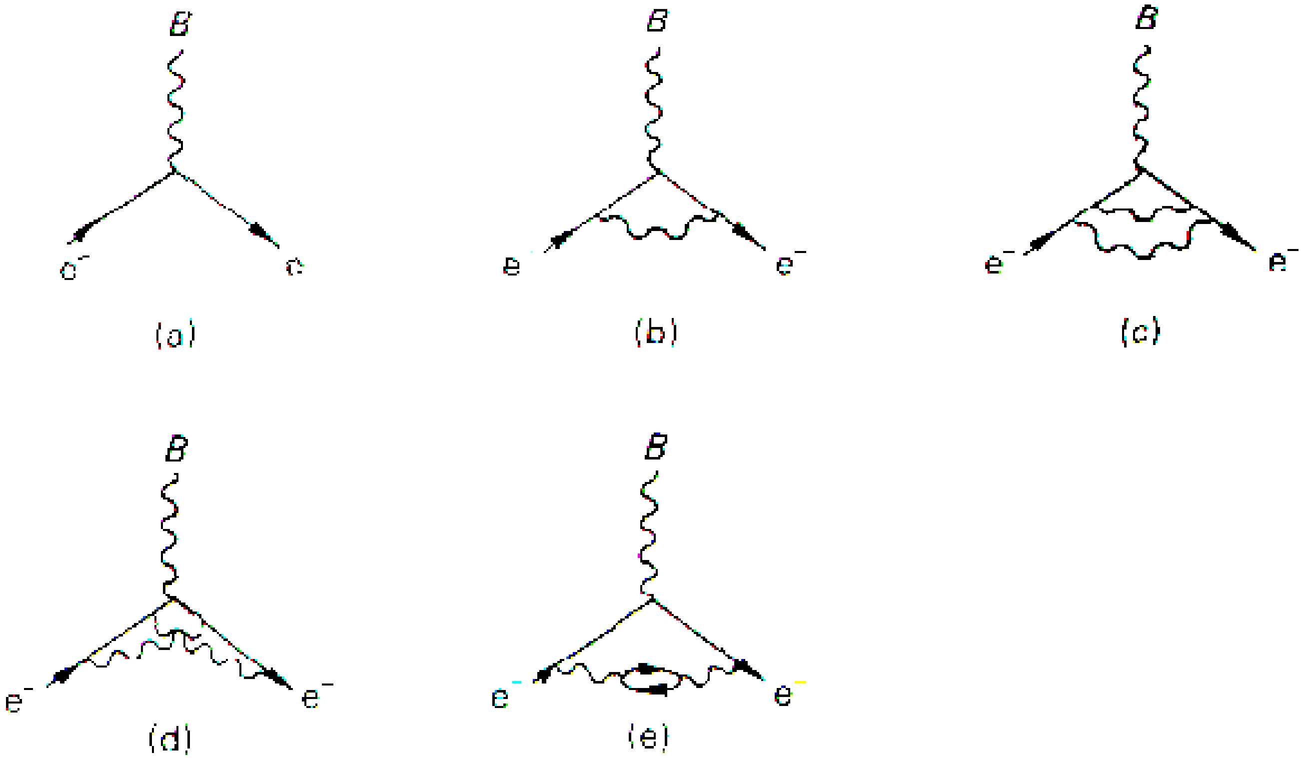
Lecture 5: QED, Symmetry

- The Bohr Magneton
- Off-Shell Electrons
- Vacuum Polarization
- Divergences, Running Coupling & Renormalization
- Symmetry & Unifying Electricity/Magnetism
- Space-Time Symmetries
- Gauge Invariance in Electromagnetism
- Noether's Theorem

Useful Sections in Martin & Shaw:

Section 4.1, Section 4.2, Section 7.1.2

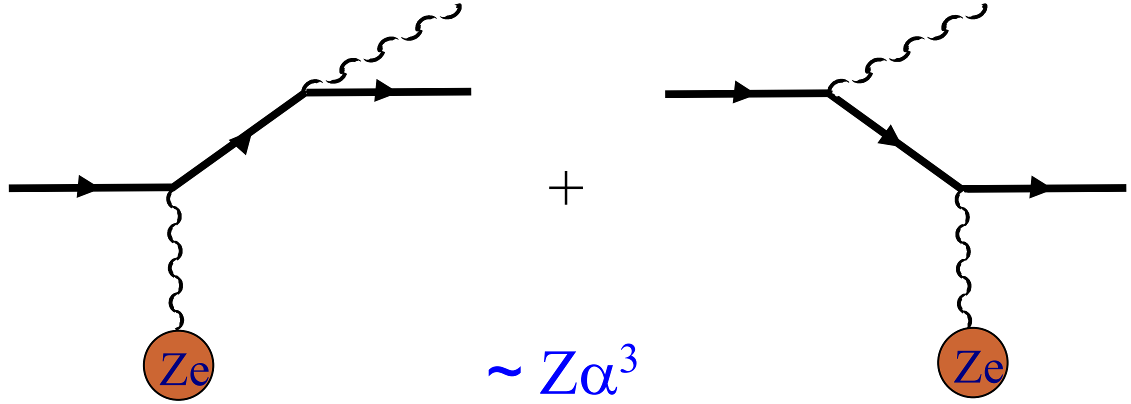
Bohr Magneton



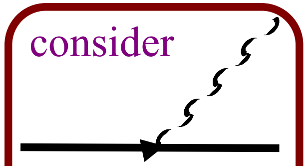
$$1+\tfrac{1}{2}(\alpha/\pi)-0.328478966(\alpha/\pi)^2+1.1765(\alpha/\pi)^3-0.8(\alpha/\pi)^4$$
$$=1.001\,159\,652\,307(110).$$

