Lecture 4
Beam Optics 1

Problem Solutions from last week

Introduction

Modification of E- and B- Fields by Materials

Ferromagnetism

Guide Fields of Particle Beams

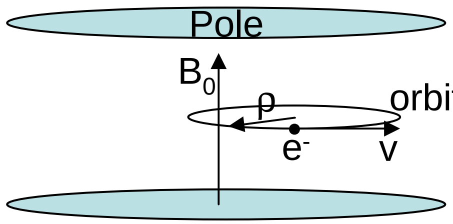
Betatron Oscillations / Weak Focusing

Strong Focusing

Problems

Introduction

- Simple guide field: uniform field



  - Lorentz force: centripetal accel.:

$$\frac{v^2}{\rho} = \frac{evB_0}{m} \Rightarrow \frac{1}{\rho} = \frac{eB_0}{mv} \text{ or}$$

$$\frac{1}{\rho}[m^{-1}] = 0.2998\frac{B_0}{p}\left[\frac{T}{GeV/c}\right]$$

  - Magnetic rigidity for single charge:

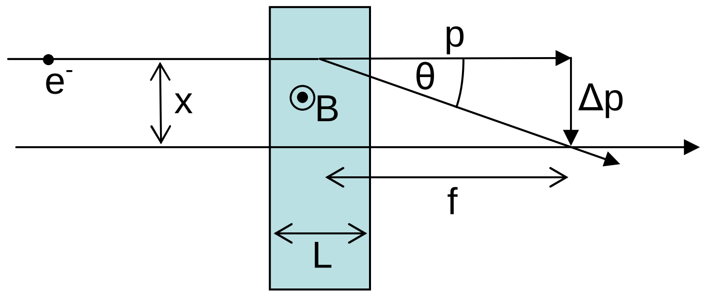
$$B\rho\ [T \cdot m] = \frac{p}{0.2998}[GeV/c]$$

  - Cyclotron frequency

$$\omega = \dot{\theta} = \frac{qB}{m}$$

- Focusing fields are however required to produce/keep "focused" beams

- Magnetic Lens



  - $$\Delta p = F\Delta t = evB_y \frac{L}{v} = eB_y L$$

$$\theta \cong \frac{\Delta p}{p} = \frac{eB_y L}{p} = \frac{x}{f}$$

$$\text{quadrupole: } B_y = \frac{\partial B_y}{\partial x} x = B'x$$

$$\Rightarrow \theta = \frac{eB'xL}{p} \text{ or } \frac{1}{f} \cong \frac{eB'L}{p} = \frac{B'L}{B\rho}$$

    - quadrupole magnet acts like focusing lens of focal length f.

Modification of E- and B- Fields by Materials

Dielectrics

- Can store more electrostatic field energy than vacuum.
- Storage densitiy of water (80 larger than vacuum): Basis for much of modern pulsed power technology.
- Reduce v_g of el. mag. waves. Helps to match v_{rf} with $v_{high\text{-}energy\ particles}$

Ferromagnetic materials

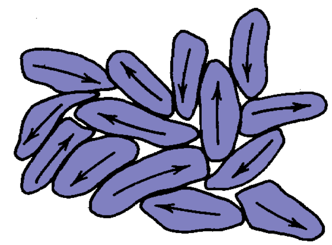
- Used by all high-energy accelerators
- Shape magnetic fields
  - Role analogous to electrodes in electrostatics
  - Shaped Iron surfaces (poles): generate complex field distributions
- Amplify flux change produced by a real current (important for transformers)

Paramagnetic materials

- With no external field molecules are randomly aligned
- With external field neighbouring molecules align with opposite polarity to minimize field energy.
- $B_{parmagnetic}=\mu/\mu_0\ B_{external}$
- μ/μ_0 is relative permeability
- $\mu_{paramagnet}$ less than $1+10^{-6}$

