The quark model and deep inelastic scattering

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1 Symmetry, patterns and substructure

The proton and the neutron have rather similar masses. They are distinguished from one another by at least their different electromagnetic interactions, since the proton is charged, while the neutron is electrically neutral, but they have identical properties under the strong interaction. This provokes the question as to whether the proton and neutron might have some sort of common substructure. The substructure hypothesis can be investigated by searching for other similar patterns of multiplets of particles.

$$\begin{aligned} m_p &= 938.3\,\text{MeV}/c^2 \\ m_n &= 939.6\,\text{MeV}/c^2 \end{aligned}$$

There exists a zoo of other strongly-interacting particles. Exotic particles are observed coming from the upper atmosphere in cosmic rays. They can also be created in the labortatory, provided that we can create beams of sufficient energy. The Quark Model allows us to apply a classification to those many strongly interacting states, and to understand the constituents from which they are made.

1.1 Pions

The lightest strongly interacting particles are the *pions* (π). These can be produced by firing protons with energies of order GeV into material. Different pion creation interactions are observed to occur, such as

$$\begin{aligned} p+p &\rightarrow p+p+\pi^0 \\ p+p &\rightarrow p+p+\pi^+ + \pi^- \\ p+n &\rightarrow p+n+\pi^0+\pi^+ + \pi^-. \end{aligned}$$

There are three different pions with charges, $+1$, 0 and -1 (π^+, π^0 and π^- respectively). In each of these pion production interactions electric charge is conserved. However some of the energy of the incident particle(s) is turned into creation of new pion particles.

The three pions have masses

$$\begin{aligned} m_{\pi^+} = m_{\pi^-} &= 139.6\,\text{MeV}/c^2 \\ m_{\pi^0} &= 135.0\,\text{MeV}/c^2. \end{aligned}$$

Again we see an interesting pattern – all three pions have similar masses, in this case that mass is about one seventh of that of the proton or neutron.

In fact the two charged pions have exactly the same mass. This is because the π^+ and π^- are anti-particles of one another. Anti-particles share the same mass, but have opposite charges. The π^0 has no charge, and is its own anti-particle.

Collisions also produce negatively charged anti-protons, $\bar{p}$.

$$p + p \to p + p + p + \bar{p}.$$

There is also an anti-neutron $\bar{n}$, with the same mass m_n as the neutron, and which which can also be produced in collisions e.g.

$$p + p \to p + p + n + \bar{n}.$$

Though the neutron has no charge it is *not* its own anti-particle. We can tell the two are different because the anti-neutron decays differently from the neutron:

$$\begin{aligned} n &\to p + e^- + \nu \\ \bar{n} &\to \bar{p} + e^+ + \bar{\nu}. \end{aligned}$$

Another piece of evidence that neutrons are not the same as anti-neutrons is that they do not annihilate against one another inside nuclei.

1.2 Baryon number conservation

In all of the reactions above, we observe that the total number of protons and neutrons less anti-protons and anti-neutrons

$$N(p) + N(n) - N(\bar{p}) - N(\bar{n})$$

is conserved. This rule is a special case of the conservation of **baryon number**, which is a quantum number carried by protons and neutrons, but not by pions. Protons and neutrons each have baryon number $+1$, while their anti-particles have baryon number -1.

Baryon number conservation keeps the proton stable, since it forbids the decay of the proton to e.g. a π^0 and a π^+ each of which have baryon number of zero. Experimental lower bound on the lifetime of the proton can be made by close observation of large tanks of water underground, yielding

$$\tau_p > 1.6 \times 10^{25}\,\text{years}.$$

This is very much longer than the lifetime of the universe ($\approx 1.4 \times 10^{10}$ years) so we would expect protons created in the early universe still to be around today. Thankfully they are – as you can easily verify experimentally.

1.3 Delta baryons

Other groups of strongly interacting particles are also observed. Charged pions live long enough to be made into beams, and so we can study their reactions with

Magnetic moments *Non examinable*

Hints about proton and neutron substructure can also be found in their magnetic dipole moments. It is a prediction of the Dirac theory that any fundamental spin-half fermion with charge $\mathcal{Q}$ and mass m should have a magnetic moment

$$\mu = \frac{\mathcal{Q}\hbar}{2m}.$$

This would predict that if the proton was a fundamental particle it would have magnetic moment equal to the nuclear magneton

$$\mu_N = \frac{e\hbar}{2m_p}$$

However the proton has a magnetic moment of 2.79 μ_N, in disagreement with the Dirac prediction for a fundamental particle. The neutron, which would have no magnetic moment in the Dirac theory, has magnetic moment equal to -1.91 μ_N. These observations suggest that protons and neutrons are not fundamental particles, but are made of something smaller.

protons and neutrons. Examples of reactions observed include the production and decay of a the Δ multiplet of particles, which are observed as resonances in the cross-sections for processes such as

$$\begin{aligned}
\pi^- + n &\rightarrow \quad \Delta^- \quad \rightarrow \pi^- + n \\
\pi^- + p &\rightarrow \quad \Delta^0 \quad \rightarrow \pi^0 + n \\
\pi^+ + n &\rightarrow \quad \Delta^+ \quad \rightarrow \pi^0 + p \\
\pi^+ + p &\rightarrow \Delta^{++} \rightarrow \pi^+ + p
\end{aligned}$$

The four short-lived delta particles Δ have different charges ($+2, +1, 0$, -1), including a double-positively charged particle, Δ^{++}. All have rest-mass-energy close to 1232 MeV. All are produced in charge-conserving reactions. All have spin quantum number $s = 3/2$. They decay in a very short time — of order 10^{-22} s — so cannot be observed as propagating particles. Instead they are observed as resonances. From the width Γ of the resonance we can infer the lifetime of the corresponding particle.

From the reactions above we can see that all four deltas must have baryon number $+1$, in order to conserve baryon number throughout each reaction — these Δ particles are baryons. Conservation of baryon number implies that none of the Δ particles can be anti-particles of one another – they must have separate anti-particles, which would be created in reactions with anti-protons or anti-neutrons.