Bremsstrahlung

Internal electron line since real electron cannot emit a real photon and still conserve energy and momentum:



consider



in electron rest frame

$$m_e = m_e' + p_e'^2/2m_e' + E_\gamma \qquad \text{(conservation of energy)}$$

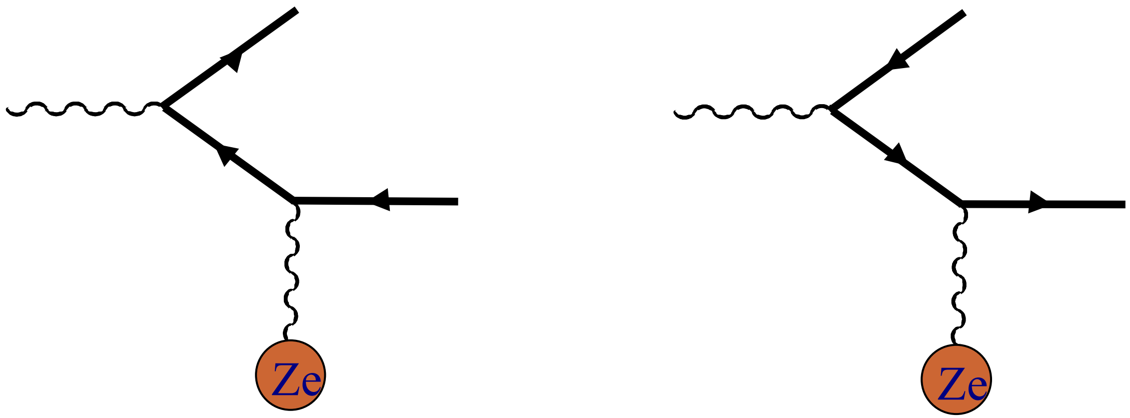
$$|p_e'| = |p_\gamma| = E_\gamma \qquad \text{(conservation of momentum)}$$

$$\Rightarrow \quad m_e - m_e' = E_\gamma^2/2m_e' + E_\gamma$$

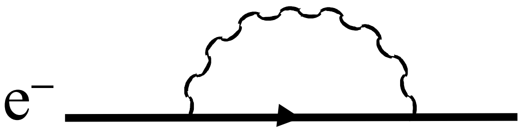
So, for a positive energy photon to be produced, the electron has to effectively "lose mass" $\Rightarrow$ virtual electron goes "off mass shell"

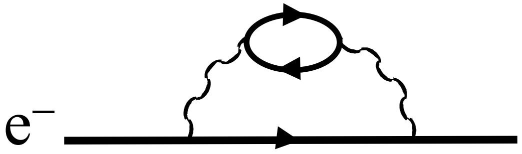
Pair Production

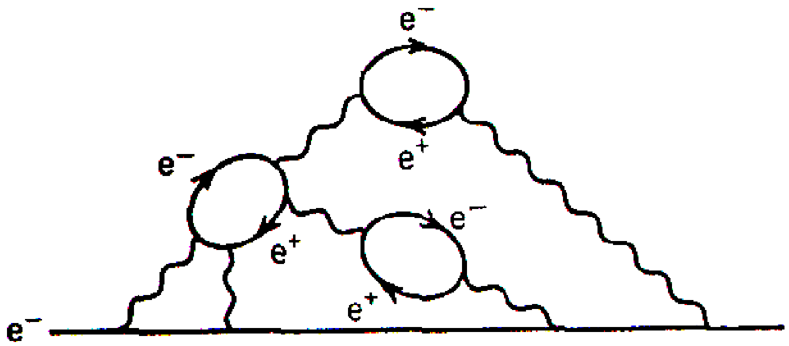
"twisted Bremsstrahlung"
Matrix element will be the same; Cross-section will only differ by a kinematic factor



Vacuum Polarization

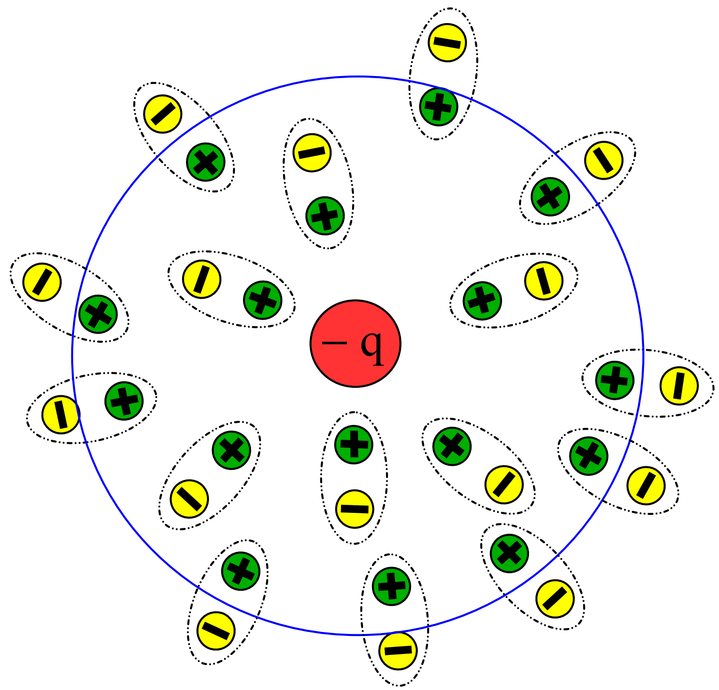






Thus, we never actually ever see a "bare" charge, only an effective charge shielded by polarized virtual electron/positron pairs. A larger charge **(or, equivalently, α)** will be seen in interactions involving a high momemtum transfer as they probe closer to the central charge.

