Pu}$. Plutonium-239 does not occur naturally, since it has only a 24 kyr half life. It is produced as a by-product in nuclear reactors, when ${}^{239}\mathrm{U}$, created via reaction (2), followed by successive β decays

$$\begin{aligned} {}^{239}_{92}\mathrm{U} &\longrightarrow {}^{239}_{93}\mathrm{Np} + e^- + \bar{\nu}_e \\ {}^{239}_{93}\mathrm{Np} &\longrightarrow {}^{239}_{94}\mathrm{Pu} + e^- + \bar{\nu}_e. \end{aligned}$$

Reactors designed specifically to produce and burn fissionable ${}^{239}\mathrm{Pu}$ fuel during their operation are called **breeder** reactors.

1.2 Cross sections for fission reactions

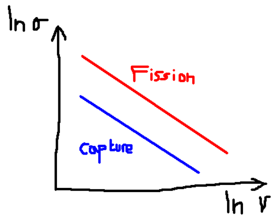
The important reactions for fissionable reactors are elastic scattering of neutrons (n, n) radiative absorption of neutrons (n, γ) and neutron-induced fission (n, f). The radiative absorption cross section is important because it removes neutrons which would otherwise be able to induce fission. For fission to proceed we will need to know the cross sections for each of these processes, and their energy dependences.

The elastic (n, n) scattering cross section is found to be approximately independent of the speed of the neutron – other than where resonant scattering occurs. By contrast, for the (n, γ) radiative capture reaction, the cross section falls rapidly with the speed v_{in} of the incoming neutron,

$$\boxed{\sigma_{\text{capture}} \propto \frac{1}{v_{\text{in}}}.}$$

The same fall-off with v_{in} is seen for the fission reaction

$$\boxed{\sigma_{\text{fission}} \propto \frac{1}{v_{\text{in}}}.} \tag{3}$$

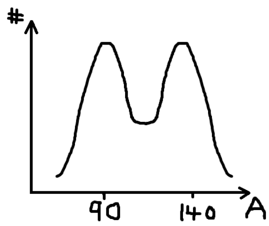


Graph of $\ln \sigma$ against $\ln v$

1.3 Chain reactions

When ${}^{235}\mathrm{U}$ fissions, it releases about 200 MeV of energy. The products are two fission fragments, and several neutrons. The reason for the emission of neutrons is as follows. Heavy nuclei, such as the parent, are more neutron-rich than lighter ones (recall the curve in the valley of stability in §**??**). Hence the fission products, if they had the same ratio of protons-to-neutrons as their parent, would be too neutron for their value of A. This leads to the direct emission of on average 2.5 neutrons per fission.

These fission fragments are also **neutron rich** and hence they must **beta-decay** towards the valley of stability.



The fission tends to be asymmetric, resulting in fragments peaked around $A \sim 90$ and $A \sim 140$.

Neutron scattering, capture and fission cross sections.
We can understand the functional dependence of the cross sections σ_{elastic}, σ_{capture} σ_{fission} on neutron speed as follows. The rate is governed by the Fermi golden rule (**??**). The matrix element is M_{fi} and the density of states will be proportional to

$$p_f^2 \frac{dp_f}{dE}.$$

Now, cross section is rate divided by incoming neutron flux, and flux is proportional to v_{in}. Hence the cross section has the following dependence on v_{in} and p_{f}:

$$\sigma \propto \frac{1}{v_{\text{in}}} |M_{fi}|^2 \; p_f^2 \frac{dp_f}{dE}, \tag{4}$$

We can simplify the equation by noting that the energy-momentum relation $E^2 = p^2 + m^2$ implies that[a]

$$\frac{dp}{dE} = \frac{E}{p} = \frac{1}{v}$$

For elastic (n,n) reactions, the incoming and outgoing speeds are the same ($v_f = v_{\text{f}}$) in the centre-of-mass frame, hence for a low energy neutron with $p = mv$ the energy dependence simplifies to

$$\sigma_{\text{elastic}} \propto \frac{1}{v_{\text{in}}} |M|^2 \frac{m_n^2 v_f^2}{v_f} = |M|^2 m_n^2$$

Hence we expect the elastic cross section to be approximately **constant**, except perhaps where resonant scattering causes sharp peaks.