2 Accelerating protons – linear accelerators

We needed protons with kinetic energy of order GeV to perform these experiments. Unless we are willing to wait for the occasional high-energy cosmic ray coming from space, we'll need to accelerate them. Since the magnetic field changes only the direction of $\mathbf{p}$, it is the electrical field which is used to increase their energy of the particles.

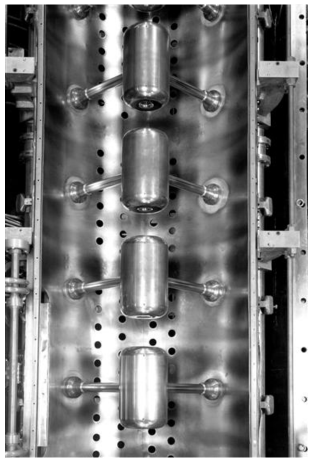
$$\boxed{\frac{d\mathbf{p}}{dt} = Q[\mathbf{E} + \mathbf{v} \times \mathbf{B}]}$$

Reminder of the Lorentz force law.

The problem we encounter if we try to use a constant electric field to do our acceleration is that to get these very high energies (of order GeV) we need to pass them through an enormous potential difference – of order 10^9 volts. Van der Graaff generators can reach potentials of order ten million volts, but then tend to break down because of electrical discharge (sparking) to nearby objects. For the particle creation reactions above, we're looking for about two orders of magnitude more energy than this.

We can get around the limitations of a static potential difference by realizing that only that only the local $\mathbb{E}$ field needs to be aligned along $\mathbf{v}$, and only during the period in which the particle is in that particular part of space.

We can use then use time-varying electric fields. In the margin is a picture of a **linear accelerator** or *linac*. In this device we have a series of cylindrical electrodes with holes through the middle of each which allow the beam to pass through. Electrodes are attached alternately to either pole of an alternating potential. The particles are accelerated in bunches. As each bunch travels along we reverse the potential while the bunch is inside electrode (where this is no field). We can then ensure that when the bunch is in the gap between the electrodes the field is always in the correct direction to keep it accelerating.



The linear accelerator injector to the CERN proton synchrotron. ©CERN

The oscillating potential on the electrodes may be created by connecting wires directly from the electrodes to an oscillator. For radio frequency AC oscillations we can instead bathe the whole system in an electromagnetic standing wave, such that the protons always 'surf' the wave and are continually accelerated.

3 Hadrons – symmetries as evidence for quarks

We have noted the existence of a variety of strongly-interacting particles coming in multiplets with similar masses. The generic name for all these strongly interacting particles is **hadrons**. There are many more of them than we have listed. We therefore need an organising principle – a model that can explain why the strongly interacting particles should come in these multiplets with similar properties. That model is the **quark model**.

3.1 Baryons

In the quark model, we can explain the properties of the nucleons, the delta particles, and other similar states as being composites — bound states of smaller, fundamental particles, called quarks. The quarks are spin-half fermions, and are point-like. No internal structure has ever been observed for a quark.

If we try to build a state out of two spin-half fermions, then quantum mechanical angular momentum addition formulae tell us that the four resulting states will be a spin-1 triplet and a spin-0 singlet. These are the wrong spins for our baryons, so baryons can not be made of pairs of spin-half constituents. However if we build a state out of **three** spin-half fermions, then the eight resulting states are two spin-$\frac{1}{2}$ doublets and a spin-$\frac{3}{2}$ quadruplet. These are the right spins for the the baryons we observe.

	Charge	Spin	Parity
u	$+\frac{2}{3}e$	$\frac{1}{2}$	$+$
d	$-\frac{1}{3}e$	$\frac{1}{2}$	$+$
$\bar{u}$	$-\frac{2}{3}e$	$\frac{1}{2}$	$-$
$\bar{d}$	$+\frac{1}{3}e$	$\frac{1}{2}$	$-$

Up and down quarks, their antiparticles, and quantum numbers.

Baryons are made out of **triplets** of spin-half fermions called **quarks**.

If sets of three constituent quarks are to explain all of the charge states discussed above, then we will need them to come in two distinct types or **flavour**, with electric charges of $+2/3e$ and -$1/3e$. As shown in Table 1, the proton is made of two up-quarks and a down-quark. The neutron is made of two down-quarks and an up quark.

Particle	Quarks	Spin	Charge	Mass / MeV
p	uud	$\frac{1}{2}$	+1	938.3
n	udd	$\frac{1}{2}$	0	939.6
Δ^{++}	uuu	$\frac{3}{2}$	+2	~ 1232
Δ^{+}	uud	$\frac{3}{2}$	+1	~ 1232
Δ^{0}	udd	$\frac{3}{2}$	0	~ 1232
Δ^{-}	ddd	$\frac{3}{2}$	-1	~ 1232

Table 1: Properties of the nucleons and Δ baryons as explained by the quark model. The charges of the baryons are equal to the sum of the charges of their constituent quarks. The proton and neutron are the spin-half angular momentum combinations, while the heavier, unstable delta baryons form the spin-$\frac{3}{2}$ combinations.

3.2 Mesons

The pions have spin zero. If they are made out of quarks, it must be from an even number of them. The simplest hypothesis is to use only two quarks. How can we build the triplet of pion charges $\{-1, 0, +1\}$ out of pairs quarks of charge $\mathcal{Q}_u = +\frac{2}{3}$ and $\mathcal{Q}_d = -\frac{1}{3}$? We can do so if we also use anti-quarks, which have the opposite charges to their respective quarks. The positively charged pion is composed of an up quark and an anti-down quark. The negatively charged pion is a anti-up quark and a down quark.

Composite hadrons formed from a quark and an anti-quark are known as **mesons**.

	Quarks	Spin	Charge	Mass / MeV
π^+	$u\bar{d}$	0	$+1$	139.6
π^0	$u\bar{u}$, $d\bar{d}$	0	0	135.0
π^-	$d\bar{u}$	0	-1	139.6

Table 2: Properties of the pions as explained in the quark model. The neutral pion exists in a superposition of the $u\bar{u}$ and $d\bar{d}$ states.

