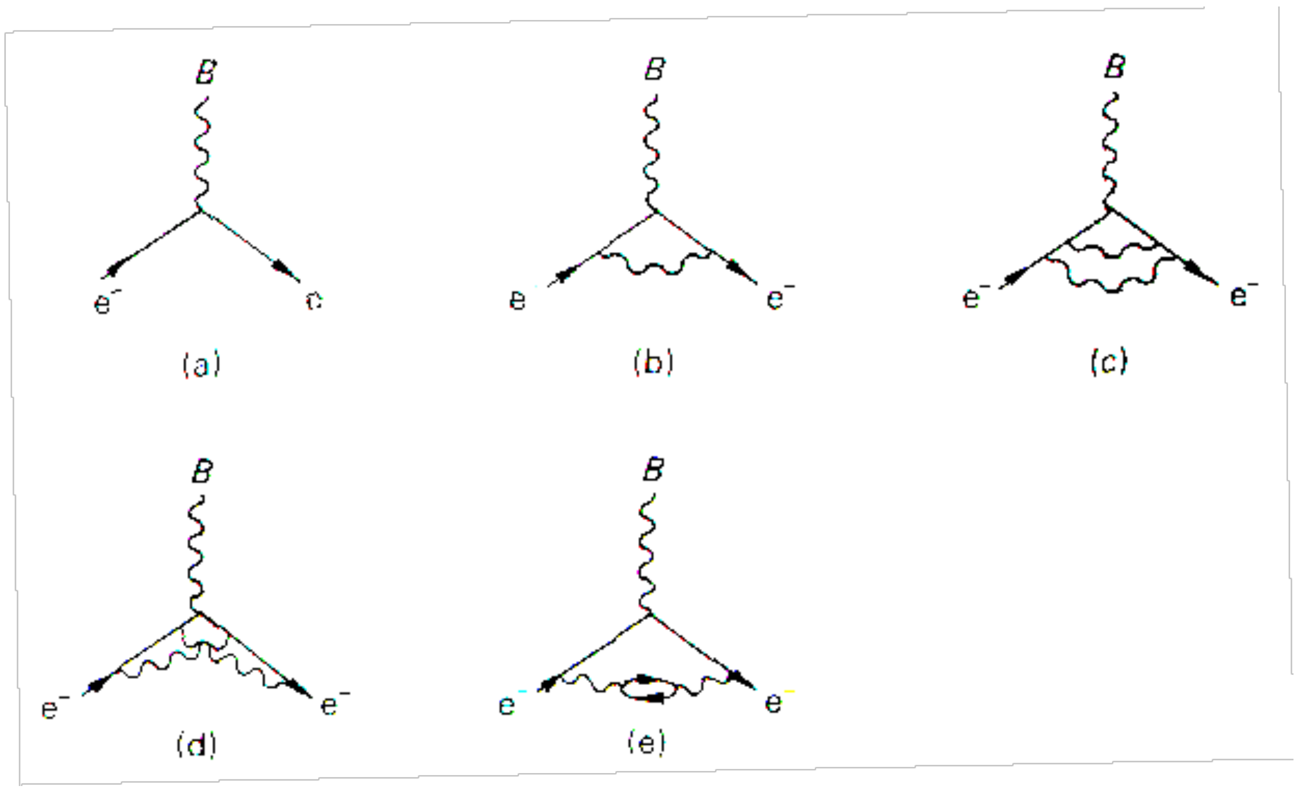
Lecture 5: QED

- The Bohr Magneton
- Off-Shell Electrons
- Vacuum Polarization
- Divergences, Running Coupling & Renormalization
- Yukawa Scattering and the Propagator

Useful Sections in Martin & Shaw:

Section 5.1, Section 5.2, Section 7.1.2

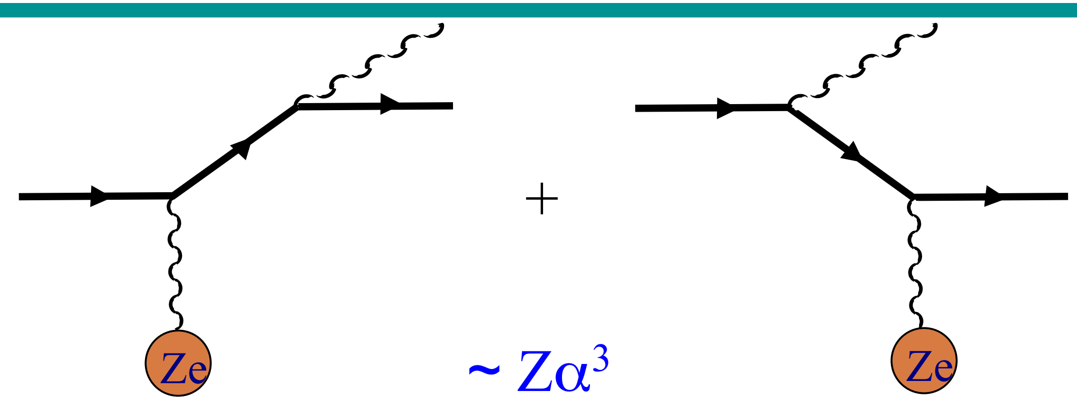
Bohr Magneton

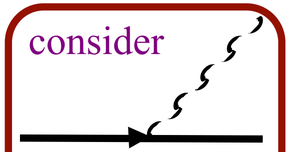


$$1 + \tfrac{1}{2}(\alpha/\pi) - 0.328478966(\alpha/\pi)^2 + 1.1765(\alpha/\pi)^3 - 0.8(\alpha/\pi)^4$$
$$= 1.001\,159\,652\,307(110).$$

Bremsstrahlung

Internal electron line since real electron cannot emit a real photon and still conserve energy and momentum:





$$m_e = m_e' + p_e'^2/2m_e' + E_\gamma \quad \text{(conservation of energy)}$$

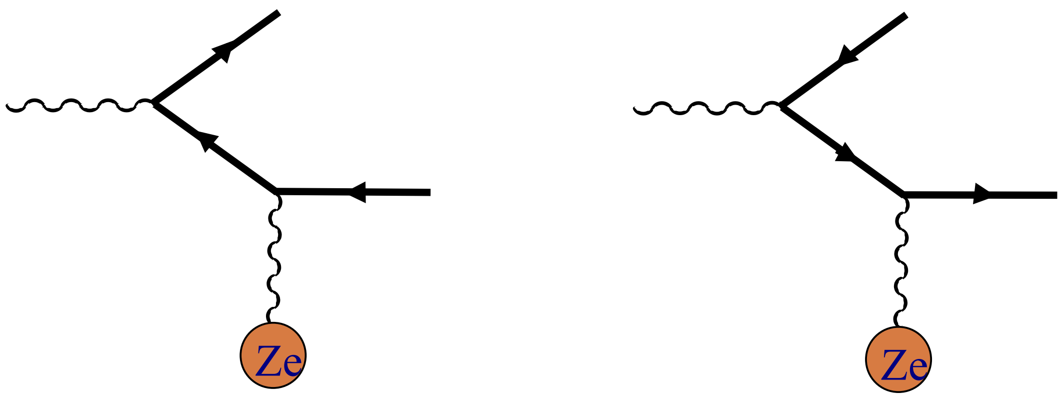
$$|p_e'| = |p_\gamma| = E_\gamma \quad \text{(conservation of momentum)}$$

$$\Rightarrow \quad m_e - m_e' = E_\gamma^2/2m_e' + E_\gamma$$

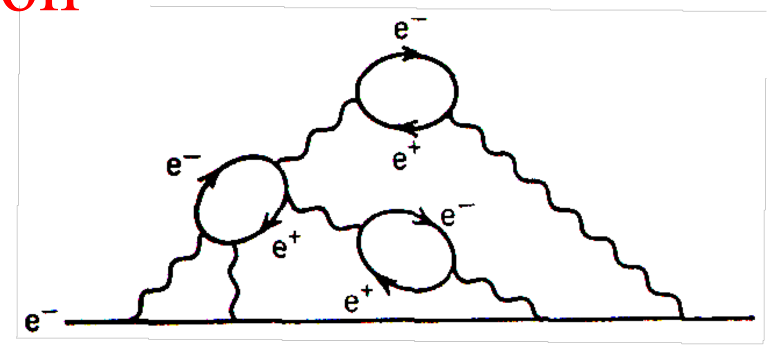
So, for a positive energy photon to be produced, the electron has to effectively "lose mass" $\Rightarrow$ virtual electron goes "off mass shell"