For the very exothermic (n,γ) and (n,f) reactions, the starting point (4) is the same, but the density of states factor is now completely dominated by the energy released to the decay products – the photon for the radiative case, or the fission products. The density of final states is now almost completely independent of the incoming neutron speed. Aside from any energy dependence of the matrix element, the capture and fission cross sections will be proportional to the reciprocal of the flux, i.e.

$$\sigma_{\text{exothermic}} \propto 1/v_{\text{in}}.$$

[a] The same v dependence is found using the non-relativistic formula $E = p^2/(2m)$.

If the ejected neutrons can be induced to cause further (n,f) reactions, then a **chain reaction** can occur in which neutrons produced in one generation of decays initiate the next.

For a power station, the chain reaction must proceed in a controlled manner. The rate of fissions, and hence the number density of neutrons at fissionable energies, must be controlled. Possible fates of neutrons within a reactor are:

- Neutrons can decay with an average lifetime of $\tau_n = 885$ s
- Lost from the reactor core
- Radiatively captured on the fuel
- Induce further fissions

We might try to improve reaction rates by using pure uranium-235. In pure $^{235}\mathrm{U}$, the mean-free path travelled by a neutron before it will induce fission is

$$\lambda_{\text{fission}} = \frac{1}{n_{235}\sigma_{\text{fission}}} \approx 10 \text{ cm}$$

where n_{235} is the number density of $^{235}\mathrm{U}$ nuclei. We would need to make our pure-235 reactor at least this large. Neutrons will be emitted from fissions with $\sim$ MeV energies and so will be travelling at $v = \sqrt{2m_nE_n} \approx 0.1c$. Since these speeds are much faster than the speed of sound in the material, the emitted neutrons will induce fission in further nuclei the before the material structure is disrupted by the release of energy. A chain reaction started in a critical mass of pure $^{235}\mathrm{U}$ will therefore exponentially increase in number of neutrons emitted and energy released. This fast release of a large amount of energy in a short time will not provide the controlled release of energy desired for a power station.

1.4 Fission reactor principles

Naturally-occurring uranium is approximately only 0.07% $^{235}\mathrm{U}$ with the rest made up from $^{238}\mathrm{U}$.

The majority isotope $^{238}\mathrm{U}$ has a series of sharp resonances in (n,γ) reactions in the approximate energy range 10 eV to 10 keV. These resonances absorb neutrons and make it difficult to sustain a chain reaction. To keep a reaction going one must increase the probability for fission relative to absorption by: (a) increasing the fraction of uranium-235 in the fuel or (b) increasing σ_{fission} compared to σ_{capture}, or both.

Increasing the fraction of uranium-235 is known as **enrichment**. It can only be achieved using the difference in the physical properties caused by the mass differences of the isotopes — the chemical properties of the two isotopes are identical. Enrichment can be achieved by e.g. mass spectrometers for small amounts of material, or by exploiting differential gaseous UF_6 diffusion rates, or with centrifuges.

It is possible to further reduce the radiative capture on 238U by rapidly cooling the $\sim$ MeV energy neutrons. Cooling of neutrons is known as **moderation**. Cooling neutrons to thermal temperatures (< 0.1 eV) reduces their energies below the energy at which the resonances in 238U lead to radiative neutron capture. Cooling also increases the fission cross-section since, as shown in (3), the fission cross section $\sigma_{\text{fission}} \propto 1/v_n$. To avoid captures, and to increase efficiency, we wish to cool the neutrons in a space away from the fuel. This requirement leads to a heterogeneous reactor design, in which lumps (usually rods) of fuel are embedded in a matrix of moderating material[1].

The moderating material must have low neutron capture cross section and, for efficient cooling, should contain nuclei with small A. Typical moderating materials

[1] There is also a more subtle reason why lumps of fuel are better. Since the de Broglie wavelength of the thermal neutron is larger $\lambda_{\text{thermal}} > \lambda_{\text{capture}}$, more of the fuel lump is 'seen' by the thermal neutron than by the higher energy, shorter wavelength, ~ 100 eV ready-to-capture neutron.

