Lecture 3
RF and Linear Acceleration

RF Circuit Analogy

Reentrant Cavity

Resonant Modes

Cavity Properties

Figures of Merit for Cavities

Travelling Wave Cavities

Superconducting Cavities

Linear Accelerators

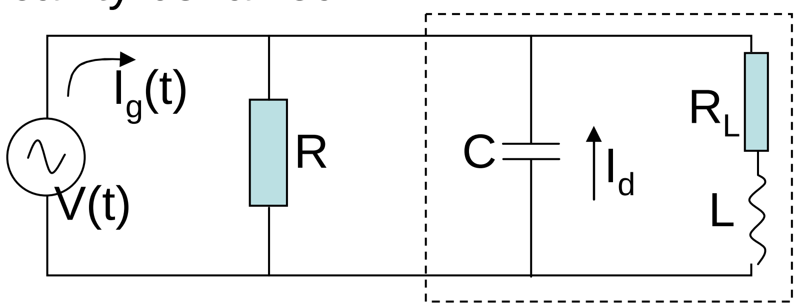
RF: Circuit Analogy

- Resonant Cavity:
  - Volume enclosed by metal walls
  - Supports electromagnetic oscillations

RF-Cavity for RHIC
(Relativistic Heavy Ion Collider at
Broohaven National Laboratory)

RF: Circuit Analogy

- Resonant Cavity:
  - Volume enclosed by metal walls
  - Supports electromagnetic oscillations
- Use circuit analogy to understand cavity behaviour:



- Dashed box:

$$Z(\omega) = (j\omega C + \frac{1}{j\omega L})^{-1} = \frac{j\omega L}{1-\omega^2 LC}$$

ν_{low}: inductive ν_{high}: capacitive

- At $\omega_0 = 1/\sqrt{LC}$: Resonance ($Z \rightarrow \infty$)
  - Current minimized!

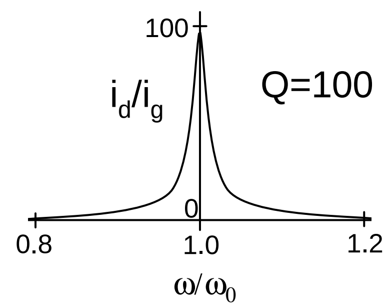
- Imperfect Inductor:
  - Resistive losses→damping current I_d

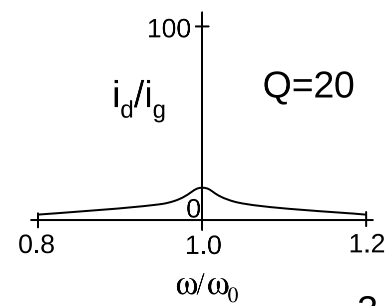
$$Z(\omega) = (j\omega C + \frac{1}{j\omega L + R})^{-1} = \frac{j\omega L + R}{1-\omega^2 LC + j\omega RC}$$

$$\text{Magnitude: } Z(\omega) \sim \frac{1}{(1-\frac{\omega^2}{\omega_0^2})^2 + (wRC)^2}$$