We have found that the pions are spin-0 states of u and d quark/anti-quark pairs. What happens to the spin-1 states? We would expect to see a set of mesons with spin-1, and indeed we do. The ρ^+, ρ^0 and ρ^- mesons are the equivalent spin-1 combinations. They all have mass of about 770 MeV.

The mesons and baryons are eigenstates of the parity operator, which inverts the spatial coordinates. The eigen-values be found as follows. The Dirac equation describing the relativistic propagation of spin-half particles requires that a particle and its anti-particles have opposite parity quantum numbers. The parity of the quark is set to be positive $(+1)$ by convention, so the anti-quark has negative parity (-1). The parity of the meson state is therefore

$$P_{\text{meson}} = (+1)(-1)(-1)^L = (-1)^{L+1}$$

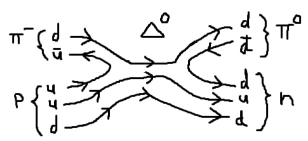
where the term $(-1)^L$ is the spatial parity for a state with orbital angular momentum quantum number L. The lowest-lying meson states for any quark content all have $L = 0$ so we can expect them to have negative parity – this is indeed what we observe for the pions.

The term symbol for the mesons is written J^P, where J is the angular momentum quantum number of the meson, and P is its parity quantum number. The pions have $J^P = 0^-$ and so are called pseudoscalars.

3.3 Quark flow diagrams

We're now in a position to understand the production and decay reactions of the Δ baryons at the quark level. Let us take the example of the Δ^0 and examine the reaction as a flow of quarks:

$$\begin{array}{ccccc} \pi^- + & p & \rightarrow \Delta^0 & \rightarrow \pi^0 + & n \\ d\bar{u} + & uud & \rightarrow udd & \rightarrow d\bar{d} + & udd \end{array}$$



In the first part of the reaction a $\bar{u}$ antiquark in the pion annihilates against a u quark in the proton, leaving a udd state in the correct configuration to form a Δ^0 baryon. In the decay, a quark—anti-quark pair is created to form a neutral pion and a neutron. In the quark model, the conservation of baryon number is a consequence of the conservation of quark number. Each quark has baryon number of $\frac{1}{3}$, and each anti-quark has baryon number of $-\frac{1}{3}$. This leads to the correct baryon numbers: $+1$ for qqq baryons, -1 for $\bar{q}\bar{q}\bar{q}$ anti-baryons, and 0 for $q\bar{q}$ mesons.

Since quarks can only annihilate against antiquarks of the same flavour, quark flavour is conserved throughout the strong reaction[1]. This is a characteristic property of all of the strong interactions:

$$\boxed{\text{Strong interactions conserve quark flavour}}$$

3.4 Strangeness

The up and down quarks are sufficient to describe the proton, neutron, pions and delta baryons. However the story does not stop there. Other particles are also created in strong interactions — particles which did not fit into the two-quark-flavour model and were called ‘strange particles’. Bubble chamber experiments were used to examine the properties of beams of strange particles and demonstrated that the strange particles could travel macroscopic distances before decaying. Their stability could be explained if there was a new, almost-conserved, quantum number associated with these strange particles. This ‘strangeness’ quantum number is conserved in strong interaction. In order to decay the particles had to undergo a weak interaction, which changed the strangeness.

For example some strong interactions produce charged Kaon particles, $K^{\pm}$ with masses just less than 500 MeV, and which carry the strange quantum number

$$p + p \to p + p + K^+ + K^-.$$

The positively charged Kaon is said to have strangeness $+1$, while the negatively charged particle has strangeness -1.

Each kaon can decay to a final state consisting only of pions (e.g. $K^+ \to \pi^0\pi^+$). Considering the baryon number of the final state pions is zero, this tells us that kaons must also have zero baryon number and so must be mesons rather than baryons, since the pions carry no baryon number. Within the quark model we expect

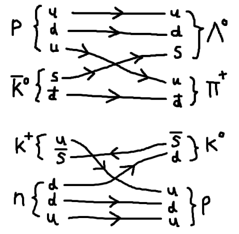
The strangeness can then be transferred to other particles in other strong interactions. For example

$$\bar{K}^0 + p \to \pi^+ + \Lambda^0$$

or

$$K^+ + n \to K^0 + p$$

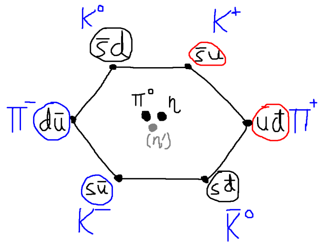
These various different interactions can be understood if we introduce a third quark s, to join u and d. This strange quark must have charge $-\frac{1}{3}$. Due to an accident of history the strange quark carries strangeness quantum number of -1 rather than $+1$. It's anti-particle, the anti-strange quark $\bar{s}$ has charge $+\frac{1}{3}$ and carries strangeness $+1$. We can now see that the positively charged kaon can be a $u\bar{s}$ meson, and the negatively charged kaon a $s\bar{u}$ meson.



[1] Quark flavour is conserved in strong and electromagnetic interactions, but not in weak interactions. For example the beta decay process $n \to p + e^- + \bar{\nu}$ does not conserve quark flavour number, so must be mediated by the weak interaction.

Drawing quark flow diagrams we see that the K^0 must be a $d\bar{s}$ meson – a neutral particle with strangeness $+1$. The Λ^0 must be a baryon with quark content uds.

3.5 The light pseudoscalar octet



The octet of pseudoscalar mesons along with the η' singlet. All have $J^P = 0^-$. The mesons are positioned according to their strangeness (vertical) and the third component of their isospin (horizontal).

We can list the meson states it's possible form with three quarks, u, d and s and their anti-quarks. There are three flavours of quarks and three (anti-)flavours of anti-quarks so we should find 3×3 states. These states break down into an octet and a singlet $(3 \times 3 = 8 + 1)$.