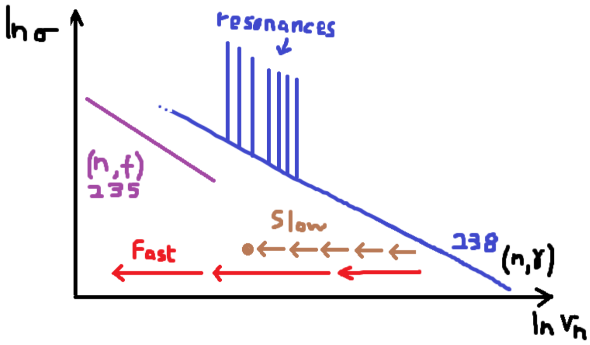


Figure 1: If a neutron can be cooled rapidly, there is a decreased probability for it to be lost to radiative capture.

are graphite $^{12}\mathrm{C}$ or heavy water D_2O, where $D = {}^2_1\mathrm{H}$ is the deuteron. Rapid cooling — in a small number of collision steps — reduces the probability of a neutron having energy close to the $^{238}\mathrm{U}(n,\gamma)$ resonant peaks (Figure 1).

In a chain reaction the number of neutrons at any time will be given by

$$n = n_0 e^{(k-1)t/T}$$

where k is the average number of neutrons produced per fission less the number lost through decays, radiative captures, or loss from the core. T is the characteristic time for one generation of fissions. Since the neutron transit time between reactions is of order nanoseconds, even if $k = 1.001$ such a chain reaction can lead to exponential growth with a short doubling time. In order for the reaction to be controlled one must maintain k very close to unity.

Loss of neutrons to the surrounding material can be reduced by increasing the size of the reactor, and/or by surrounding it with a neutron reflector – a material with a large elastic scattering cross section. The reactor core is usually held within a steel pressure vessel, which is itself within a concrete shield. The steel reflects neutrons, and absorbs γ radiation. The concrete absorbs residual γ radiation and provides physical protection.

Energy is extracted from the reactor by circulating a fluid inside the core. Air, water or liquid sodium coolants have all been used. The thermal energy is transferred through a heat exchanger, used to boiling water, and to generate electricity using steam turbines.

The fission fragments are neutron rich, and so waste products include isotopes unstable to beta decay. Products with half-lives less than a day decay rapidly and do not present a problem. At the other extreme, products with half-lives larger than about 10^6 years have such small decay rates that they too are safe. In between lie the more awkward waste products. Small quantities of $^{90}\mathrm{Sr}$ ($T_{1/2} = 29\,\mathrm{yr}$), $^{137}\mathrm{Cs}$ ($30\,\mathrm{yr}$) and $^{99}\mathrm{Tc}$ ($200\,\mathrm{kyr}$) must be dealt with. There are proposals to use proton beams to transmute these into other safe isotopes, but for the moment such

Moderators and energy loss

Consider an elastic collision between a moving body of mass m with another, initially stationary, body of mass M. In the zero momentum frame (ZMF), let the speed of the first mass be u and that of the second be $U = um/M$, and let the scattering angle in the ZMF be θ. In the lab the speed of m before the collision is

$$v_{\text{in}} = U + u,$$

whereas after the collision it is given by v_{out} where

$$\begin{aligned} v_{\text{out}}^2 &= v_{\text{out}\|}^2 + v_{\text{out}\perp}^2 \\ &= (U + u\cos\theta)^2 + (u\sin\theta)^2 \\ &= U^2 + u^2 + 2Uu\cos\theta \end{aligned}$$

The fraction of energy lost by m by in the lab frame is $v_{\text{out}}^2/v_{\text{in}}^2$. To find the average energy loss, we need to average over the scattering angle θ. For isotropic scattering

$$\langle\cos\theta\rangle = -\int_{-1}^{+1} \cos\theta \, d(\cos\theta) = 0$$

so the average fraction of energy lost by m is

$$\left\langle \frac{E_{\text{out}}}{E_{\text{in}}} \right\rangle = \frac{U^2 + u^2}{(U+u)^2} = \frac{m^2 + M^2}{(M+m)^2}$$

A moderator material will be maximally efficient when M is equal to m, meaning that we want materials with small A. At maximum efficiency, with $A = 1$, half of the initial neutron energy will be lost on average. An initial 1 MeV neutron will then cool to 0.1 eV after about $\log_2(\text{MeV}/0.1\,\text{eV}) \approx 23$ collisions.

isotopes are typically encased in glass (vitrified) and held in secure storage.