Pair Production

"twisted Bremsstrahlung"
Matrix element will be the same; Cross-section will only differ by a kinematic factor

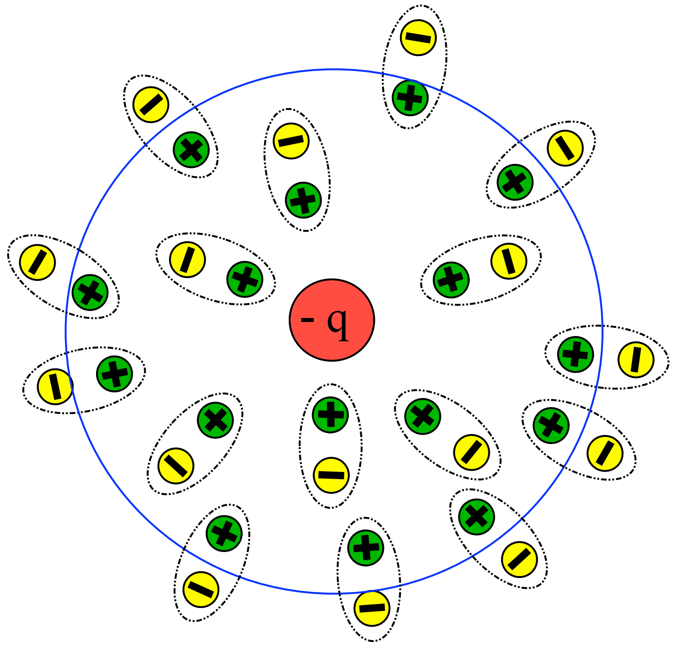


Vacuum Polarization



Thus, we never actually ever see a "bare" charge, only an effective charge shielded by polarized virtual electron/positron pairs. A larger charge **(or, equivalently, α)** will be seen in interactions involving a high momemtum transfer as they probe closer to the central charge.

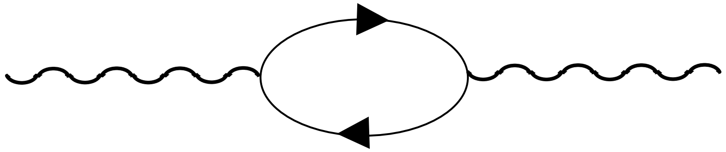
$\Rightarrow$ "running coupling constant"



In QED, the bare charge of an electron is actually infinite !!!

Note: due to the field-energy near an infinite charge, the bare mass of the electron ($E=mc^2$) is also infinite, but the effective mass is brought back into line by the virtual pairs again !!

Another way...



Consider the integral corresponding to the loop diagram above:

At large q the product of the propagators will go as $\sim 1/q^4$

Integration is over the 4-momenta of all internal lines
where $d^4q = q^3\, dq\, d\Omega$ (just like the 2-D area integral is $r\, dr\, d\theta$ and the 3-D volume integral is $r^2\, dr\, d\cos\theta\, d\phi$)

so the q integral goes like:

$$\int_{q_{min}}^{\infty} (1/q^4)\, q^3\, dq = \int_{q_{min}}^{\infty} dq/q = \ln q \,\Big]_{q_{min}}^{\infty}$$

$\Rightarrow$ logarithmically divergent !!

With some fiddling, these divergences can always be shuffled around to be associated with a bare charge or mass. But we only ever measure a "**dressed**" (screened) charge or mass, which is finite. Thus, we "**renormalize**" by replacing the bare values of α_0 and m_0 by running parameters.