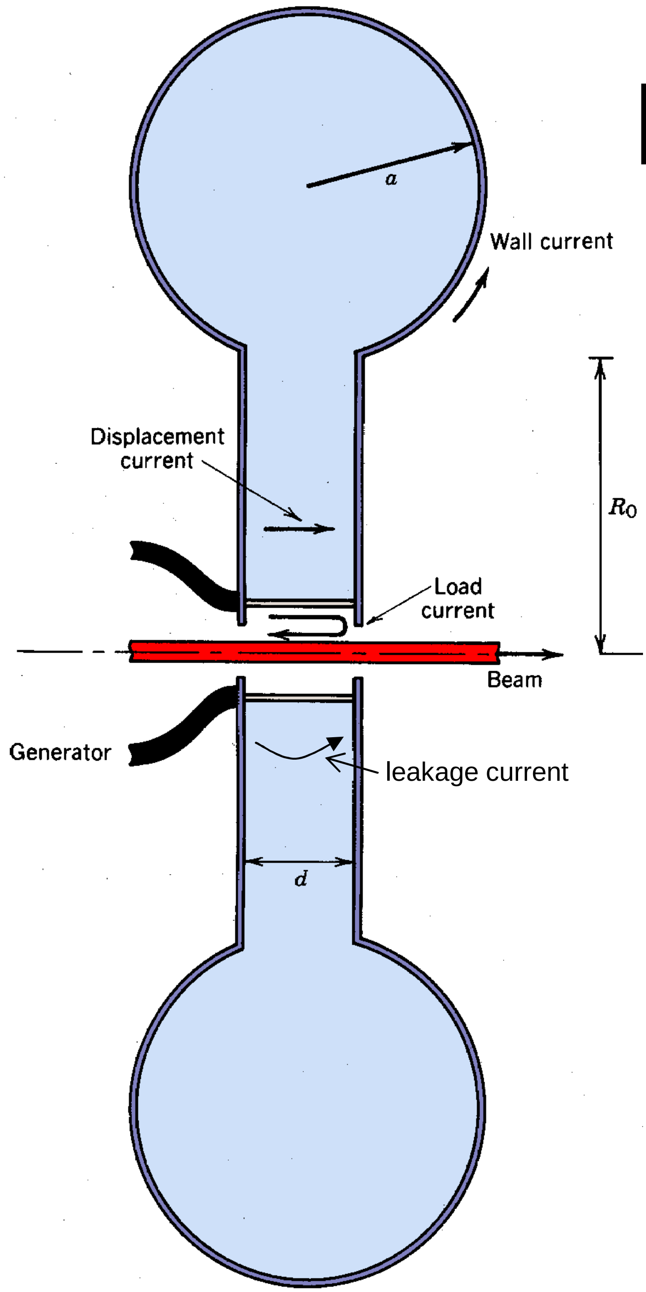
$$\text{Bandwidth } \frac{\Delta\omega}{\omega} = \frac{R}{\sqrt{LC}} = \frac{1}{Q}$$

$$Q = \frac{\omega_0 \mathrm{E_s}(\text{stored in resonant circuit})}{\text{time averaged power loss}} \cong \frac{\sqrt{LC}}{R}$$





RF: Reentrant Cavity



Example of a Standing wave cavity:

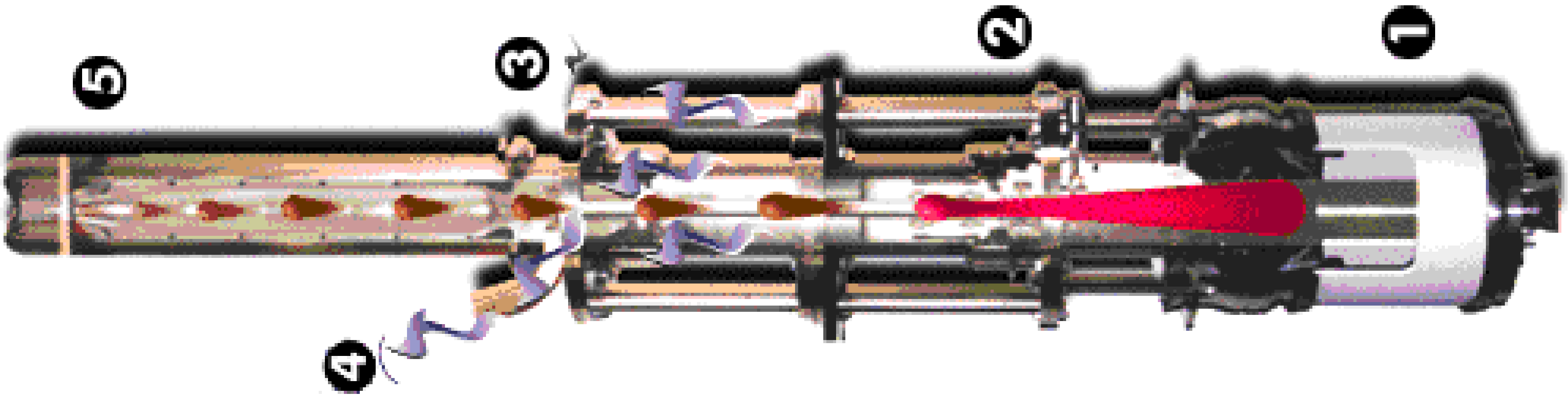
- Narrow gap d $\rightarrow$ C large, L small