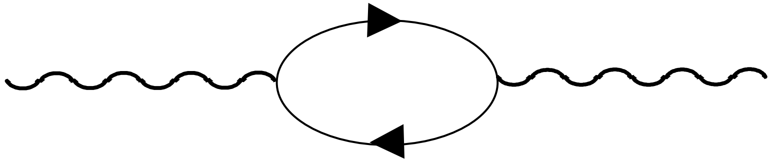
$\Rightarrow$ "running coupling constant"



In QED, the bare charge of an electron is actually infinite !!!

Note: due to the field-energy near an infinite charge, the bare mass of the electron ($E=mc^2$) is also infinite, but the effective mass is brought back into line by the virtual pairs again !!

Another way...



Consider the integral corresponding to the loop diagram above:

At large q the product of the propagators will go as $\sim 1/q^4$

Integration is over the 4-momenta of all internal lines
where $d^4q = q^3\, dq\, d\Omega$ (just like the 2-D area integral is $r\, dr\, d\theta$ and the 3-D volume integral is $r^2\, dr\, d\cos\theta\, d\phi$)

so the q integral goes like:

$$\int_{q_{min}}^{\infty} (1/q^4)\, q^3\, dq = \int_{q_{min}}^{\infty} dq/q = \ln q \Big]_{q_{min}}^{\infty}$$

$\Rightarrow$ logarithmically divergent !!

With some fiddling, these divergences can always be shuffled around to be associated with a bare charge or mass. but we only ever measure a "**dressed**" (screened) charge or mass, which are finite. Thus, we "**renormalize**" by replacing the bare values of α_0 and m_0 by running parameters.

Analogy for Renormalization:

Instead of changing the equations of motion, you could instead (in principle) find the "effective" mass of the cannonball by shaking it back and forth in the water to see how much force it takes to accelerate it. This "mass" would no longer be a true constant as it would clearly depend on how quickly you shake the ball.

Still unhappy? Well, if it’s any comfort, note that the electrostatic potential of a classical point charge, $\mathbf{e^2/r}$, is also infinite as $\mathbf{r \to 0}$

(perhaps this all just means that there are really no true point particles... **strings?? something else??**)

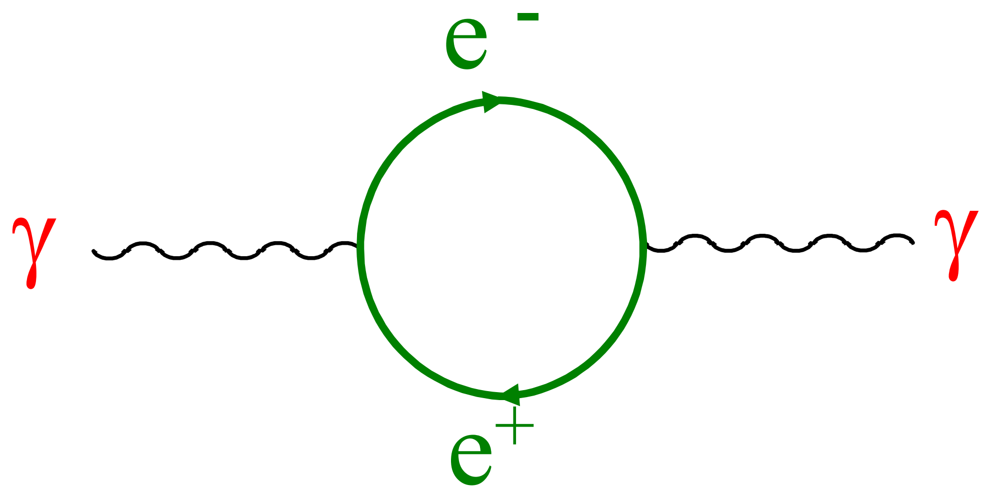
Does an electron feel it’s own field ???

"Ven you svet, you smell you’re own stink, Ja? Zo..."