The octet contains three pions, four kaons, and the η meson. The singlet η' is largely a $s\bar{s}$ state, and is heavier than the other mesons.

The four kaons, K^+, K^-, K^0 and $\bar{K}^0$ are the lowest lying strange mesons, and therefore have $S = L = 0$. They must then have 'spin' $J = 0$ and negative parity $(J^P = 0^-)$, just like the three pions we have already encountered.

The flavour content of the K mesons and the $\pi^\pm$ mesons is uniquely determined from their strangeness and charge. There are three uncharged mesons with zero strageness, which are mixtures of $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ states. That makes the full set of $S = 0$, $L = 0$, $J = 0$ mesons made from u, d, s quarks and their anti-quarks. All of these lightest states have orbital angular momentum $L = 0$, and so parity quantum number equal to the product of the quark and anti-quark parities, which according to the Dirac equation is -1.

We should expect another nine states, also with $L = 0$ but with $S = 1$ and hence $J = 1$. Those states are also observed, the ρ mesons mentioned above, being three of them. These states also have negative parity as expected from the value of L. These spin-1 states are known as the vector mesons.

3.6 The light baryon octet

Baryon parity

Using similar arguments to those used in calculating meson parity, we can calculate the spin and parity of the lightest baryons. The spin is $\frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2}$ which is either $\frac{1}{2}$ or $\frac{3}{2}$. The parity is $(+1)(+1)(+1) \times (-1)^L$ which is positive for the lightest $L = 0$ states. We therefore expect to have $J^P = \frac{3}{2}^+$ and $J^P = \frac{1}{2}^+$ states. The anti-baryon partners have the same spins and negative parity.

(Note J^P is a term symbol specifying both J and P. It should not be confused with an exponent and does not mean 'J to the power of P'.)

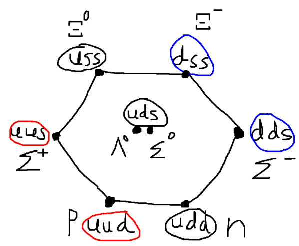
We also expect to be able to form various baryons using quark combinations that include strange quarks. Such baryons do indeed exist. If we examine the spin-half baryons we find a triplet of strangeness -1 baryons:

$$
\begin{aligned}
\Sigma^{+} &= uus \\
\Sigma^{0} &= uds \\
\Sigma^{-} &= dds
\end{aligned}
$$

We also find a doublet of strangeness -2 baryons:

$$
\begin{aligned}
\Xi^{0} &= uss \\
\Xi^{-} &= dss
\end{aligned}
$$

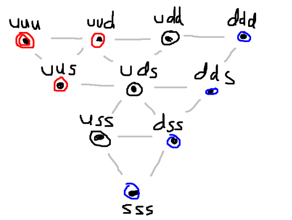
The Λ_0 singlet is a uds state, so shares the same quark content as the Σ^0 but has a different internal organisation of those quarks. The light $J^P = \frac{1}{2}^+$ baryons are therefore organised into an octet comprising: two Ξ baryons, two nucleons, three Σ baryons and the Λ^0.



The $\frac{1}{2}^+$ octet of baryons made from triplets u, d and s quarks.

We can find out more about the masses of the constituent quarks by examining the masses of the composite baryons. The masses of the Σ baryons, with one strange quark, are around 1200 MeV. The masses of the Ξ baryons with two such quarks are about 1300 MeV. The proton and neutron have masses close to 940 MeV. The baryon masses lead us to the conclusion that the strange quark mass must be of the order of 100 to 150 MeV. The u and d quark masses are so small that they are in fact very hard to measure. Almost all the rest mass energy of their host hadrons is tied up in the energy of the strong interaction field in which they reside.

The $J^P = \frac{3}{2}^+$ multiplet of u, d and s baryons contains ten different states – it is a decuplet. It is noticable that, unlike the lighter $\frac{1}{2}^+$ baryons, the $\frac{3}{2}^+$ multiplet includes states with the triplets of quarks of the same flavours.



The $\frac{3}{2}^+$ baryon decuplet.

4 Colour

A closer investigation of the $J = \frac{3}{2}$ baryons shows an interesting problem when we consider the symmetry – under exchange of labels – of the three quarks in the uuu, ddd and sss baryons. The problem will be found to be resolved when we consider the 'charges' of the quarks under the strong force that bind them together.

The quarks in these baryons are identical fermions, so from the spin statistics theorem, the state-vector $|\psi\rangle$ should be antisymmetric under interchange of any pair of labels:

$$|\psi(1,2,3)\rangle = -|\psi(2,1,3)\rangle \quad \text{etc.}$$

Let's test this taking the Δ^{++} baryon as an example. The state vector must describe the spin, the spatial wave-function, and the flavour. If the separate parts of the state vector can be written as a direct product, then we might expect

$$|\psi_{\rm trial}\rangle \overset{??}{=} |\psi_{\rm flavour}\rangle \times |\psi_{\rm space}\rangle \times |\psi_{\rm spin}\rangle. \tag{1}$$

Spin statistics

Given a system of particles, the state vector $|\psi\rangle$ must be symmetric under interchange of labels of any pair of identical bosons. It must be anti-symmetric under interchange of labels of any pair of identical fermions.

Let us examine the exchange symmetry of each part of (1) in turn, taking the example of the the Δ^{++} baryon.

The Δ^{++} is composed of three up-type quarks, so we expect that

$$|\psi_{\rm flavour}\rangle = |u_1\rangle|u_2\rangle|u_3\rangle.$$

The flavour part of the state vector is symmetric under interchange of any pair of labels.

The spin of the Δ baryons is $\frac{3}{2}$, which means that the *spin part* of its state vector must also be symmetric under interchange of labels. For example the $m = \frac{3}{2}$ spin state can be written in terms of the quark spins as

$$|s = \frac{3}{2}, m_s = \frac{3}{2}\rangle \;=\; |\uparrow_1\rangle|\uparrow_2\rangle|\uparrow_3\rangle$$

which is symmetric under exchange of any pair of labels. The three other $s = \frac{3}{2}$ states (which have $m_s = \frac{1}{2}, -\frac{1}{2}$ and $-\frac{3}{2}$) can be created from $|\frac{3}{2}, \frac{3}{2}\rangle$ using the lowering operator

$$\hat{S}_- = \hat{S}_{1-} + \hat{S}_{2-} + \hat{S}_{3-}$$

which is also symmetric under interchange of any pair of labels. This means that all of the $s = \frac{3}{2}$ states have a spin part which is symmetric under interchange of any pair of labels.