  $\rightarrow C_{cavity} = \varepsilon_0 \pi R_0^2 / d$

- Outer region: C small, $L_{cavity} = \mu_0 \pi a^2 / 2\pi(R_0 + a)$
- ν_{low}: leakage current, short circuited
- ν_{high}: displacement current flows across capacitor
- ν_{res}: L_{cavity} infinite $\rightarrow$ generator energy transferred into load $\rightarrow$ acceleration
- resonance frequency:

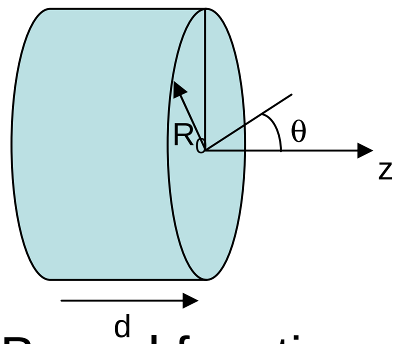
$$\omega_0 = \frac{1}{\sqrt{LC}} = c\sqrt{2(R_0 + a)\frac{d}{R_0^2 a^2 \pi}}$$

Klystron



1. Electron gun produces a flow of electrons
2. Bunching Cavities regulate speed of e-→arrive in bunches at output cavity
3. Bunches of electrons excite output cavity
4. Microwaves flow into waveguide, which transports them to the accelerator
5. Electrons are absorbed in beam stop

Resonant Modes



- Cylindrical (pillbox) cavity
  - Difference to Reentrant:
    - Regions of E - and B fields mixed
  - Solve Maxwell equations:

$$\nabla^2 \vec{E} - \frac{1}{v^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \text{ with } v = \frac{c}{\sqrt{\varepsilon\mu / \varepsilon_0\mu_0}}$$

$$\nabla^2 \vec{B} - \frac{1}{v^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

  - Assumptions:
    - Azimuthal symmetry (∂/∂θ=0)
    - ∂E/∂z=0, E // E_z
    - $\vec{E} = E_z(r) e^{j\omega t} \overrightarrow{u_z}$
  - $\frac{d^2 E_z(r)}{dr^2} + \frac{1}{r}\frac{dE_z(r)}{dr} + \frac{\omega^2}{v^2} E_z(r) = 0$
    - Bessel equation!

- Solutions: 0-order Bessel function:

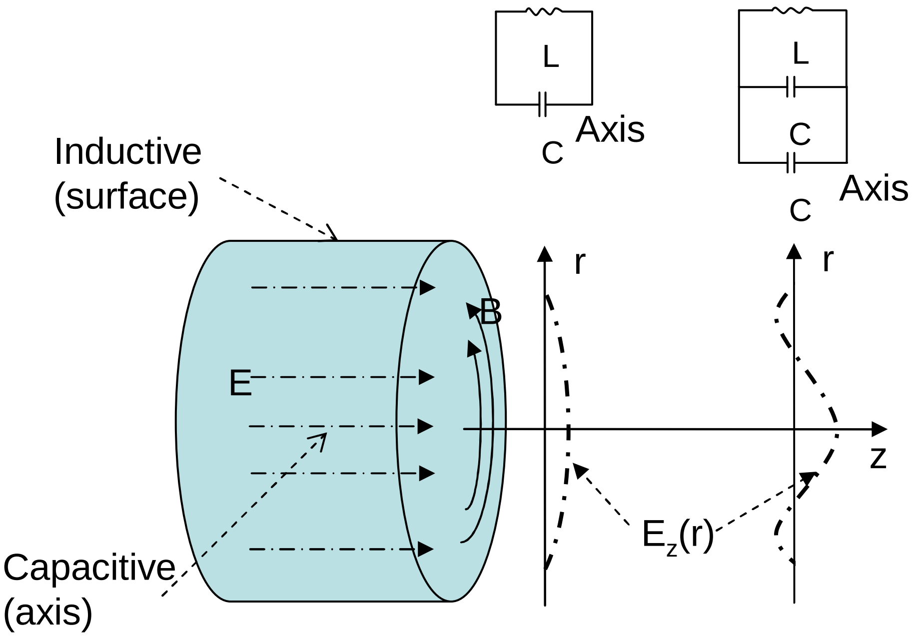
$$E_{zn}(r,t) = E_{0n} J_0(k_n r)\cos(\omega t);\ \sigma = \varepsilon_0 E_\perp$$

$$B_{\Theta n}(r,t) = \frac{E_{0n}}{c} J_1(k_n r)\sin(\omega t);\ K = B_\parallel / \mu_0$$