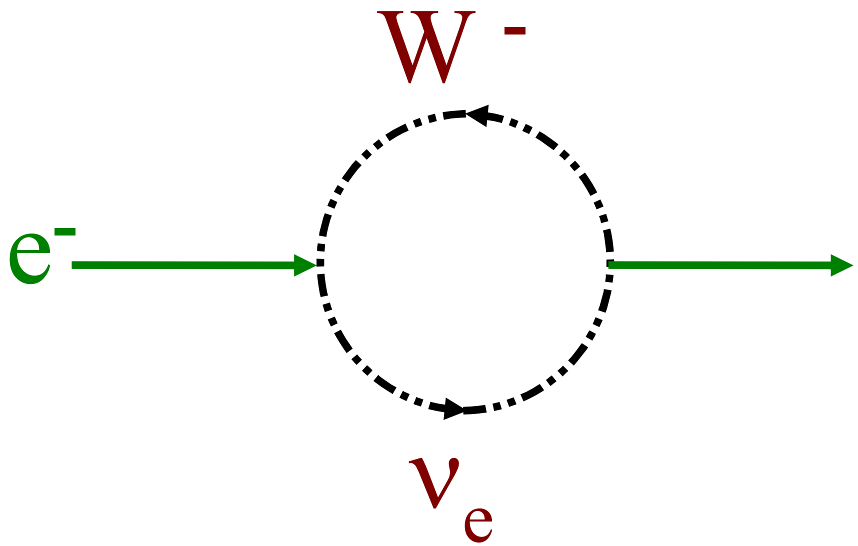
M. Veltman

Steve's Tips for Becoming a Particle Physicist

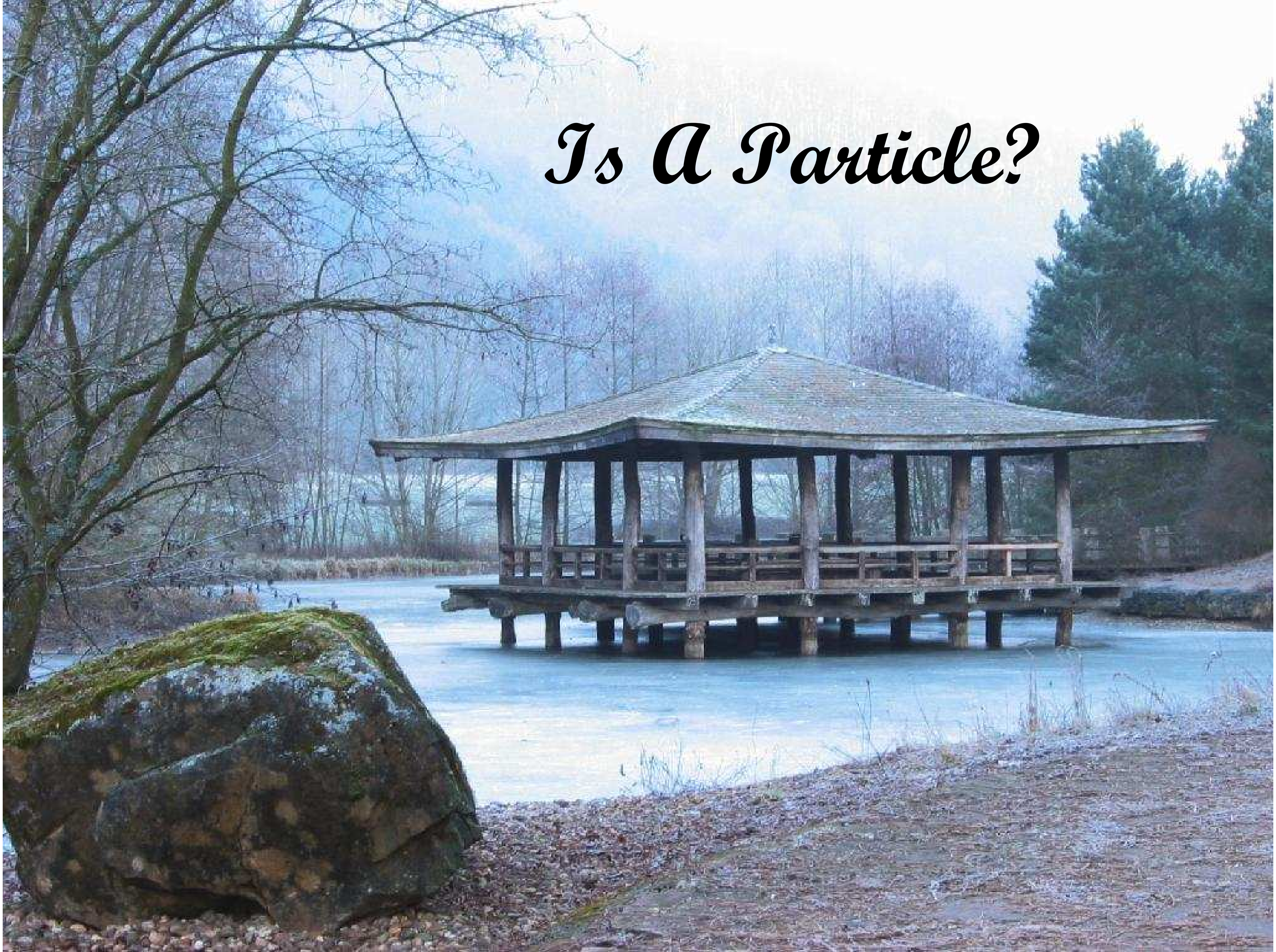
1) Be Lazy
2) Start Lying
3) Sweat Freely