Analogy for Renormalization:

Instead of changing the equations of motion, you could instead
(in principle) find the "effective" mass of the cannonball by shaking
it back and forth in the water to see how much force it takes to accelerate it.
This "mass" would no longer be a true constant as it would clearly depend
on how quickly you shake the ball.

Still unhappy? Well, if it’s any comfort, note that the electrostatic potential of a classical point charge, $\mathbf{e^2/r}$, is also infinite as $\mathbf{r \rightarrow 0}$

(perhaps this all just means that there are really no true point particles... **strings?? something else??**)

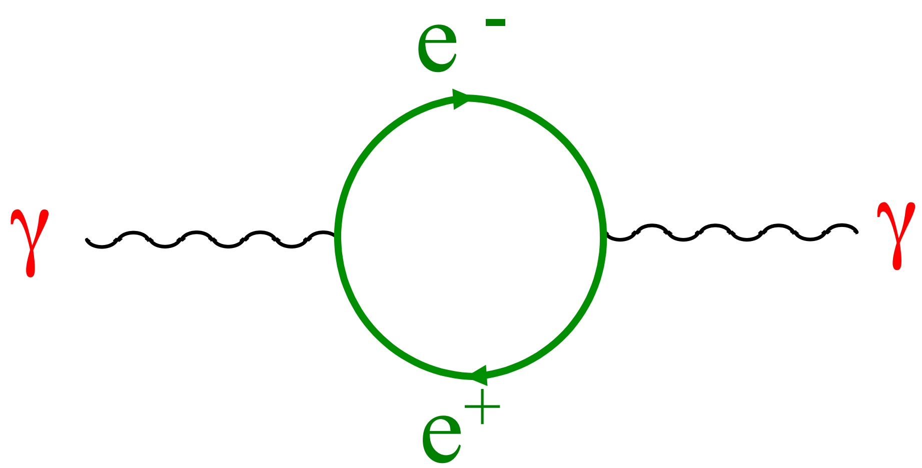
Does an electron feel it’s own field ???

"Ven you svet, you smell you’re own stink, Ja? Zo..."

