SECOND PUBLIC EXAMINATION

Honour School of Physics – Part B: 3 and 4 Year Courses

Honour School of Physics and Philosophy Part B

B2 IV: Subatomic physics

45 minutes per question

EXAMPLE QUESTIONS

Start the answer to each question on a fresh page.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners expect to assign to each part of the question.

Do NOT turn over until told that you may do so.

1. The semi-empirical mass formula for the mass of a nucleus $M(A,Z)$ is

$$M(A,Z)c^2 = m_\mathrm{p}c^2 Z + m_\mathrm{n}c^2(A-Z) - \alpha A + \beta A^{\frac{2}{3}} + \frac{\epsilon Z^2}{A^{\frac{1}{3}}} + \gamma\frac{(A-2Z)^2}{A} + \delta.$$

Give a brief justification of the form of the term involving ϵ in terms of physical properties of the model of the nucleus including the assumptions used to estimate the nuclear radius. [5]

For constant A, show that the most stable isotope is given by

$$Z \simeq \frac{A}{2}\left(1 + \frac{\epsilon}{4\gamma}A^{\frac{2}{3}}\right)^{-1}.$$

[5]

The following table lists properties of the nuclides with $Z = 17$ (chlorine) and $Z = 95$ (americium). The most prominent observed decay modes and their Q values are given in the table (energy from γ radiation following the disintegrations shown are not given; EC=Electron capture).

	Half life or abundance	Decay modes and energies (MeV)		Half life or abundance	Decay modes and energies (MeV)
^{32}Cl	0.31 s	β^+ 10, 8	^{237}Am	4700 s	EC; α 6.01
^{33}Cl	2.5 s	β^+ 4.5	^{238}Am	6700 s	EC
^{34}Cl	1.5 s	β^+ 4.5	^{239}Am	4.32×10^5 s	EC; α 5.77
^{35}Cl	Stable 76% abundance		^{240}Am	1.84×10^6 s	EC
^{36}Cl	9.5×10^{12} s	β^- 0.71; EC	^{241}Am	1.45×10^{11} s	α 5.48
^{37}Cl	Stable 24% abundance		^{242}Am	5760 s	β^- 0.63; EC; α
^{38}Cl	2240 s	β^- 4.8, 1.1, 2.8	^{243}Am	2.41×10^{12} s	α 5.27, 5.22, 5.17
^{39}Cl	3300 s	β^- 1.91, 2.18, 3.43	^{244}Am	2.60×10^4 s	β^- 0.39
^{40}Cl	84 s	β^- 3.2, 7.5	^{245}Am	7450 s	β^- 0.91
			^{246}Am	1500 s	β^- 1.31, 1.60, 2.10

Explain why

(a) there is a general trend for the middle of the group to be more stable and that the type of β decay changes between the top and bottom half of each table.

(b) it is possible for ^{36}Cl and ^{242}Am to decay by both β^- and electron capture. [6]

Discuss why electron capture is more prevalent for americium decays than for chlorine as shown in the table. [4]

Domestic smoke detectors use ^{241}Am to ionise air. In clean air the ionisation generates a steady ion current in the presence of a small potential difference. The presence of fire is inferred by the capture of ions by smoke in the air which reduces the ionisation current. Estimate the order of magnitude of the number of electron-ion pairs a single alpha particle might create. Could a single alpha particle be measurable? Hence roughly estimate the amount of ^{241}Am required in such detectors. [5]

[2008 B1 Q3 modified]

2. A particle of mass m and wavevector $\mathbf{k}$ is incident on a real potential $V(\mathbf{r})$ that is negligible outside a finite region $|\mathbf{r}| < R$ around the origin. The diffracted wave function $\psi_k(\mathbf{r})$ satisfies the equation

$$\Psi_k^{\pm}(\mathbf{r}) = e^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} - \frac{m}{2\pi\hbar^2}\int \frac{e^{\pm \mathrm{i}k|\mathbf{r}-\mathbf{r}'|}}{|\mathbf{r}-\mathbf{r}'|} V(\mathbf{r}')\Psi_k^{\pm}(\mathbf{r}')\,\mathrm{d}^3\mathbf{r}',$$