The space part of the state vector is also symmetric under interchange of any pair of quark labels, since for this 'ground state' baryon all of the quarks are in the lowest-lying $l = 0$ state. The result is that $|\psi_{\rm trial}\rangle$ is overall symmetric under interchange of any pair of labels of quarks, and does *not* satisfy the spin statistics theorem. Something is wrong with equation (1).

The resolution to this dilemma is that there must be some other contribution to the state vector which is anti-symmetric under interchange of particles. What is missing is the description of the strongly interacting charges – also known as the 'colour'.

To describe the baryon state we need to extend the space of our quantum model to include a colour part $|\psi_{\rm colour}\rangle$ to the state vector,

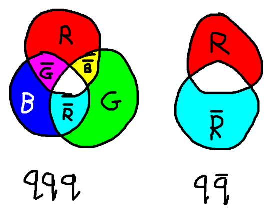
$$\boxed{|\psi_{\rm baryon}\rangle = |\psi_{\rm flavour}\rangle \times |\psi_{\rm space}\rangle \times |\psi_{\rm spin}\rangle \times |\psi_{\rm colour}\rangle.} \tag{2}$$

The flavour, space and spin parts remain symmetric under interchange of any pair of labels, provided that the colour part is totally antisymmetric under interchange.

We can arrange for total antisymmetry by using a determinant[2]

$$|\psi_{\rm colour}\rangle = \frac{1}{\sqrt{6}} \left\| \begin{pmatrix} r_1 & g_1 & b_1 \\ r_2 & g_2 & b_2 \\ r_3 & g_3 & b_3 \end{pmatrix} \right\|, \tag{3}$$

which will change sign under interchange of any two rows – a procedure equivalent to swapping the corresponding labels.



Baryon and mesons are both colourless states.

For us to be able to build such a determinant we require that there must be three different colour charges, which we have labelled 'r', 'g' and 'b', following the convention that they are known as red, green and blue. The need for three such colour 'charges' has since been proven in very many other experimental measurements. The antisymmetric colour combination (3) is the only combination of three quark states that has no net colour.

Quarks carry colour, while anti-quarks carry anti-colour. The colour in the mesons is contained in quark-antiquark combinations in the superposition

$$|\psi^{\rm meson}_{\rm colour}\rangle = \frac{1}{\sqrt{3}} \left(|r\bar{r}\rangle + |g\bar{g}\rangle + |b\bar{b}\rangle \right)$$

which is also colourless.

The strongly interacting particles observed – the qqq baryons and the $q\bar{q}$ mesons have no net colour. Quarks, which do have net colour have never been observed in isolation.

$\boxed{\text{No coloured object has ever been observed in isolation.}}$

Quarks only occur within the colourless combinations consisting of three quarks qqq for baryons and a quark and an anti-quark $q\bar{q}$ for mesons.

The quarks are **confined** within hadrons by the strong force, and are unable to exist as free particles. If we attempt to knock a u quark out of a proton (for example by hitting the proton with a high-energy electron, as we shall discuss on page **??**) we do not observe a free u quark in the final state. Instead the struck u-quark uses part of its kinetic energy to create other $q\bar{q}$ pairs out of the vacuum, and joins together with them so that the final state contains only colour-neutral hadrons.

The particles that carry the strong force between quarks are known as **gluons**. Each gluon carries both colour and anti-colour. There are eight gluons, since of the nine possible othogonal colour–anti-colour combinations, one is colourless. We will later find (§**??**) that the fact that the gluon also carries colour charge itself makes the strong force very different from the electromagnetic force, which is mediated by neutral photons.

[2] This can be compared to the more familiar case of the two-particle spin state

$$|\psi(S=0)\rangle = \frac{1}{\sqrt{2}} \left\| \begin{pmatrix} \uparrow_1 & \downarrow_1 \\ \uparrow_2 & \downarrow_2 \end{pmatrix} \right\| = \frac{1}{\sqrt{2}} \left(|\uparrow_1\rangle|\downarrow_2\rangle - |\downarrow_1\rangle|\uparrow_2\rangle \right)$$

which has $S = 0$ and hence no net spin.

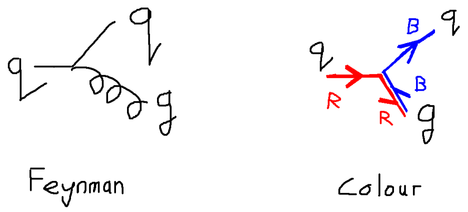


Figure 1: The diagram on the left shows the emission of a gluon from a quark. The right hand side shows a possible colour-flow. The gluon changes the colour of the quark, and itself carries both colour and anti-colour.

5 Heavier quarks

We have so far discussed hadrons made from three flavours of quarks, u, d and s. In fact these are only half of the total number which are found in nature. The full set of six quarks is as follows:

Name	Symbol	Charge	Mass [GeV]
down	d	-$\frac{1}{3}$	∼0.005
up	u	$+\frac{2}{3}$	∼0.003
strange	s	-$\frac{1}{3}$	0.1
charm	c	$+\frac{2}{3}$	1.2
bottom	b	-$\frac{1}{3}$	4.2
top	t	$+\frac{2}{3}$	172

It's very difficult to obtain good values for the masses of the light quarks, since they are always bound up inside much heavier hadrons.