Ferromagnetic materials

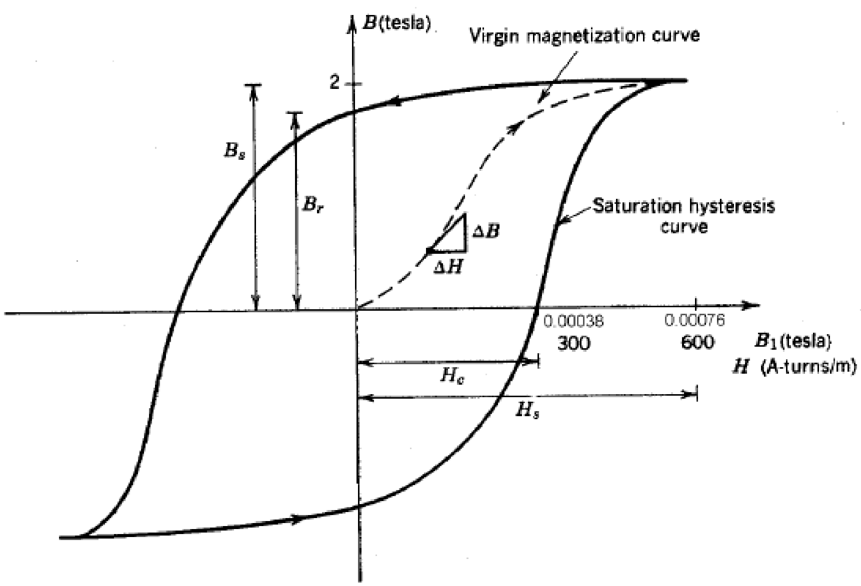
- Respond to $B_{external}$ by shifting domain boundaries.
- With no $B_{external}$ domains randomized



- Size of domain ~ 1000 atoms
- $\mu_{ferromagnet}$ as high as 10000!
- Saturation (complete alignment) can occur at field strengths (~2T)

Ferromagnetism

- Unmagnetized material: B=H=0
  - Use current along a loop around a iron core for example to raise B-field
  - Domains become aligned when B-field raised along dashed line called virgin magnetization curve
  - Unless material is demagnetized

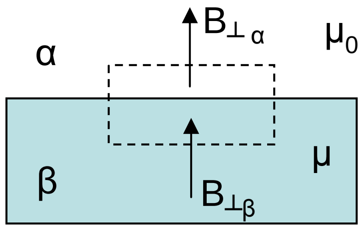


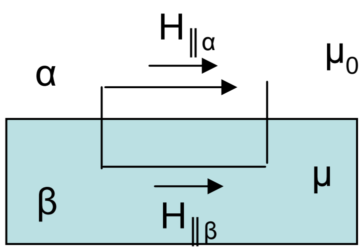
- Typically magnets are switched on and off all the times:
  - Curve does not follow virgin magnetization because of energy needed to shift domain boundaries
  - Reverse current applied: magnetic field driven to reverse saturation.
  - Figure shows special case: saturated hysteresis curve.

Soft (ferro) magnetic material:
  - Small force needed to drive to saturation (~10Oe)
  - Used to conduct field lines

Hard (ferro) magnetic material:
  - Large force needed (up to 8000Oe)
  - Used for permanent magnets