M. Veltman

Steve's Tips for Becoming a Particle Physicist

1) Be Lazy
2) Start Lying
3) Sweat Freely


e -
γ
γ
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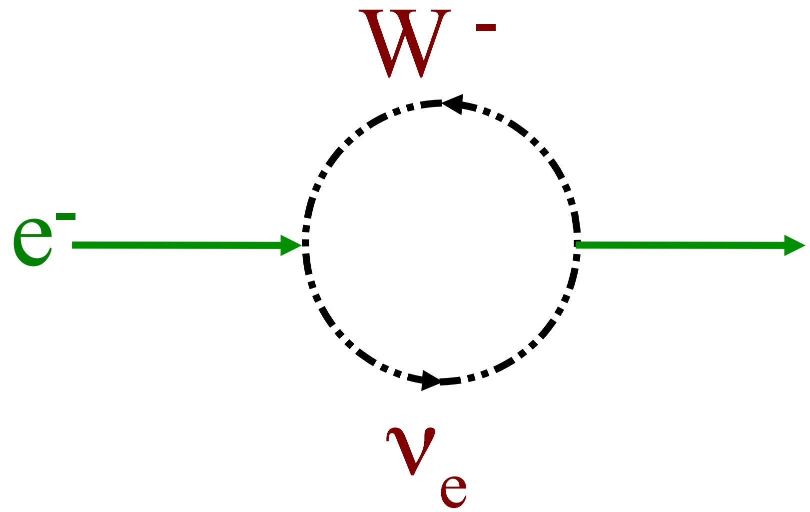
W -
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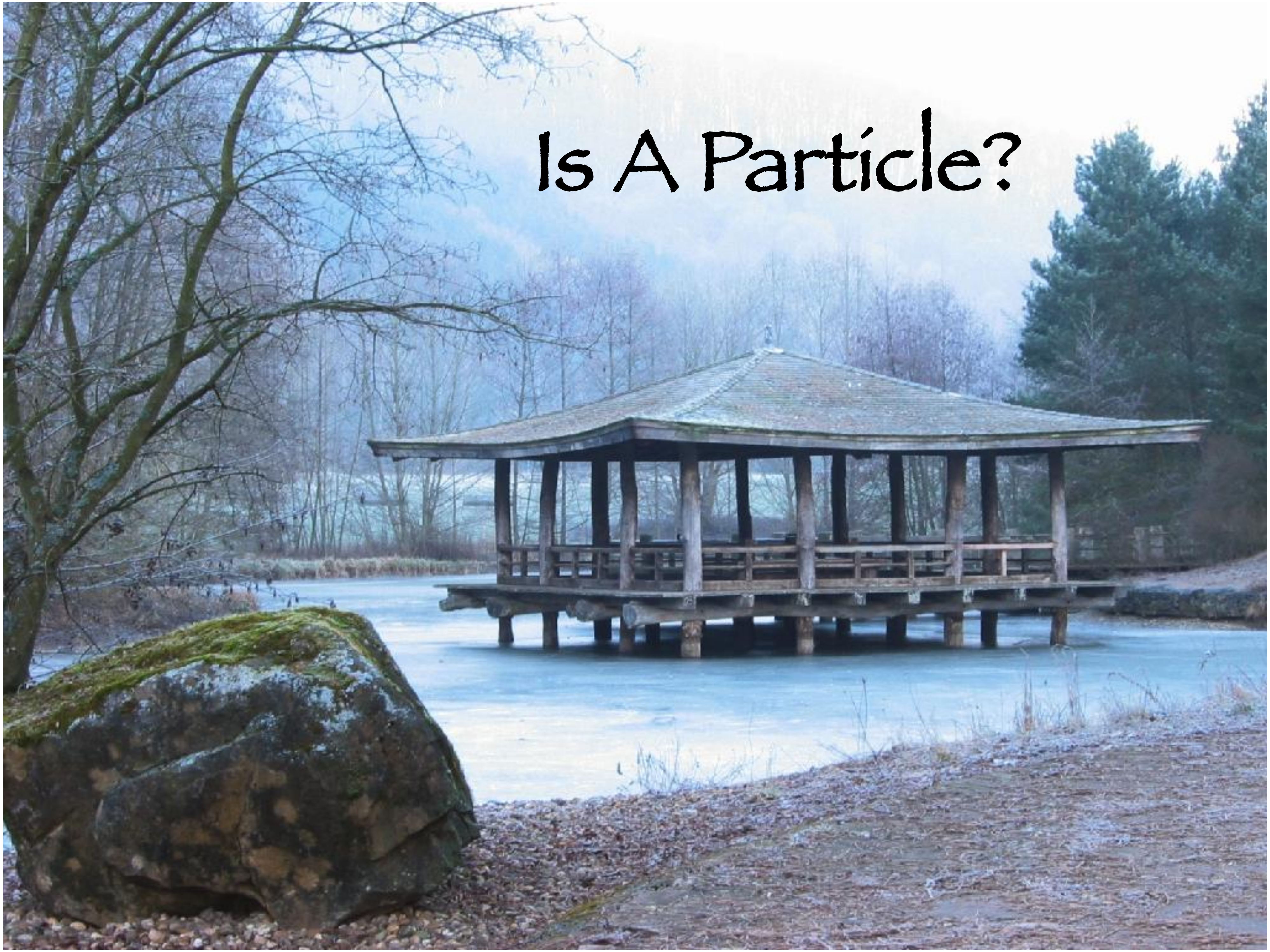
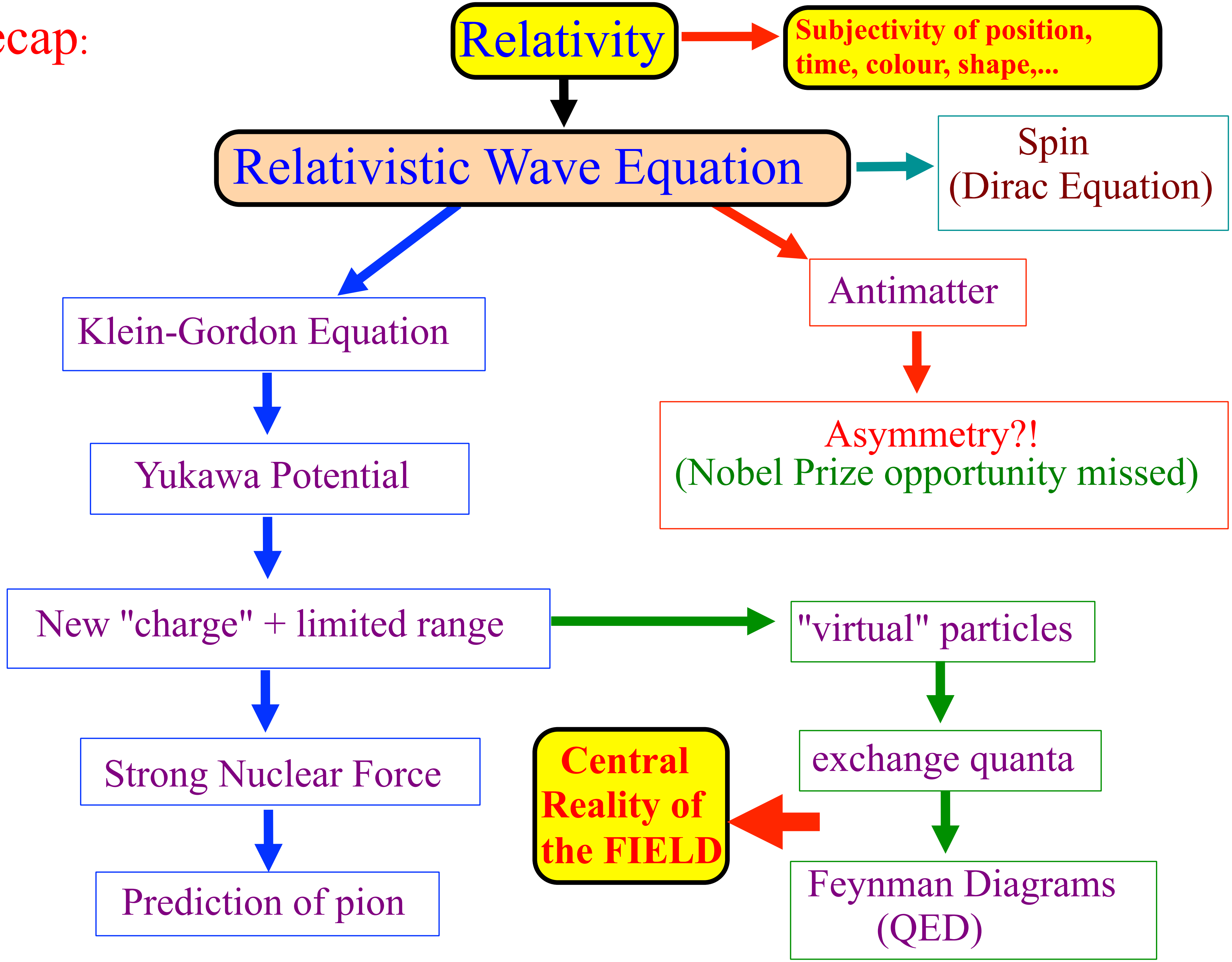