2 Fusion

Since the most stable elements are found in the middle of the table of nuclides, energy can also be released by fusing together nuclei with very small A. The fusion process is responsible for the power of the stars, including the sun. Fusion is also a necessary step in the formation of the chemical elements. It also offers the potential of providing clean and abundant power for the future.

An example of a reaction which would liberate a large amount of energy is

$$4H \to {}^4_2\text{He} + 2e^+ + 2\nu_e \quad [Q = 28\,\text{MeV}]. \tag{5}$$

The isotope ${}^4\text{He}$ is particularly tightly bound[2], so this reaction is very energetically favourable. Despite being energetically favourable it is inhibited by the requirement that four protons need to come together overcoming Coulomb repulsion. In addition two factors of the Fermi coupling constant G_F enter the matrix element, one for each of the proton to neutron transitions.

	Nuclide		$T_{1/2}$
D	Deuterium	${}^2_1\text{H}$	stable
T	Tritium	${}^3_1\text{H}$	12.3 yr

[2] ${}^4\text{He}$ is a doubly magic nucleus, with the special property that all four nucleons occupy essentially the same spatial wave function.

Reactor control

Control on timescales of order seconds is possible by inserting and withdrawing control rods with high neutron-capture cross section. Nuclides with large capture cross sections include the boron isotope $^{10}\mathrm{B}$ and the cadmium isotope $^{113}\mathrm{Cd}$.

How is it possible to control reactors by moving rods over timescales of seconds when the typical time between fission reaction generations is of order nanoseconds? Fortunately about 1% of neutrons emitted after fission come not from fission fragments themselves, but from their daughters after beta decay.

$$^{\mathrm{A}}_{\mathrm{Z}}\mathrm{X} \xrightarrow{\beta^-} {}^{\mathrm{A}}_{\mathrm{Z+1}}\mathrm{Y} \longrightarrow {}^{\mathrm{A-1}}_{\mathrm{Z+1}}\mathrm{Y} + n.$$

These neutron emissions are **delayed** by time taken for the the beta decays, which have typical time constants of 0.1 s – 1 s. The reactor is then operated at such that it is subcritical ($k < 1$) with fast neutrons alone. The reactor is then critical only because of the delayed neutrons, and the effective time constant for control increases to of order seconds.

What fuel to use in man-made fusion reactions?

Prototype fusion reactors achieve best fusion rates using a duterium-tritium fuel mixture.

The barrier height for $D + D$ and $D + T$ fusion would appear to be the same as for H+H using our naive calculation. In fact the barrier is reduced since the nucleons within these composite nuclei have some freedom to arrange themselves to reduce the height of the Coulomb barrier.

The rate for $D + T$ is further enhanced by a resonant reaction involving an intermediate excited state of $^5\mathrm{He}$:

$$^3_1\mathrm{T} + {}^2_1\mathrm{D} \longrightarrow {}^5\mathrm{He}^* \longrightarrow {}^4\mathrm{He} + n \quad [+17.6\ \mathrm{MeV}]$$

The D+T reaction is used because it combines a large resonant cross section and a large Q value.

The simpler reaction

$$p + p \to D + e^+ + \nu_e,$$

has the advantage of not requiring the coincidence of four particles, but it is also inhibited by Coulomb repulsion. The Coulomb barrier is of size

$$E_b \approx \alpha_{\mathsf{EM}} \frac{\hbar c}{2\,\mathrm{fm}} = \frac{1}{137} \frac{(197\,\mathrm{MeV\,fm})}{2\,\mathrm{fm}} \approx 0.7\ \mathrm{MeV}.$$

Fusion will therefore only proceed uninhibited by this barrier at temperatures where each proton has energy of order $E_b/2$. We can use the Boltzman constant k_B to convert this to a temperature, which is of order $E_b/(2k_B) \approx 4 \times 10^9$ K. Uninhibited fusion therefore requires extremely high temperatures.