It can be seen that the quarks only come in charges of $-\frac{1}{3}$ and $+\frac{2}{3}$. Their anti-quark partners have the opposite charges. It is useful to group the quarks into three generations, each containing a $+\frac{2}{3}$ and a $-\frac{1}{3}$ partner:

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix} \qquad \begin{matrix} \leftarrow Q = +\frac{2}{3} \\ \leftarrow Q = -\frac{1}{3} \end{matrix}$$

where the up and down form the first generation, the strange and charm quarks the second, and the top and bottom quarks form the third generation. The pairings are those favoured by the weak interaction (§**??**), and mean that (for example) when a t quark decays it does so dominantly to a b quark.

What further hadrons may we expect from these additional quarks? All these quarks – other than the top quark – form hadrons in both meson ($q\bar{q}$) and baryon (qqq) combinations. The top quark is so heavy that it decays almost immediately, before it can form hadrons. An example of a charmed meson is the $c\bar{d}$ state with $J = 0$ known as the D^+ meson. Similarly there are mesons containing b quarks, such as the $b\bar{b}$ meson known as the Υ.

We can put these quarks and anti-quarks together to form colourless hadrons in any qqq or $q\bar{q}$ flavour combinations we choose, so long as we ensure that the final state-vector is antisymmetric with respect to exchange of labels of any identical quarks. So for example valid combinations are:

$$cds,\ b\bar{u},\ c\bar{c},\ uud,\qquad \text{etc.}$$

6 Charmonium

The charm quark was first discovered in $c\bar{c}$ bound states. These 'charmonium' mesons are interesting because bound states of heavy quarks can tell us about the properties of the strong nuclear force which binds them.

The hadrons containing only the lightest quarks, u, d and s, have masses that tell us only a little about the mass of their constituent quarks. Most of the energy of the lightest baryons and mesons is stored in the strong-interaction field.

The charm quark (and to an even greater extent the bottom quark) is sufficiently heavy that mesons containing $c\bar{c}$ combinations are dominated by the mass of the constituent quarks. The energy in the field is now a relatively small correction to the rest mass energy of the baryons, and the whole two-particle system can be reasonably well described by non-relativistic quantum mechanics. If we model the system as a two-body quantum system, with reduced mass $\mu = m_c/2$ then we can write down the Schrödinger equation for the energy eigenstates of the system,

$$\left(\frac{P^2}{2\mu} + V\right)|\Psi\rangle = E|\psi\rangle.$$

The potential V due to the strong force between a quark and its anticolour partner is well described by the function

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \frac{r}{a^2}, \tag{4}$$

The first term is the strong-force equivalent to the Coulomb potential. The electromagnetic fine structure constant (α) has replaced by the strong-force constant α_s, and the factor of $4/3$ has its origin in the three colour 'charges' rather than the single one electromagnetic charge. The term linear in r means that V continues growing as $r \rightarrow \infty$. It is this linear term that leads to quark confinement, since an infinite amount of energy would be required to separate the quarks to infinity.

$$E_n = -\frac{\mu\alpha^2}{2n^2}$$

Hydrogen atomic energy levels.

For $c\bar{c}$ or 'charmonium' mesons, the typical separation r is rather smaller than a. In these states the linear term can be neglected, and the potential takes the $1/r$ form familiar from atomic physics. We then expect that the energy eigenstates should follow the pattern of the hydrogenic states.

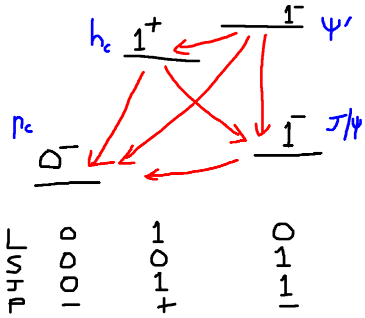


Figure 2: Some of the lowest-lying charmonium ($c\bar{c}$) states. Radiative transitions between states are indicated by arrows.

The energy levels should then be given by the strong-force equivalent of the hydrogenic energies:

$$E_n = -\frac{\mu}{2n^2}\left(\frac{4}{3}\alpha_s\right)^2. \tag{5}$$

Therefore we expect to see charmonium states with energies equal to $2m_c + E_n$. The observed charmonium spectrum bears out these predictions (Figure 2).

The lowest-lying state again has $L = S = 0$, and hence $J = 0$ and parity $(+1)(-1)(-1)^L = (-1)^{L+1} = -1$. This state is labelled η_C in Figure 2.

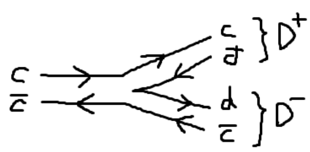
$n\,^{2S+1}L_J$	j^P	Name
1^1S_1	0^-	η_c
1^1L_1	1^+	h_c
1^3S_1	1^-	J/Ψ
2^3S_1	1^-	Ψ'

Some charmonium states and their quantum numbers

The first meson to be discovered was not the lightest one η but the slightly heavier J/Ψ. The J/Ψ has spin 1 and negative parity resulting from $S = 1$ and $L = 0$. These are exactly the right quantum numbers to allow it to be made in electron-positron collisions, via an intermediate (virtual) photon, since the photon also has quantum numbers $J^P = 1^-$

$$e^- + e^+ \to \gamma^* \to J/\psi.$$

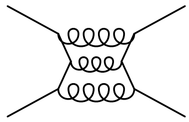
The transitions in the plot indicate possible electromagnetic transitions between charmonium states. Measurement of the gamma-ray photon energies allows us to make precision measurement of mass differences, and hence of the predictions of (5). Charmonium states which are heavier than $2m_D$ can decay rapidly via the strong force to either a D^0 and a $\bar{D}^0$ meson or to final state consisting of a D^+ and a D^- meson.



Those charmonium states which are lighter than $2m_D$ cannot decay to a pair of charmed D mesons. Instead the charm and anti-charm quarks must annihilate either via the electromagnetic force, or via a suppressed version of the strong interation. [3] This unusally suppressed strong decay is known as an 'OZI suppression' and is a feature of decays in which the intermediate state consists only of gluons. The

[3]The reason for this suppression is that a single-gluon intermediate state cannot be colourless, so is forbidden. A two-gluon final state has positive parity under the charge conjucation operator so is forbidden for any state, such as the J/ψ, which is negative under charge conjugation. Hence a three-gluon intermediate state is required.