Is A Particle?


No: All Are But
Shadows Of The Field

Recap:


Relativity
Subjectivity of position, time, colour, shape,...
Relativistic Wave Equation
Spin (Dirac Equation)
Klein-Gordon Equation
Antimatter
Yukawa Potential
Asymmetry?!
(Nobel Prize opportunity missed)
New "charge" + limited range
"virtual" particles
Strong Nuclear Force
exchange quanta
Central Reality of the FIELD
Prediction of pion
Feynman Diagrams (QED)

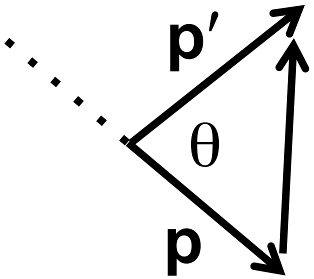
6
sheet 2

Consider the scattering of one nucleon by another via the Yukawa potential: $V(r) = \frac{-g^2}{4\pi r} e^{-Mcr/\hbar}$

In the CM, both nucleons have equal energy and equal & opposite momenta, $\boldsymbol{p}$. *Take the incoming state of the 1st nucleon (normalised per unit volume) to be:* $\Psi_o = e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$

After scattering, the final state will have a new momentum vector, $\boldsymbol{p}'$, *where* $|\boldsymbol{p}| = |\boldsymbol{p}'|$ *:* $\Psi_f = e^{i\mathbf{p}'\cdot\mathbf{r}/\hbar}$

If θ *is the angle between* $\boldsymbol{p}$ *and* $\boldsymbol{p}'$, *write an expression for the momentum transfer,* $\boldsymbol{q}$, *in terms of* $\boldsymbol{p}$.



$\mathbf{q} = \mathbf{p}' - \mathbf{p}$

magnitude of q = $2p\sin(\theta/2)$

Compute the matrix element for the transition.

$$<\Psi_f|V(r)|\Psi_o> = \int e^{-i\mathbf{p}'\bullet\mathbf{r}/\hbar}\left(\frac{-g^2}{4\pi r}e^{-Mcr/\hbar}\right)e^{i\mathbf{p}\bullet\mathbf{r}/\hbar}\ r^2\,dr\,d\phi\,d(\cos\theta)$$

$$= \frac{-g^2}{2}\int e^{-i\mathbf{q}\bullet\mathbf{r}/\hbar}\,e^{-Mcr/\hbar}\ r\,dr\,d(\cos\theta)$$

$$= \frac{-g^2}{2}\int e^{-iqr\cos\theta/\hbar}\ e^{-Mcr/\hbar}\ r\,dr\,d(\cos\theta)$$

$$= \frac{-i\hbar g^2}{2q}\int \left(e^{-(iq+Mc)r/\hbar} - e^{(iq-Mc)r/\hbar}\right)\,dr$$

$$= \frac{i\hbar^2 g^2}{2q}\left[\frac{e^{-(iq+Mc)r/\hbar}}{iq+Mc} + \frac{e^{(iq-Mc)r/\hbar}}{iq-Mc}\right]_{r=0}^{r=\infty}$$

$$= \frac{i\hbar^2 g^2}{2q}\left[\frac{e^{-(iq+Mc)r/\hbar}}{iq+Mc} + \frac{e^{(iq-Mc)r/\hbar}}{iq-Mc}\right]_{r=0}^{r=\infty}$$

This will go to zero as $r \rightarrow \infty$, so we're left with:

$$<\Psi_f|V(r)|\Psi_o> = \frac{-i\hbar^2 g^2}{2q}\left[\frac{1}{iq+Mc} + \frac{1}{iq-Mc}\right]$$

$$= \frac{-i\hbar^2 g^2}{2q}\left[\frac{iq-Mc \quad + \quad iq+Mc}{(iq+Mc)(iq-Mc)}\right]$$

$$= \frac{\hbar^2 g^2}{(q^2+M^2c^2)} \qquad \sim \quad \frac{1}{q^2+M^2} \quad \text{(natural units)}$$

NOTE:

This is all really a semi-relativistic approximation!

Really, we want to consider a 4-momentum transfer

Recall that $\mathbf{P}^2 = E^2 - p^2$

so, $\dfrac{1}{q^2+M^2} \rightarrow \dfrac{1}{q_4^{\,2} - M^2}$ (choosing appropriate sign convention)

Known as the "propagator" associated with the field characterised by an exchange particle of mass M

Show that the relation $dp/dE = 1/v$, where v is the velocity, holds for both relativistic and non-relativistic limits.