Fusion can proceed at lower temperatures — indeed the temperature in the centre of the sun is a relatively 'mild' 1.6×10^7 K. Two factors enable fusion to happen at temperatures significantly lower than 10^9 K. The first is that fusion may occur via quantum tunnelling through the Coulomb barrier. The calculation is analogous to that performed during the consideration of α decay (§**??**). The second factor that

$$p(v) \propto \exp\left(-\frac{mv^2}{2k_BT}\right)$$

The Maxwell-Boltzmann velocity distribution.

assists fusion at lower temperatures is the high-energy tail of the Maxwell-Boltzmann velocity distribution.

In man-made fusion reactors, a duterium-tritium plasma is held within a toriodal volume using magnetic confinement. In these **Tokamaks**, the plasma is heated to temperature of 1.5×10^8 K. The best reaction rates and largest energy release is found at the lowest temperatures by using the reaction

$$ {}^3_1\mathrm{T} + {}^2_1\mathrm{D} \longrightarrow {}^4\mathrm{He} + n. \tag{6} $$

Duterium fuel can be efficiently extracted from sea water. Tritium is unstable, with a half-life of about 12 years, so must be artificially produced. The main waste product is inert helium gas. The 14 MeV neutrons from the fusion reaction can be used to produce extra tritium. We can place a 'blanket' of lithium in the wall of the reactor vessel. Tritium is then generated within the blanket through the reactions

$$ n + {}^6_3\mathrm{Li} \to {}^3_1\mathrm{T} + {}^4_2\mathrm{He} $$

and

$$ n + {}^7_3\mathrm{Li} \to {}^3_1\mathrm{T} + {}^4_2\mathrm{He} + n. $$

The ${}^6_3\mathrm{Li}$ reaction is exothermic, so contributes to the energy that can be extracted from the reactor. The ${}^7_3\mathrm{Li}$ reaction is endothermic, but has the advantage of recycling the neutron. We require more than one triton to be produced per neutron to maintain a continuous supply of tritium, since the fusion reaction (6) consumes a triton for each neutron it produces. The ${}^7_3\mathrm{Li}$ reaction meets this need, and allows a sufficient supply of tritium fuel to be maintained.

3 Solar reactions and nucleosynethesis

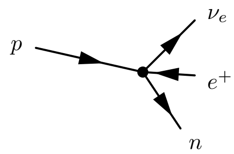
Nuclear reactions in stars are important not only because they generate light and heat, but also because they are the only method by which nuclei heavier that lithium can exist. All of the carbon, oxygen, nitrogen in our bodies is the result of nucleus building within stars, in a series of processes called **nucleosynthesis**.

The sun predominantly burns hydrogen to form helium. We have argued that the direct combination (5) of four protons to form ${}^4\mathrm{He}$ is very improbable, but the reaction can occur via a series of steps as follows.

First two protons fuse to form a deuteron:

$$ p + p \to {}^2\mathrm{D} + e^+ + \nu_e. \tag{7} $$

This first pp fusion reaction involves the transmutation of a proton into a neutron, and so must proceed via the four-fermion vertex of the Fermi beta-decay theory (§**??**). It involves both tunnelling through a Coulomb barrier, and a Fermi matrix element containing a factor of G_F, so occurs at a fairly small rate — it is the rate limiting step. Next is radiative capture of a proton on a deuteron:



Hydrostatic equilibrium and stellar temperatures

We can find the temperature and pressure inside a star of mass M and radius R as follows. Consider an element of a stellar shell, at radius r, of thickness δr and of area A. The gravitational force F_G on that element is of size

$$F_G = \frac{Gm\rho}{r^2} A\,\delta r,$$

where $m(r)$ is the mass contained with the sphere of radius r and $\rho(r)$ is the local density. For hydrostatic equilibrium the gravitational attraction must be balanced by a repulsive force caused the pressure gradient of size

$$F_P = A\Delta P = A\frac{dP}{dr}\delta r.$$

By equating these two forces we can find a bound on the pressure at the centre of the star as follows. First we recognise that

$$\frac{dm}{dr} = 4\pi r^2 \rho(r)$$

allowing us to re-express the equilibrium condition as

$$\frac{dP}{dm} = \frac{Gm}{4\pi r^4}.$$

The integral over the whole star

$$P(M) - P(0) = -\int_{m=0}^{M} \frac{Gm\,dm}{4\pi r^4}$$

cannot be exactly evaluated without knowing how the density varies with radius, but nevertheless it must be larger than

$$-\int_{m=0}^{M} \frac{Gm\,dm}{4\pi R^4} = \frac{GM^2}{8\pi R^4}.$$

where R is the radius of the star.