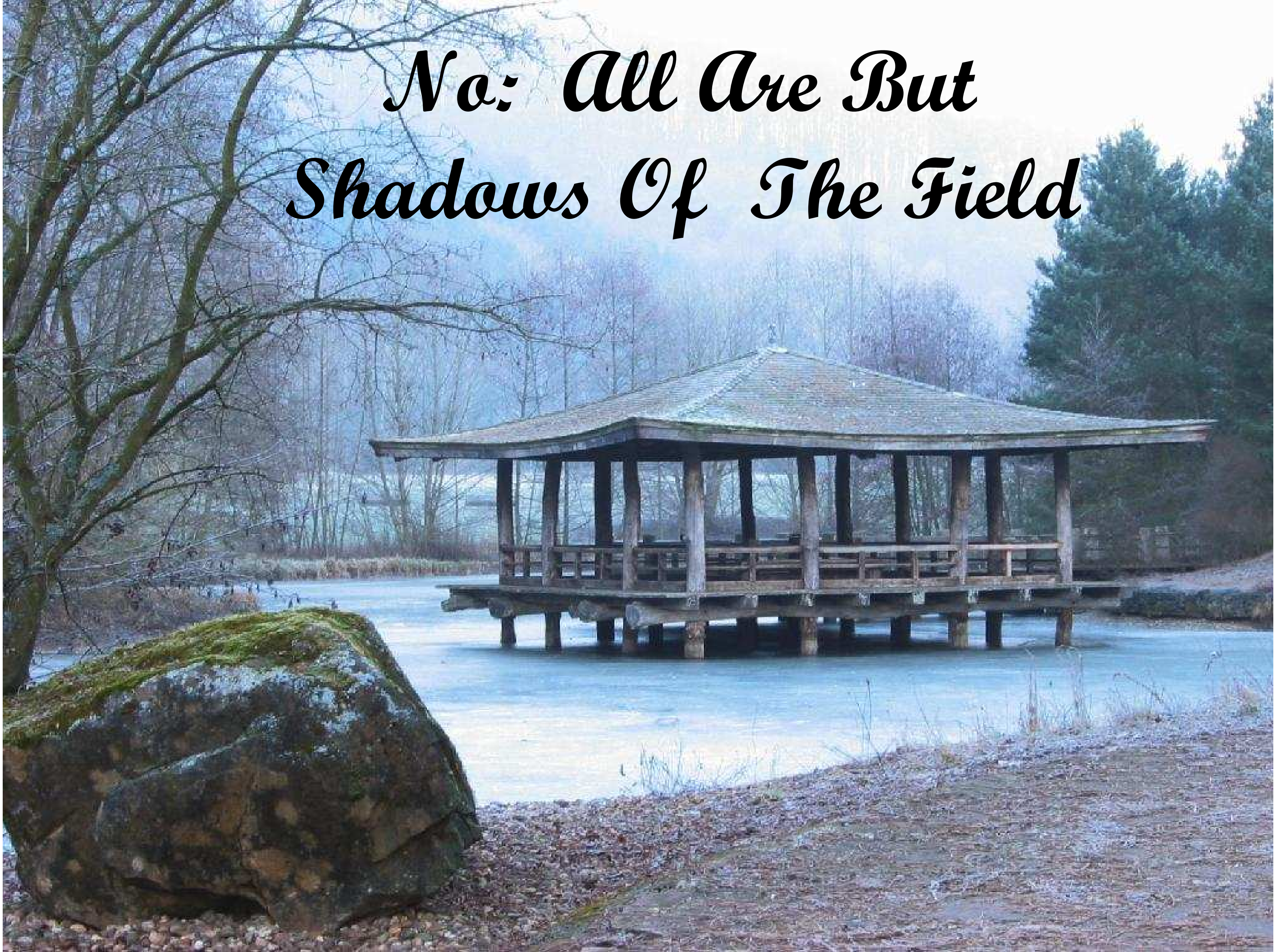
e -
γ
γ
e+



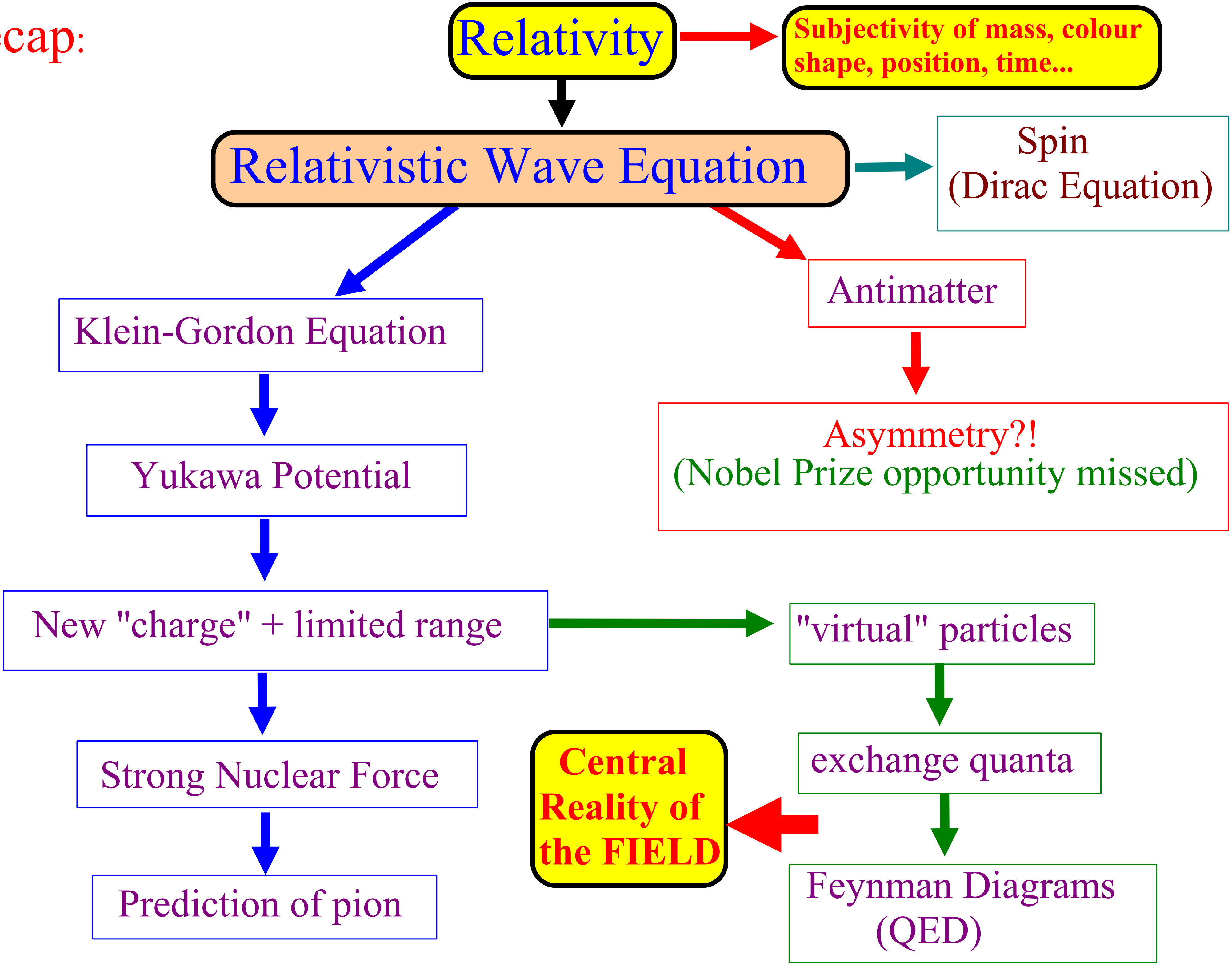
W -
e-
νe

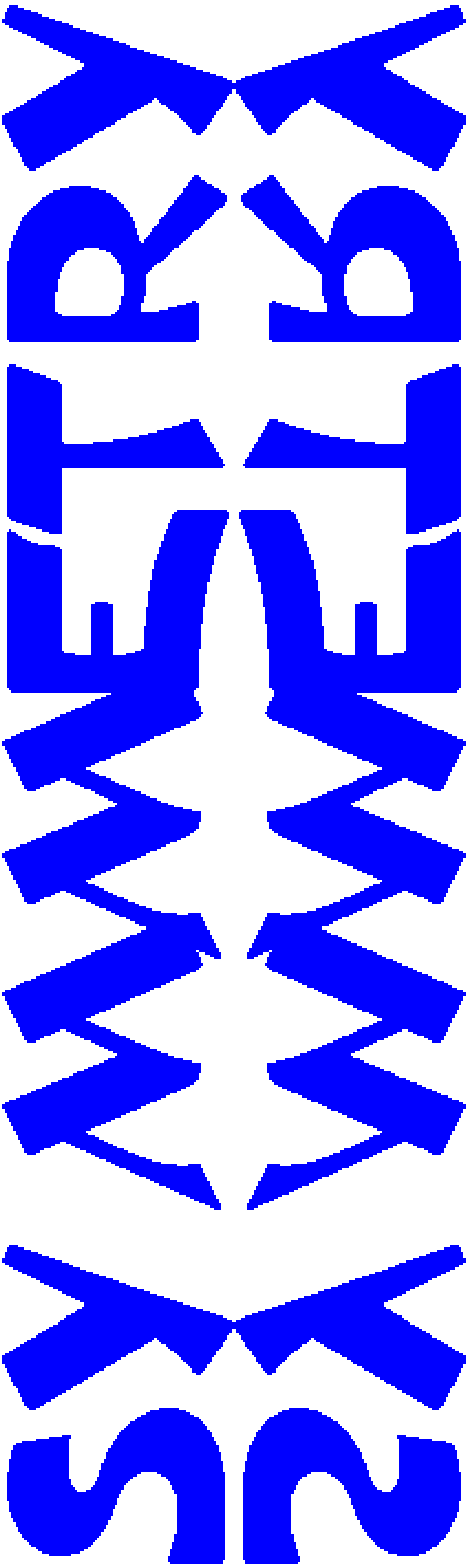

Is A Particle?

No: All Are But Shadows Of The Field

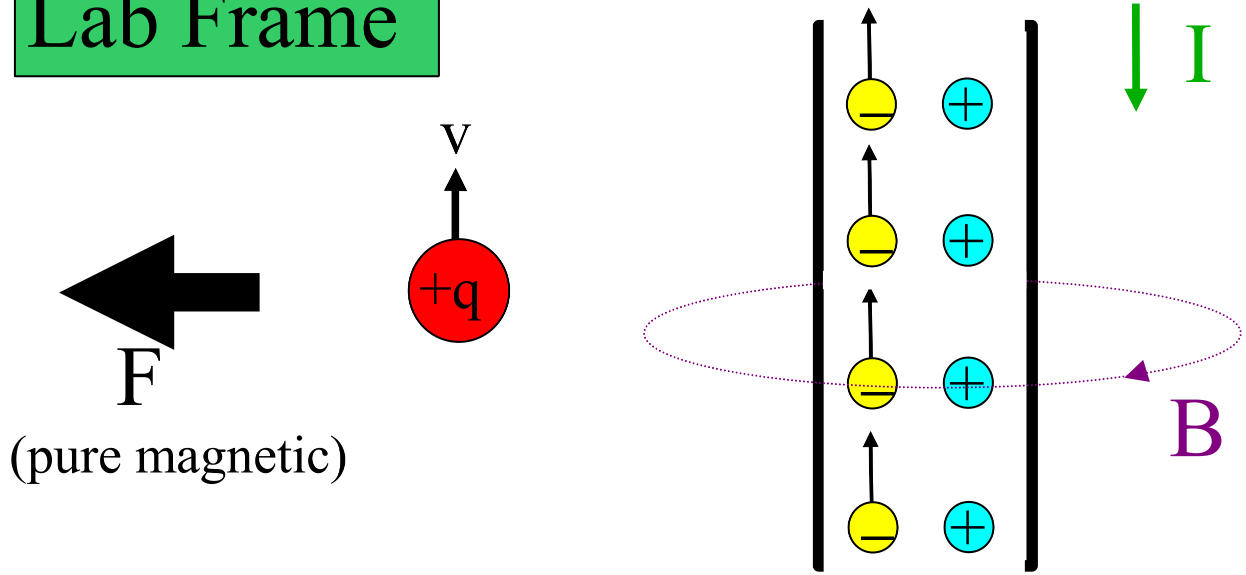


Recap:


Relativity
Subjectivity of mass, colour shape, position, time...
Relativistic Wave Equation
Spin (Dirac Equation)
Klein-Gordon Equation
Antimatter
Yukawa Potential
Asymmetry?! (Nobel Prize opportunity missed)
New "charge" + limited range
"virtual" particles
exchange quanta
Central Reality of the FIELD
Strong Nuclear Force
Feynman Diagrams (QED)
Prediction of pion