Classically:

$$E = p^2/2m$$

$$dE = (p/m)\, dp$$

$$dp/dE = (m/mv) = 1/v$$

Relativistically:

$$E^2 = p^2 + m^2$$

$$dp/dE = E/p = (\gamma m/\gamma mv) = 1/v$$

Also show that, for the present case: $p^2 d(\cos\theta) = -\tfrac{1}{2}\, dq^2$

$$q = 2p \sin(\theta/2)$$

$$q^2 = 4p^2 \sin^2(\theta/2)$$

$$\cos\theta = \cos^2(\theta/2) - \sin^2(\theta/2)$$
$$= 1 - 2\sin^2(\theta/2)$$

$$\sin^2(\theta/2) = (1 - \cos\theta)/2$$

$$q^2 = 2p^2 (1 - \cos\theta)$$

$$dq^2 = -2p^2\, d\cos\theta$$

$$p^2 d(\cos\theta) = -\tfrac{1}{2}\, dq^2$$

Now, from the definition of cross-section in terms of a rate, the relative velocities of the nucleons in the CM, and using Fermi's Golden Rule, derive the differential cross-section $d\sigma/d\Omega$.

$$\text{Rate} = \frac{v_{beam} N_{beam} N_{target}\, \sigma}{\text{Volume}}$$

in our case

$N_{beam} = N_{target} = 1$

$v_{beam} = 2v$ (relative velocity in CM)

Normalised Volume (= 1)

$$\text{so,} \quad d\sigma = \frac{1}{2v} d\text{Rate}$$

given by FGR

$$d\sigma = \frac{1}{2v} \frac{2\pi}{\hbar} |\langle\Psi_f|V(r)|\Psi_o\rangle|^2 \frac{1}{(2\pi\hbar)^3} \frac{p^2 dp\, d\phi\, d\cos\theta}{dE_{tot}}$$

$$= \frac{1}{2v} \frac{2\pi}{\hbar} \left(\frac{\hbar^2 g^2}{(q^2+M^2c^2)}\right)^2 \frac{1}{(2\pi\hbar)^3} \frac{1}{2v} \left(-\tfrac{1}{2}\, dq^2\right) d\phi$$

$$= \frac{\pi}{4v^2\hbar} \left(\frac{\hbar^2 g^2}{(q^2+M^2c^2)}\right)^2 \frac{dq^2 d\phi}{(2\pi\hbar)^3}$$

$$d\sigma = \frac{\pi}{4v^2\hbar}\left(\frac{\hbar^2 g^2}{(q^2+M^2c^2)}\right)^2 \frac{dq^2 d\phi}{(2\pi\hbar)^3}$$

$$= \frac{\pi}{4v^2\hbar}\frac{\hbar^4 g^4}{(q^2+M^2c^2)^2}\frac{dq^2 d\phi}{8\pi^3\hbar^3} = \frac{1}{(4\pi)^2}\frac{g^4}{2v^2}\frac{dq^2 d\phi}{(q^2+M^2c^2)^2}$$

Integrate this expression and take the limit as $v \rightarrow c$

$$\sigma = \frac{g^4}{2v^2(4\pi)^2}\int\frac{dq^2 d\phi}{(q^2+M^2c^2)^2} = \frac{\pi g^4}{v^2(4\pi)^2}\int\frac{dq^2}{(q^2+M^2c^2)^2}$$

$$= \frac{\pi g^4}{v^2(4\pi)^2}\left[-\frac{1}{(q^2+M^2c^2)}\right]_{q^2=\infty}^{q^2=0}$$

since integral was from $\cos\theta = -1$ to 1 (where 1 = zero momentum transfer)

$$\sigma = \frac{\pi g^4}{Mc^2(4\pi)^2} = \pi\left(\frac{g^2}{4\pi\hbar c}\right)^2\left(\frac{\hbar}{Mc}\right)^2$$

taking v ~ c

strong coupling constant (~1)

"range" of Yukawa potential (~1fm)

$\sigma = \pi \times 10^{-30}\ m^2$

~ 30 mb

(within a factor of 2-3)