The calculation places a lower bound on the pressure at the centre of the sun of 4.4×10^{13} Pa, which is equivalent to 450 million atmospheres. We may then use the ideal gas law $PV = nk_BT$ to calculate a lower bound on the temperature of the centre of the sun. The lower limit we obtain will be found to be somewhat lower than the value (of about $T_c = 1.6 \times 10^7$ K) that we would have obtained from a more exact numerical calculation.

$$D + p \rightarrow {}^3\mathrm{He} + \gamma. \tag{8}$$

Finally, two of these ${}^3\mathrm{He}$ nuclei may fuse, generating an alpha particle and recycling two protons:

$${}^3\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^4\mathrm{He} + 2p. \tag{9}$$

The net effect of the three reactions (7)-(9) is that of reaction (5) above. The combined set of reactions (7)-(9) is known as the **pp-I chain**.

3.1 The pp-II and pp-III chains

Several other competing reactions also occur which have the net effect of turning Hydrogen into Helium. At temperatures above that at which the pp-I chain occurs, two helium isotopes (produced as described in §3 above) may fuse to form Beryllium-7,

$${}^3_2\mathrm{He} + {}^4_2\mathrm{He} \rightarrow {}^7_4\mathrm{Be} + \gamma.$$

At this point the reaction can take one of two branches.

In the **pp-II branch** the subsequent series of reactions is electron capture

$${}^7_4\mathrm{Be} + e^- \rightarrow {}^7_3\mathrm{Li} + \nu,$$

followed by the lithium-7 absorbs a further proton

$${}^7_3\mathrm{Li} + p \rightarrow 2\,{}^4_2\mathrm{He}$$

generating two further Helium-4 nuclei.

In the **pp-III chain** a different series of reactions leads to the same end result. First the Beryllium-7 absorbs a proton

$${}^7_4\mathrm{Be} + p \rightarrow {}^8_5\mathrm{B} + \gamma.$$

The resulting ${}^8_5\mathrm{B}$ is unstable to β^+ decay

$${}^8_5\mathrm{B} \rightarrow {}^8_4\mathrm{Be} + e^+ + \nu_e$$

generating ${}^8_4\mathrm{Be}$. Because of the particularly large binding energy of the Helium nucleus, Beryllium-8 spontaneously splits in two

$${}^8_4\mathrm{Be} \rightarrow 2\,{}^4_2\mathrm{He}$$

generating two Helium nuclei.

In each of these two chains — pp-II and pp-III — the net reaction is

$${}^3_2\mathrm{He} + {}^4_2\mathrm{He} + {}^1_1\mathrm{p} \rightarrow {}^4_2\mathrm{He} + {}^4_2\mathrm{He} + e^+ + \nu_e.$$

Producing carbon – Helium burning

There are roadblocks in forming elements heavier than Helium caused by the absence of any stable isotopes with mass numbers 5 or 8. Because Helium is particularly tightly bound, the candidate isotopes with $A = 5$ and $A = 8$ decay in the following manners:

$$\begin{aligned} {}^5_3\mathrm{Li} &\longrightarrow {}^4_2\mathrm{He} + p \\ {}^5_2\mathrm{He} &\longrightarrow {}^4_2\mathrm{He} + n \\ {}^8_4\mathrm{Be} &\longrightarrow 2\;{}^4_2\mathrm{He} \end{aligned}$$

Fortunately, in thermal equilibrium there will exist a small but non-zero population of ${}^8_4\mathrm{Be}$ through the equilibrium process

$${}^8_4\mathrm{Be} \rightleftharpoons 2\;{}^4_2\mathrm{He}$$

A further ${}^4_2\mathrm{He}$ nucleus can react with the equilibrium population of ${}^8_4\mathrm{Be}$ to generate ${}^{12}\mathrm{C}$. We are fortunate that there is an excited state of ${}^{12}\mathrm{C}^*$ at just the right energy to cause resonant production via

$${}^8_4\mathrm{Be} + {}^4_2\mathrm{He} \rightarrow {}^{12}\mathrm{C}^* \longrightarrow {}^{12}\mathrm{C} + \gamma$$

3.2 The CNO cycles

Protons are also burned to helium through carbon-nitrogen-oxygen catalysis. These reactions are catalysed by ${}^{12}\mathrm{C}$, which must therefore be formed within stars.