SYMMETRY

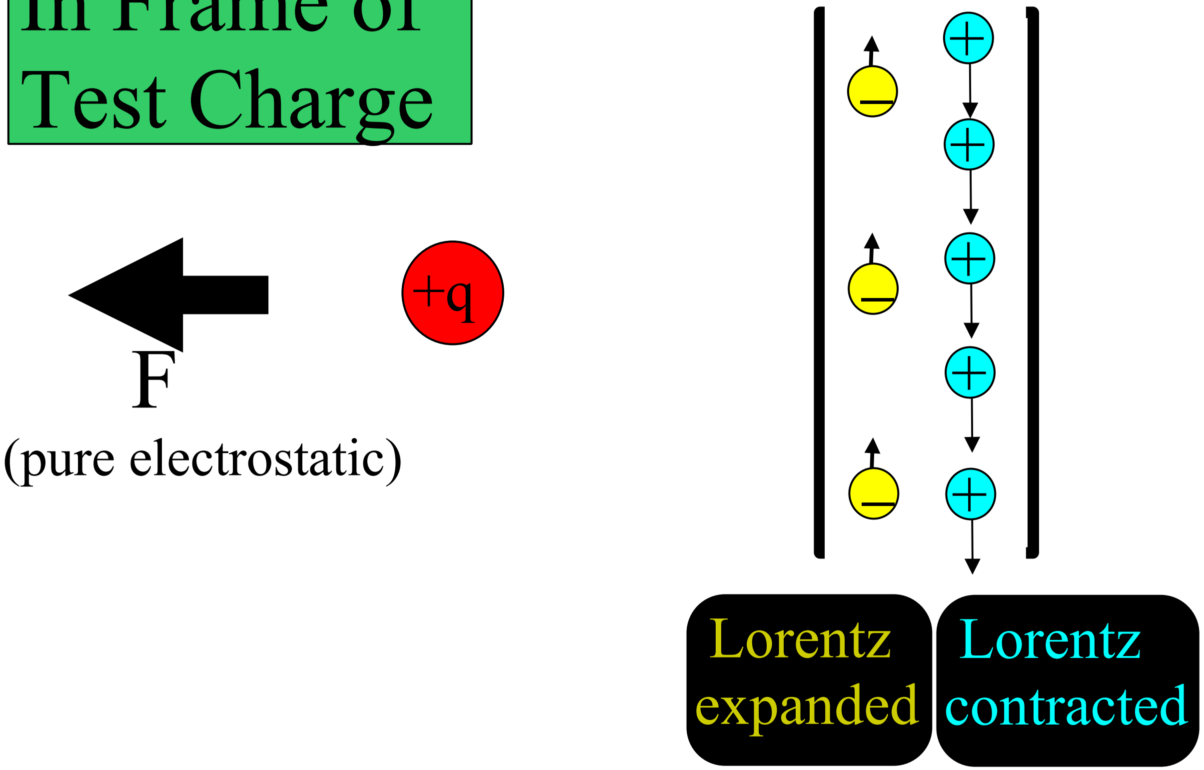
Lab Frame



Symmetry:
The magnitude of a force looks the same when viewed from reference frames boosted in the perpendicular direction

$\Downarrow$

In Frame of Test Charge



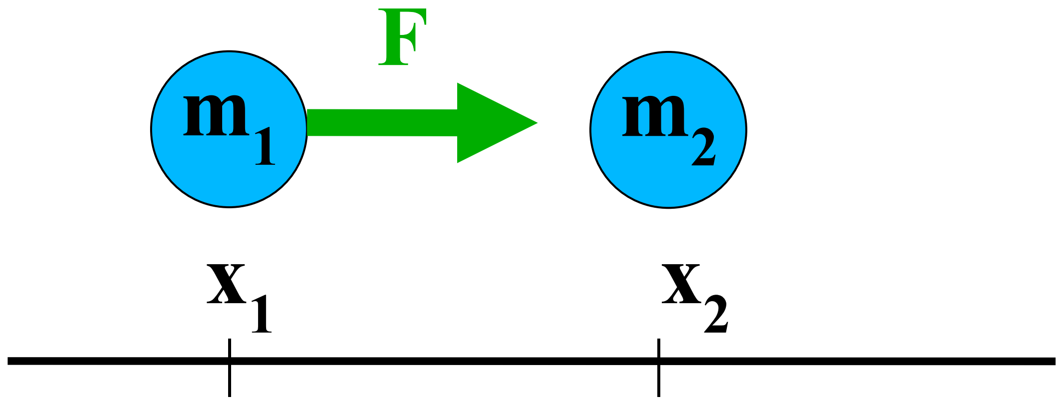
Electricity & Magnetism are identically the same force, just viewed from different reference frames

$\Downarrow$

UNIFICATION !!

(thanks to Lorentz invariance)

Space-Time Symmetries



If the force does not change as a function of position, then

force felt at x_2 : $F = m_2 \ddot{x}_2$

recoil felt at x_1 : $F = -m_1 \ddot{x}_1$

subtracting: $m_2 \ddot{x}_2 + m_1 \ddot{x}_1 = 0$

$$d/dt \, [\, m_2 \dot{x}_2 + m_1 \dot{x}_1 \,] = 0$$

$$m_2 v_2 + m_1 v_1 = \text{constant}$$

Translational Invariance ⇔ Conservation of Linear Momentum

Consider a system with total energy

$$E = \frac{1}{2} m \dot{x}^2 + V$$

assume this basic description also holds at other times

$$\frac{dE}{dt} = m \dot{x} \ddot{x} + \frac{dV}{dx} \frac{dx}{dt}$$

$$= m \dot{x} \ddot{x} + \frac{dV}{dx} \dot{x}$$

but $dV/dx = -F = -m\ddot{x}$ (Newton's 2nd law)

$$\Rightarrow \quad \frac{dE}{dt} = m \dot{x} \ddot{x} - m \dot{x} \ddot{x} = 0$$

$$\Rightarrow \quad E = \text{constant}$$

Time Invariance $\Leftrightarrow$ Conservation of Energy

Gauge Invariance in Electromagnetism:

"local"
symmetry

$$\mathbf{A} \rightarrow \mathbf{A} + \nabla \chi(x,t) \qquad \Phi \rightarrow \Phi - \frac{\partial}{\partial t}\chi(x,t)$$

$$E = -\nabla \Phi - \frac{\partial}{\partial t}\mathbf{A} \quad \rightarrow \quad -\nabla\left[\Phi - \frac{\partial}{\partial t}\chi(x,t)\right] - \frac{\partial}{\partial t}\left[\mathbf{A} + \nabla \chi(x,t)\right]$$

$$= -\nabla \Phi - \frac{\partial}{\partial t}\mathbf{A} \quad = \mathbf{E}$$

$$B = \nabla \times \mathbf{A} \quad \rightarrow \quad \nabla \times \left[\mathbf{A} + \nabla \chi(x,t)\right]$$

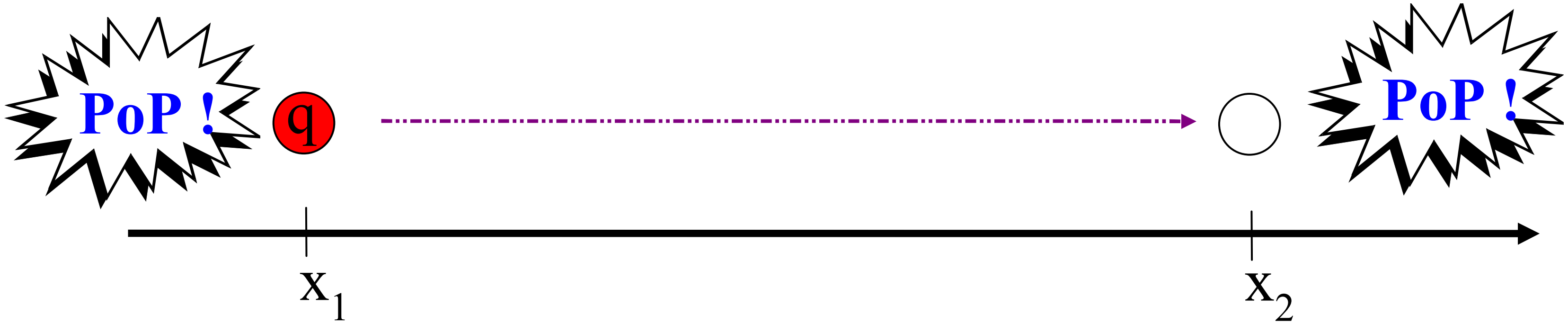
$$= \nabla \times \mathbf{A} \quad = \mathbf{B}$$

Gauge Invariance $\Leftrightarrow$ Conservation of Charge

(Wigner, 1949) To see this, assume charge were not conserved

So a charge could be created here by inputing energy E

And destroyed here, with the output of some energy E′



$E = -q\Phi(x_1)$

$E' = -q\Phi(x_2)$

Thus we will have created an overall energy $E' - E = -q\,\{\,\Phi(x_2) - \Phi(x_1)\,\}$

So, to preserve energy conservation, if Φ is allowed to vary as a function of position, charge must be conserved

Noether's Theorem

Continuous Symmetries $\Leftrightarrow$ Conserved "Currents"

(Emmy Noether, 1917)

Gauge symmetry from another angle...

Take the gauge transformation of a wavefunction to be $\Psi \rightarrow e^{iq\theta}\,\Psi$

where θ is an arbitrary "phase-shift" as a function of space and time

Say we want the Schrodinger equation to be invariant under such a transformation

$$\boxed{\frac{\partial \Psi}{\partial t} = \frac{i}{2m}\nabla^2\Psi}$$

clearly we're in trouble !

Consider the time-derivative for a simple plane wave: $\Psi = Ae^{i(px-Et)}$

$\Psi \rightarrow Ae^{i(px-Et+q\theta)}$ $\quad \partial/\partial t\,\Psi = i\,(\,-E + q\,\partial\theta/\partial t\,)\,\Psi$

here's the problem!

Note that if we now introduce an electric field, the energy level gets shifted by $-q\Phi$ $\longrightarrow$ $\partial/\partial t\,\Psi = i\,(\,-E + q\Phi + q\,\partial\theta/\partial t\,)\,\Psi$

But we can transform $\Phi \rightarrow \Phi - \partial\theta/\partial t$, thus cancelling the offending term!

(a similar argument holds for the spatial derivative and the vector potential)

Gauge invariance REQUIRES Electromagnetism !!

Another example...

Special Relativity: Invariance with respect to reference frames moving at constant velocity $\Rightarrow$ global symmetry

Generalize to allow velocity to vary arbitrarily at different points in space and time (i.e. acceleration) $\Rightarrow$ local gauge symmetry

Require an interaction to make this work

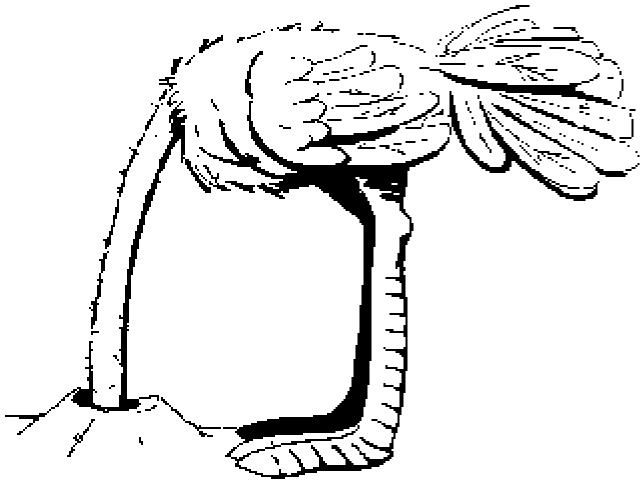
GRAVITY!

All known forces in nature are consequences of an underlying gauge symmetry !!

or perhaps

Gauge symmetries are found to result from all the known forces in nature !!

Pragmatism:



Symmetries (and asymmetries) in nature are often clear and can thus be useful in leading to dynamical descriptions of fundamental processes

↓

True even for "approximate" symmetries !