$c\bar{c}$ states with $m < 2m_D$ are therefore unusually long-lived and are visible as very narrow resonances with masses close to 3 GeV.



The 'OZI-suppressed' decay of the J/ψ proceeds via a-three gluon intermediate state.

We can go further and use the difference between the 1S and the 2S levels to find out the size of the strong 'fine structure constant'. It is found that α_s is much larger than for the electromagnetic case – in fact close to unity.

$$\boxed{\alpha_s \approx 1}$$

This much larger value of α_s compared the electromagnetic fine structure constant $\alpha \approx \frac{1}{137}$ is a reflection of the relative strengths of the two forces.

The 'bottomonium' ($b\bar{b}$) system of mesons are the corresponding set of hydrogenic states for the bottom quark. They lead to sharp resonances close to $2m(b) \approx 10$ GeV.

7 Hadron decays

The strong interaction allows reactions and decays in which quarks are interchanged between hadrons, but there is no change of net quark flavour. For example we saw in §3.3 that strong decays such as

$$\underset{udd}{\Delta^+} \rightarrow \underset{udd}{n} + \underset{u\bar{d}}{\pi^+},$$

conserve net quark content. The strong decays occur very rapidly, typically occur over lifetimes of order 10^{-22} s.

Electromagnetic interactions do not change quark flavour either. Therefore if overall quark flavour is changed, for example in the strangeness-violating reactions,

$$\underset{u\bar{s}}{K^+} \rightarrow \underset{u\bar{d}}{\pi^+} + \underset{u\bar{u},d\bar{d}}{\pi^0} \qquad [\Delta S = -1]$$

$$\underset{dds}{\Sigma^-} \rightarrow \underset{ddu}{n} + \underset{u\bar{d}}{\pi^-} \qquad [\Delta S = +1]$$

a **weak** interaction must involved.

Only **weak** interactions can change quark flavour.

Weak decays are suppressed by the Fermi coupling constant, and so weakly decaying particles are characterised by much longer lifetimes, of order 10^{-10} s. This may seem like a short life, but is twelve orders of magnitude much longer than typical strong decays.

Decay	Typical lifetime
Strong	10^{-22} s
EM	10^{-18} s
Weak	10^{-10} s

Examples of other weak decays include the decay of the charged pion to a muon[4] and an associated neutrino

$$\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$$

[4] As we will see in §?? the muon is a fundamental particle with electric charge but no strong interactions – like an electron electron but heavier.

and the beta decay of a neutron.

$$n \to p + e^- + \bar{\nu}_e$$

The neutron is unusually long-lived even for a weak decay ($\tau = 881\,\mathrm{s}$). The long life is due to the closeness in mass between the neutron and the proton, which results in a small density of states for the decay products (recall the $\Gamma \propto Q^5$ rule in §**??**).

Electromagnetic decays have typical lifetimes intermediate between those of strong and weak decays. For example the electromagnetic decay

$$\pi^0 \to \gamma + \gamma$$

has a lifetime of $8 \times 10^{-17}\,\mathrm{s}$.

Hadron lifetimes

The lifetime of hadron states depends on the force by which they decay. Typical lifetimes are as follows:

Force	Typical lifetime	Example
Strong	$10^{-22}\,\mathrm{s}$	$\Delta^- \to \pi^+ + p$
Electromagnetic	$10^{-18}\,\mathrm{s}$	$\pi^0 \to \gamma + \gamma$
Weak	$10^{-10}\,\mathrm{s}$	$K^+ \to \pi^0 + \pi^+$

Where more than one decay mode is possible, decay modes with much very small rates are often unobserved. For example consider the two baryons in the $J^P = \frac{1}{2}^+$ multiplet with quark content uds:

$$\begin{array}{ll} \Lambda^0 & (1115.7\text{ MeV}) \\ \Sigma^0 & (1192.6\text{ MeV}) \end{array}$$

The Λ^0 is the lightest neutral strange baryon. Strangeness-conserving decays to other hadrons e.g. $p + K^-$ are kinematically forbidden, hence the only way the Λ^0 can decay is via the strangeness-violating weak decays:

$$\begin{array}{lcll} \Lambda^0 & \to & p + \pi^- & (64\%) \\ \Lambda^0 & \to & n + \pi^0 & (36\%) \end{array}$$

The lifetime of the Λ^0 is therefore relatively long by subatomic stanards – $\tau \approx 2.6 \times 10^{-10}\,\mathrm{s}$.

By contrast the heavier Σ^0 is decays electromagnetically to the Λ^0 with a lifetime of $7 \times 10^{-20}\,\mathrm{s}$:

$$\Sigma^0 \to \Lambda^0 + \gamma$$

Since $\Gamma_{\mathrm{EM}} \gg \Gamma_{\mathrm{Weak}}$ the weak decay mode of the Σ^0 is not observed.

Key concepts

- Strongly interacting objects are composed of point-like spin-half objects called **quarks** q

- Quarks come in three strong-charges, r, g, b known as **colours**
- There are six different **flavours** of quark in three **generations**

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$

  each containing a $+2/3$ and a $-1/3$ charged partner.
- **Anti-quarks** $\bar{q}$ have the opposite charges and colours to their respective quarks
- The quarks are **confined** in the 'colourless' combinations called hadrons
- **Mesons** are colourless $q\bar{q}$ combinations
- **Baryons** are colourless qqq combinations

.A Isospin §

Non-examinable

We can get extra insight into the meson and baryon combinations using the concept of **isospin**. The name 'isospin' is used in analogy to the spin, since the algebra of the isospin states has the same structures as the angular momentum states of quantum mechanics. However isospin is completely separate from angular momentum – it is simply an internal quantum number of the system that tells us about the quark content.

Let us consider the u and d quarks to be the isospin-up and isospin-down states of an isospin-half system.