There are two important catalysis cycles. The first (**CNO-I**) is:

$$\begin{aligned} {}^{12}_6\mathrm{C} + p &\rightarrow {}^{13}_7\mathrm{N} + \gamma \\ {}^{13}_7\mathrm{N} &\rightarrow {}^{13}_6\mathrm{C} + e^+ + \nu_e \\ {}^{13}_6\mathrm{C} + p &\rightarrow {}^{14}_7\mathrm{N} + \gamma \\ {}^{14}_7\mathrm{N} + p &\rightarrow {}^{15}_8\mathrm{O} + \gamma \\ {}^{15}_8\mathrm{O} &\rightarrow {}^{15}_7\mathrm{N} + e^+ + \nu_e \\ {}^{15}_7\mathrm{N} + p &\rightarrow {}^{12}_6\mathrm{C} + {}^4_2\mathrm{He} \end{aligned}$$

which recycles the ${}^{12}\mathrm{C}$, and has the same net effect of converting four protons to ${}^4_2\mathrm{He}$ plus two positrons and two neutrinos – i.e. the same reaction as shown in (5).

The second cycle of the CNO set of reactions shares several of the reactions of the first:

$$\begin{aligned} {}^{14}_7\mathrm{N} + p &\rightarrow {}^{15}_8\mathrm{O} + \gamma \\ {}^{15}_8\mathrm{O} &\rightarrow {}^{15}_7\mathrm{N} + e^+ + \nu_e \end{aligned}$$

but has a different way of recycling the ${}^{14}_{7}\mathrm{N}$, via somewhat heavier elements.

$$\begin{aligned} {}^{15}_{7}\mathrm{N} + p &\rightarrow {}^{16}_{8}\mathrm{O} + \gamma \\ {}^{16}_{8}\mathrm{O} + p &\rightarrow {}^{17}_{9}\mathrm{F} + \gamma \\ {}^{17}_{9}\mathrm{F} &\rightarrow {}^{17}_{8}\mathrm{O} + e^{+} + \nu_e \\ {}^{17}_{8}\mathrm{O} + p &\rightarrow {}^{14}_{7}\mathrm{N} + {}^{4}_{2}\mathrm{He} \end{aligned}$$

The net effect is that of conversion of four protons to ${}^{4}_{2}\mathrm{He}$ plus two positrons and two neutrinos – i.e. the same reaction as shown in (5).

These reactions are most important in stars more massive than the sun, in which higher temperatures can be reached, and for which the large Coulomb barrier for these larger-Z reactants is relatively less important.

3.3 Solar neutrinos

In the reactions above β^+ decays and electron capture reactions lead to the emission of electron-type neutrinos. In a three-body decay process, such as β^+ decay, the final energy is shared out between the electron and the neutrino, and so a continuum spectrum of neutrino energies is produced (up to some maximum close to Q). For a two-body decay process, such as electron capture, there energy and momentum conservation constrain the size of the neutrino energy to a single value, equal to the Q value of the reaction minus the recoil energy of the daughter nucleus.

There is a continuum distribution of low-energy neutrinos from the initial proton fusion reaction

$$2\,p \rightarrow {}^{2}_{1}\mathrm{D} + e^{+} + \nu_e.$$

This competes with the 'pep' reaction

$$p + e^{-} + p \rightarrow {}^{2}_{1}\mathrm{D} + \nu_e.$$

which has a smaller cross-section, but which has a two-body final state, so produces a source of mono-energetic neutrinos.

We can see that there are also mono-energetic ν_e emissions from the pp-II chain from the electron capture process

$${}^{7}_{4}\mathrm{Be} + e^{-} \rightarrow {}^{7}_{3}\mathrm{Li} + \nu.$$

There is continuum of neutrino energies from the pp-III chain from the β^+ decay process

$${}^{8}_{5}\mathrm{B} \rightarrow {}^{8}_{4}\mathrm{Be} + e^{+} + \nu_e.$$

In all cases the type of neutrino produced is an electron-flavour neutrino ν_e.