where $k = \sqrt{\mathbf{k}^2}$. Show that when $|\mathbf{r}| \gg R$, the wavefunction $\psi_k(\mathbf{r})$ consists of a plane wave and a spherical wave of amplitude

$$f(\mathbf{k}', \mathbf{k}) = -\frac{m}{2\pi\hbar^2}\int e^{-\mathrm{i}\mathbf{k}'\cdot\mathbf{r}'} V(\mathbf{r}')\Psi_k^{\pm}(\mathbf{r}')\mathrm{d}^3\mathbf{r}',$$

where $\mathbf{k}' = k\,\mathbf{r}/|\mathbf{r}|$. [8]

Determine the differential cross section $\mathrm{d}\sigma(\theta)/\mathrm{d}\Omega$, where θ is the scattering angle in the laboratory frame, in the Born approximation for the case $V(\mathbf{r}) = e^{-\mu\,|\mathbf{r}|}$. [10]

Estimate the accuracy of this result in the limit $k \ll \mu$. [7]

[2008 S18 Q1]

3. The concept of an exchange particle may be used to describe forces between particles. Describe how the properties of the force change depending on whether the exchange particle has mass. [3]

Use a Feynman diagram to show how $e^+e^- \to \mu^+\mu^-$ may proceed through γ exchange. What factors appear in the expression for the cross section of the above reaction from each vertex and from the γ propagator? [3]

(a) At around 10 GeV, the cross section of e^+e^- collisions goes through three narrow resonances. Account for these peaks and their unusual widths. Calculate the ratio of cross sections R

$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

at an energy just below these peaks. [6]

(b) As the beam energy is raised to suitably high energies, the reaction $e^+e^- \to \mu^+\mu^-$ may also proceed through Z-boson exchange. Neglecting any differences in vertex factors, at what beam energy (below M_Z) does the contribution to the cross section from the γ exchange diagram become equal to the contribution from the Z exchange diagram? What contribution comes from interference between the two diagrams at this energy? [6]

(c) For each of the three resonances near 10 GeV, the total width is found to be equal to the sum of the partial widths of the individual measured final states (hadrons, e^+e^-, $\mu^+\mu^-$ and $\tau^+\tau^-$) whereas for the Z-boson resonance, this is not true. Account for this difference. [2]

(d) Describe how the momentum of the $\mu^\pm$ could be measured experimentally. [5]

[2006 B1 Q5 modified]

4. An alpha decay proceeds from an unstable parent nucleus to the daughter nucleus D and an α-particle of kinetic energy Q (neglecting recoil of D). Draw a diagram of the potential energy $V(r)$ as a function of the distance r between D and the α particle. Use your diagram to explain the mechanism by which the alpha particle escapes from the nucleus. [6]

By using a one-dimensional approximation to the Hamiltonian

$$H = \frac{p_x^2}{2m} + V,$$

and by representing the wave function by

$$\langle x|\Psi\rangle = \exp[\eta(x)],$$

show that the decay rate $\omega = fP$, where f is the frequency of attempts by the alpha particle to escape, $P = \exp(-2G)$ is the probability for the alpha particle to escape on a given attempt, and the Gamow factor G is

$$G = \frac{\sqrt{2m}}{\hbar}\int_a^b \sqrt{V(r) - Q}\,\mathrm{d}r.$$

[8]

- Sketch the form of the solutions to the Schrödinger equation in each distinct region of $V(r)$ shown on your diagram.
- Indicate how the limits of integration a and b depend on properties of the strong and/or electromagnetic forces. Give an approximate value for a and an expression for b. Define any symbols you use. [5]

Evaluating the above expression for ω yields an approximate expression for the decay rate of the form

$$\ln\omega = C_1 + \frac{C_2}{\sqrt{Q}},$$

where C_1 and C_2 are constants. ^{232}Th has half life 1.4×10^{10} years and has $Q = 4.08\,\mathrm{MeV}$, while ^{218}Th has half life 100 ns and $Q = 9.85\,\mathrm{MeV}$. Estimate the half life of ^{226}Th, which has $Q = 6.45\,\mathrm{MeV}$. [6]

[2007 B1 Q4, modified]