The quantum number I is the total isospin quantum number, with $I = \frac{1}{2}$ for the nucleon doublet. The third component of isospin, I_3 distinguishes the proton with $I_3 = \frac{1}{2}$ from the neutron with $I_3 = -\frac{1}{2}$. These are analogous to the quantum numbers s and m_s which label the eigenstates of the angular momentum operators S^2 and S_z.

We can label the quark states with their quantum numbers $|I, I_3\rangle$. The $|u\rangle$ and $|d\rangle$ quarks form a $|\frac{1}{2}, \pm\frac{1}{2}\rangle$ isospin doublet:

$$\begin{pmatrix} |u\rangle \\ |d\rangle \end{pmatrix}$$

as do the the antiquarks

$$\begin{pmatrix} -|\bar{d}\rangle \\ |\bar{u}\rangle \end{pmatrix}.$$

The minus sign in front of the $|\bar{d}\rangle$ state ensures that the anti-quark doublet has the correct transformation properties.

The ladder operators $I_\pm$ change the third component of isosipin

$$I_- |u\rangle = |d\rangle$$
$$I_+ |d\rangle = |u\rangle$$

Similarly the ladder operators act on the anti-quarks

$$I_-|\bar{d}\rangle = -|\bar{u}\rangle$$
$$I_+|\bar{u}\rangle = -|\bar{d}\rangle.$$

Using the ladder operators we can generate the other pion states from the π^+:

$$\begin{aligned} I_-|\pi^+\rangle &= I_-|u\bar{d}\rangle \\ &= |d\bar{d}\rangle - |u\bar{u}\rangle = \sqrt{2}\,|\pi^0\rangle \end{aligned}$$

Operating again with I_- will generate the state $|\pi^-\rangle = |d\bar{u}\rangle$. The three pions $\{\pi^+,\, \pi^0,\, \pi^-\}$ form a $I = 1$ triplet with $I_3 = \{+1,\, 0,\, -1\}$.

The $|0, 0\rangle$ state is the linear combination of $|u\bar{u}\rangle$ and $|d\bar{d}\rangle$ that is othogonal to $|\pi^0\rangle$,

$$|0, 0\rangle = \tfrac{1}{\sqrt{2}}(|d\bar{d}\rangle + |u\bar{u}\rangle).$$

This is the state of the η meson. Quarks other than the u and d do not carry isospin.

.B Discovery of the Omega §

Non examinable

The triply strange Ω^- baryon was discovered in the set of decays shown in Figure 3.

The weak interaction is the only interaction that can change quark flavour, so strange hadrons can and do travel macroscopic distances before they decay.

In the figure the production of the Ω^- was from the interaction of a negatively charged beam of kaons onto the hydrogen target:

$$K^- + p \;\to\; \Omega^- + K^+ + K^0.$$

and found through its three sequential weak decays:

$$\begin{aligned} \Omega^- &\to \Xi^0 + \pi^- \\ \Xi^0 &\to \Lambda^0 + \pi^0 \\ \Lambda^0 &\to p + \pi^-. \end{aligned}$$

Only the charged particles create tracks of bubbles in the chamber. The presence of the neutral pion can be inferred due to a happy accident. The π^0 particle almost always decays to a pair of photons $\pi^0 \to \gamma + \gamma$. Unusually, both of the photons produced in the pion decay have converted into $e^+ + e^-$ pairs $\gamma \to e^+ + e^-$ in the presence of the atomic nuclei, leaving vee-shaped bubble tracks.

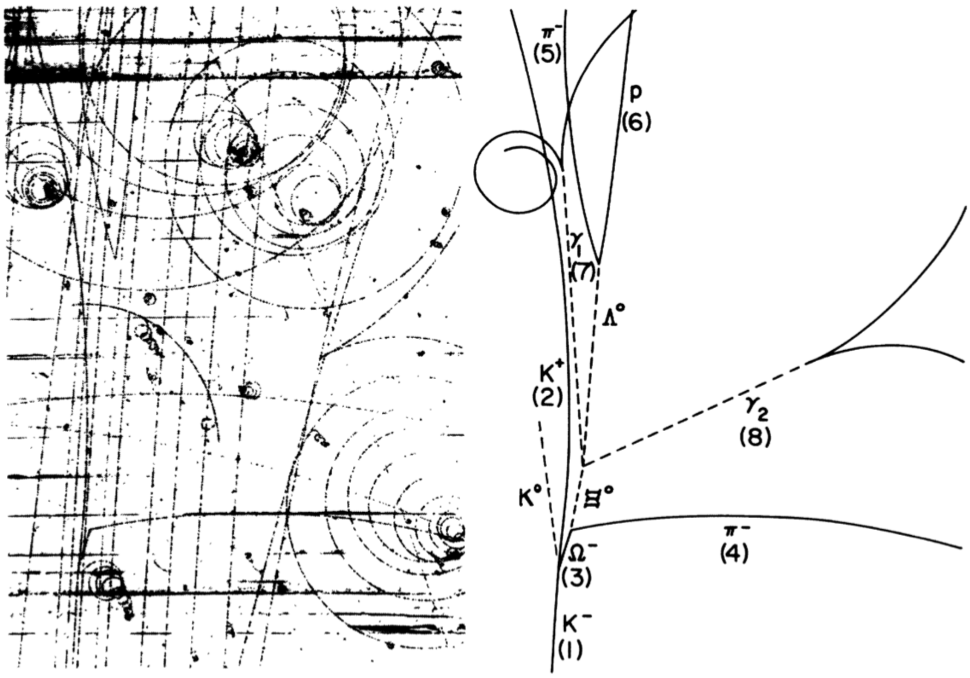


Figure 3: Bubble chamber photograph and line drawing showing the discovery of the Ω^- baryon. From [Barnes et al.(1964)].

Further reading

- B. Martin, *Nuclear and Particle Physics: An Introduction*
- W. S. C. Williams, *Nuclear and Particle Physics*
- K.S. Krane *Introductory Nuclear Physics*

References

[Barnes et al.(1964)] V. E. Barnes *et al.*, "Observation of a hyperon with strangeness minus three", *Phys. Rev. Lett.* **12** Feb (1964) 204–206.