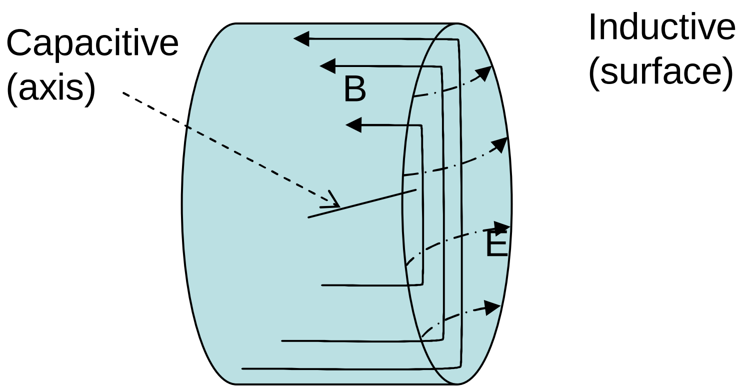
- Boundary cond: $E(R_0,t)=0$
  - k_n valid only for zeros of j_0:
  - $\omega_n = ck_n$ or $k_n = 2\pi/\lambda_n$,
    - Cavity radius determines wavelength

Mode	k_n
TM_{010}	$2.405/R_0$
TM_{020}	$5.520/R_0$
TM_{030}	$8.654/R_0$

Resonant Modes



Tm_{010} - Mode Tm_{020} - Mode



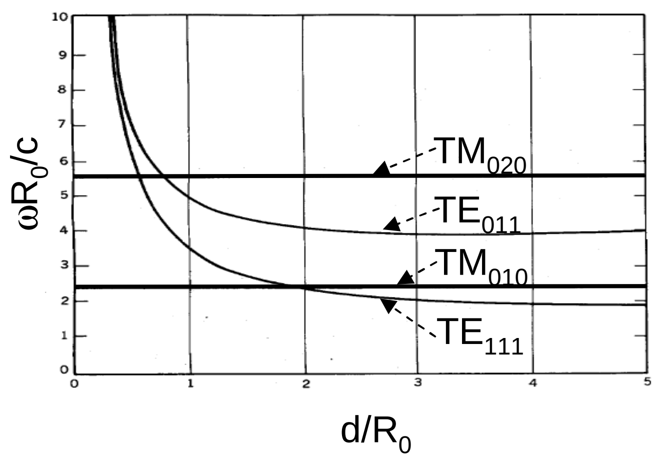
TE_{111} - Mode

- TM_{xxx} (transverse magnetic)
- TE_{xxx} (transverse electric)
- 1st index: azimuthal mode number (0 for symmetric modes)
- 2nd index: radial mode number
- 3rd index: longitudinal mode nr. zero if E_z is constant along z
- TM modes optimal for particle acceleration
  - E field // axis
  - E field maximum on axis
  - $B_{transverse}$ zero on axis (non-zero fields could deflect beam)
  - n>1 cavity divided into n resonant LC circuits
  - C and L of each circuit reduced ~1/n

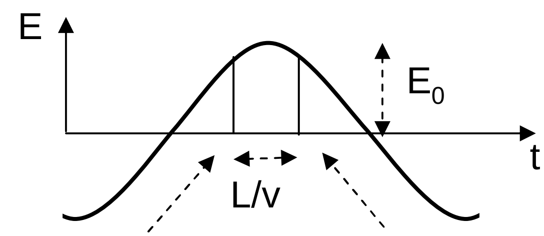
  ω increased by factor close to n

Properties

- Higher order modes
  - Undesirable; energy waster
  - Interfere with acceleration
  - Might cause beam deflection
- Geometry "chooses" modes:



- Energy gain for particle with velocity v:



$$eV_{acc} = e\int_{-L/2}^{L/2} ds E_0 \cos\omega t \xrightarrow[t=s/v]{} E_0 \frac{2\mathrm{v}}{\omega}\sin\frac{\omega L}{2\mathrm{v}}$$

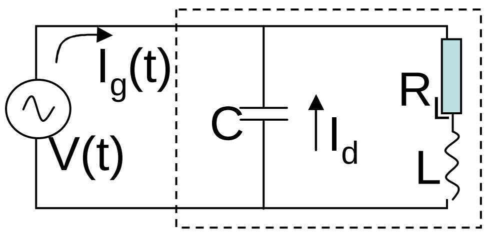
- "Transit time factor" $T = \frac{V_{acc}}{E_0 L} = \frac{\sin u}{u}, u = \frac{\omega L}{2\mathrm{v}}$

- Limit to cavity length:
  - For T=0.9
    - cavity with R=10cm (1.15 GHz)

    → L=6.7cm

Figures of Merit: Q

- Q-Value $= \dfrac{\text{stored energy}}{\text{energy loss for 1 cycle}} = \dfrac{\omega E_s}{P_l}$
- Power loss due to imperfect cavity:



Good model for imperfect cavity

Powerloss $P_l = \dfrac{1}{2}\int K^2 \dfrac{\rho}{\delta} dS = \dfrac{\omega\delta}{4\mu_0} \int_{surface} B^2 dS$

with skindepth $\delta = \sqrt{\dfrac{2\rho}{\mu\omega}}$ ρ: vol resistivity

  – Assume modes approximately same as in ideal cavity

$$Q = \frac{\omega E_s}{P_l} = \frac{\omega}{P_l}\frac{1}{2\mu_0}\int_{volume} B^2 dV = \frac{2}{\delta}\frac{\int B^2 dV}{\int B^2 dS}$$

Q depends on geometry and surface resistance

$$Q_{pillbox} = \frac{d/\delta}{1 + d/R_0}$$

- For f = 1GHz:
  – δ=2μm
    - Carefully polished walls!
  – for r=12cm, d=4cm: $Q=3\times10^4$
  – Bandwidth $\Delta f/f = 1/Q = 3\times10^{-5}$
    - requires very stable frequency!

Linac: Figures of Merit

- Shunt Impedence Z_s/unit length
  - $P_l = V_{acc}^2 / (Z_s L)$
    - P_l: power dissipated in cavity walls
    - V_{acc}: total accelerator voltage (E_{beam} in eV / particle charge)
    - L: total accelerator length
  - Typical values: 25-50MΩ/m
    - Example: e- linac, 2.5 GeV, V_{acc}: 8MV/m, L=312m, Z_s=50MΩ/m
    - $R_{tot,//}$: $1.56 \cdot 10^{10}$ Ω
    - Power to maintain acc: 400MW

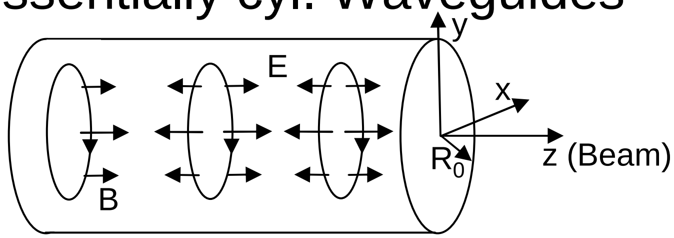
has form of resistor parallel with beam load

- Ratio $\frac{Z_s}{Q} = \frac{V_{acc}}{\omega E_s L}$:
  - depends on geometry of cavity only

- Efficiency of linear accelerator:
  - Energy efficiency = $Z_b/(Z_b+Z_s L)$
    - $Z_b = V_{acc}/i_b$: beam impedance
  - To increase efficiency reduce P_l
    - Accelerate a train of bunches in small time window
    - train repetition rate low