One can calculate the flux of neutrinos expected from these reactions at different energies. Detecting the solar neutrinos on earth requires sensitive detectors. We must also be careful to reduce the backgrounds from cosmic rays, radioactivity, and other noise sources. Solar neutrino experiments provided the early evidence for neutrino oscillations (§**??**).

3.4 Heavier elements

During the lifetime of a sun-like star, the temperature and the fuel change. When the hydrogen fuel is exhausted, the core starts to collapse under its own gravity, and it heats until the temperature is high eneough for helium to be effective as a fuel. When the helium is expended the temperature rises again and the next fuel comes into use, and so on.

Beyond ${}^{12}_{6}\mathrm{C}$ further α particle addition can generate ${}^{16}_{8}\mathrm{O}$ and ${}^{20}_{10}\mathrm{Ne}$. Fusion of two ${}^{12}_{6}\mathrm{C}$ isotopes can lead to production of ${}^{24}_{12}\mathrm{Mg}$.

The Coulomb barrier prevents fusion of large nuclei. However in the late stages of heavy stars, elements up to ${}^{56}\mathrm{Fe}$ can be produced through a process in which neutrons are radiatively captured (n, γ). The neutrons themselves are bled off from other reactions such as ${}^{13}\mathrm{C}(\alpha, n){}^{16}\mathrm{O}$.

Supernovae and the heaviest elements

Stars larger than about nine solar masses finish their lives with a bang. Their inner core collapses several times as it sequentially consumes different fuels, leading to a onion-layered structure with Hydrogen burning in the outer layers, with inner layers using He, C, Ne, O, and finally Si as fuels. Successive fuels require higher temperatures to overcome the Coulomb barrier, but liberate smaller energies, so burn increasingly rapidly. When all the silicon in the core burns to nickel and iron a cataclysmic implosion takes place over several seconds. The shock wave from this implosion detaches the outer layers of the star, and briefly provides the only natural conditions under which elements heavier than iron are produced.

The mechanism starts with photo-disintegration of Fe by high-energy gamma rays producing large fluxes of free neutrons. These neutrons bombard heavy nuclei, and are accreted with successive beta decays bringing the resulting heavy nucleus back to the valley of stability. Elements as heavy as ${}^{238}\mathrm{U}$ can be produced in this way.

Supernovae are expected to lose a large fraction of their energy through neutrinos, which pass through the outer layers relatively unimpeded. A total of 19 neutrinos from SN1987A were observed by two detectors within a 13 s interval.

Key concepts

- Neutron-induced fission of ${}^{235}\mathrm{U}$ can be controlled in self-sustaining **chain reactions**
- Cooling of the neutrons with a **moderator** to thermal temperatures allows a chain reaction since

  $$\boxed{\sigma_{\text{fission}} \propto \frac{1}{v_n}}$$

  and because such cooling reduces the loss of neutrons via resonant radiative capture ${}^{238}\mathrm{U}(n, \gamma)$.

- Fission of light elements is possible at high temperature through high-energy tail of the **Maxwell-Boltzmann** distribution and **tunnelling** through the Coulomb barrier.
- Nucleosynthesis in stars occurs via reactions including the **pp-I**, **pp-II** and **pp-III** chains, and the **CNO bi-cycle**.
- Heavier elements are created via sequential burning of larger-Z fuels as lower-Z fuels are expended.

Further reading

- W. N. Cottingham and D. A. Greenwood, *An Introduction to Nuclear Physics*
- B. Martin, *Nuclear and Particle Physics: An Introduction*
- W. S. C. Williams, *Nuclear and Particle Physics*
- K.S. Krane *Introductory Nuclear Physics*
- M.G. Bowler *Nuclear Physics*
- N.A. Jelley *Fundamentals of Nuclear Physics*
- Ed. S. Esposito and O. Pisanti, *Neutron Physics for Nuclear Reactors: Unpublished Writings by Enrico Fermi*
- D.D. Clayton, *Principles of Stellar Evolution and Nucleosynthesis* (1968).