Travelling Wave Structures

- Essentially cyl. Waveguides



  – Operated in TM_{01} mode
  – Travelling waves:

$$E_z(r,z,t) = E_0 J_0(k_c r)\cos(ks - \omega t)$$

$$E_r(r,z,t) = E_0 \frac{\sqrt{\omega^2 - \omega_c^2}}{\omega_c} J_1(k_c r)\sin(ks - \omega t)$$

$$B_\phi(r,z,t) = \frac{E_0}{c}\frac{\omega}{\omega_c} J_1(k_c r)\sin(ks - \omega t)$$

$$\text{with } k = \frac{\sqrt{\omega^2 - \omega_c^2}}{c};\ k_c = \frac{\omega_c}{c} = \frac{2.405}{R}$$

  – ω_c is called "cutoff frequency"
    - wave below ω_c does not travel

phase velocity $$\mathrm{v}_p = \frac{\omega}{k} = c\left(\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}\right)^{-1} > c,\ \frac{\omega}{k} = \frac{1}{\sqrt{LC}}$$

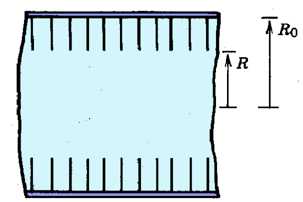
Group velocity $$\mathrm{v}_g = \frac{d\omega}{dk} = c\left(\sqrt{1 - \left(\frac{\omega_c}{\omega}\right)^2}\right) < c$$

  – for long cavities: $v_p \neq v_g$
  – Load waveguide with periodic disks:

    L: unchanged

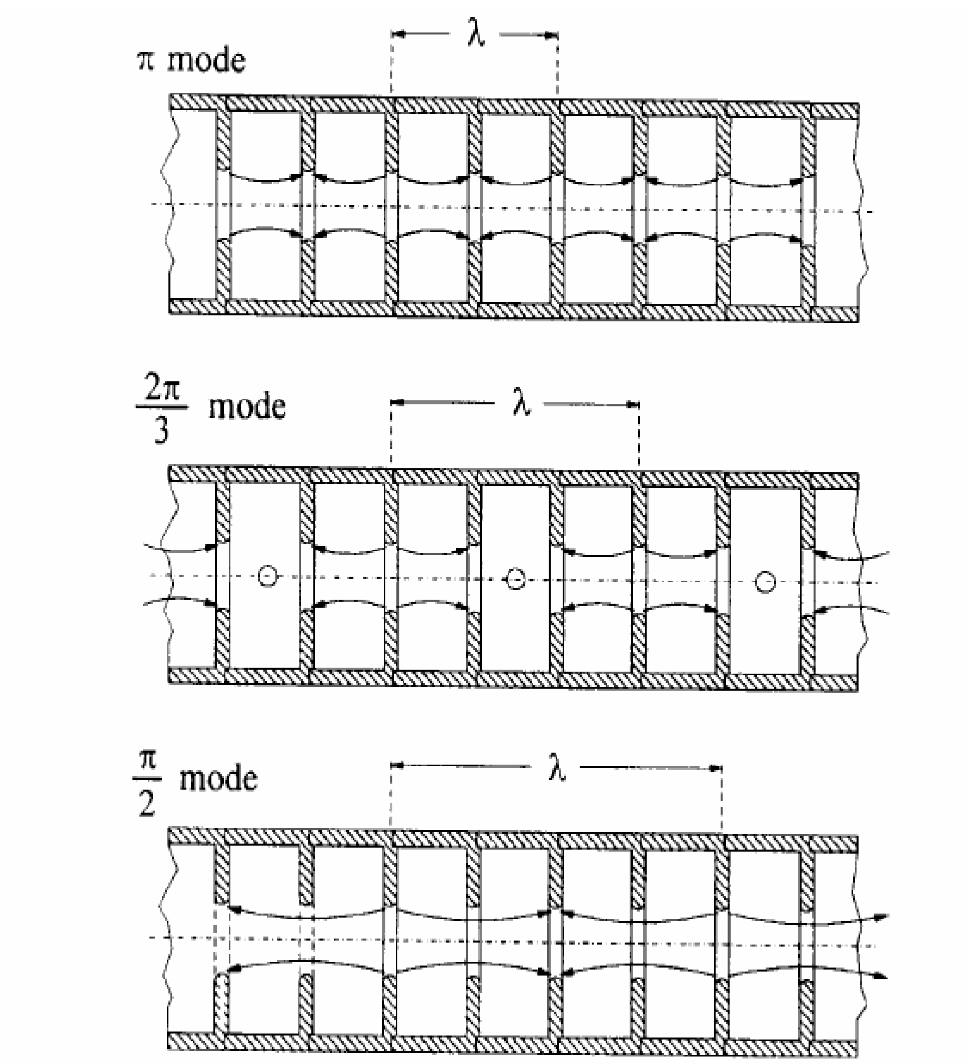
    C: reduced

    phase velocity reduced!



  – disk spacing usually for e^- linacs: $kd = 2\pi/3$
  – Advantage: small power requirement for given acceleration!

Travelling Wave Structures



phase velocity $v_p = \frac{\omega}{k} = c\left(\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}\right)^{-1} > c, \quad \frac{\omega}{k} = \frac{1}{\sqrt{LC}}$

Group velocity $v_g = \frac{d\omega}{dk} = c\left(\sqrt{1-\left(\frac{\omega_c}{\omega}\right)^2}\right) < c$

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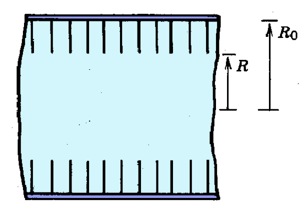

– disk spacing usually for e⁻ linacs: kd=2π/3
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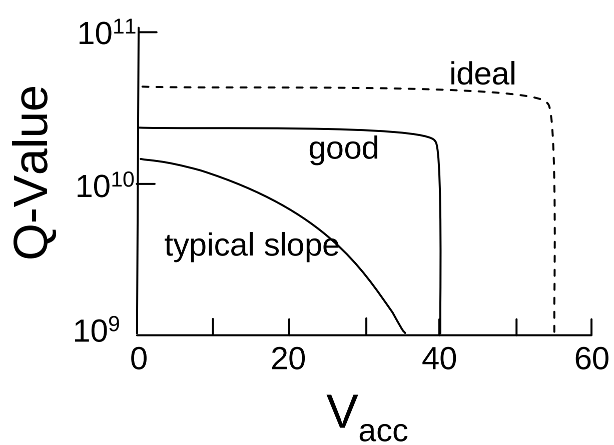
Superconducting Cavities

- Allow high duty cycle
  - Maintain high electromagnetic fields, that would melt normal conducting cavities in cw mode.
- Nearly all RF power goes to the beam
  - Zs ~ 100 GΩ!
- Much higher Q value



- Require liquid helium cooling at ~2˚K
  - No big reduction of wall plug power consumption
- Manufacturing:
  - Super high polished finish required.

Linear Acclerators:

- First stage of almost any acceleration complex after gun
- SLAC (cost)
- Neutron spallation (oak ridge)
- 4th generation light source (excellent beam quality)
- Linear collider (ring too large, excellent beam quality)

Electric field (voltage gradient):

  - $E \lambda$ = constant
  - Higher gradients for higher frequencies
  - Smaller apertures: technically more challenging

- Normal conducting (copper) cavities
  - SLAC 2.856 GHz λ = 1.7 cm
  - NLC 11.4 GHz λ = 0.42 cm
- Former CLIC: with drive beam
  - 25-30 GHz λ = 0.2 cm
  - 150-200 MeV/m
- Superconducting (Niobium) cavities
  - 1.3 GHz λ = 3.7 cm
  - Chosen for ILC (more mature technology)
  - Theoretical max gradient 55MeV/m
- Plasma: 1-100 GeV/m

Problems for lecture 3

- You would like to use a Reentrant cavity as output cavity in a klystron. The klystron should output microwaves with a frequency ν=1GHz. Find dimensions for such a cavity. (try to come up with a set of parameters you could actually use to construct a cavity)
- Cavity of defunct Adone accelerator (e^-/e^+ storage ring) in Frascati (Italy) from 1967. From the picture estimate the frequency at which this cavity operated assuming T_{010} mode.
- For pillbox cavity with r=10cm, L=6.7cm δ=2μm, shunt impedence Z_s=77MΩ/m and total drive power of 500kW calculate Q, gradient, Transit time factor and hence V_{acc}.