Ferromagnetism



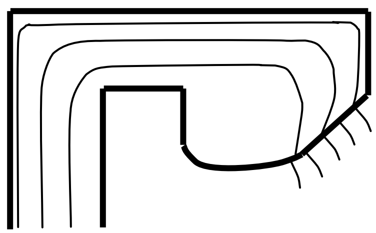


- Magnetic Poles

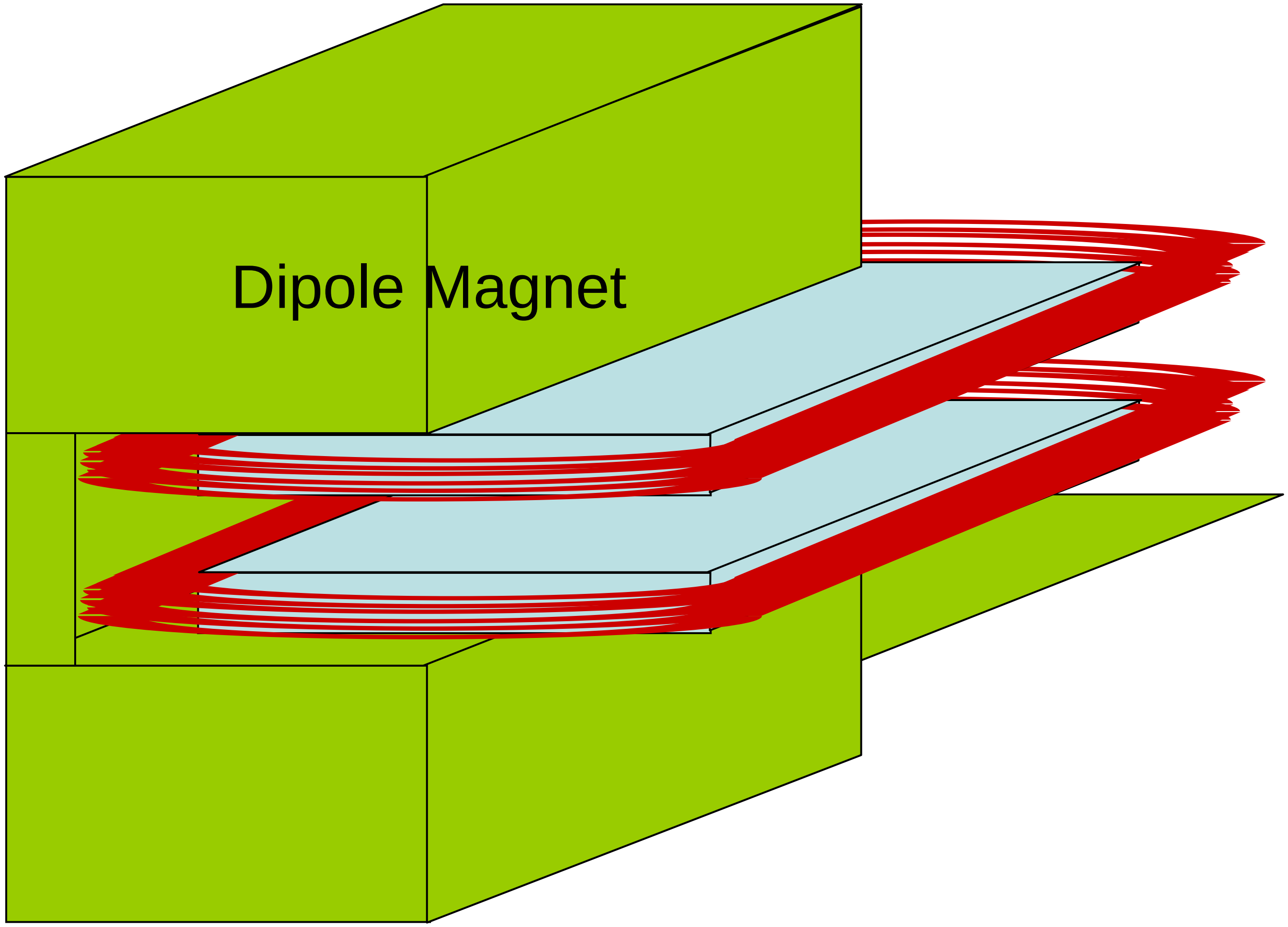
$$\nabla \cdot B = 0 \Rightarrow B_{\perp\alpha} = B_{\perp\beta}$$

$$\int (H_\alpha - H_\beta) = 0 \Rightarrow \frac{B_{\parallel\alpha}}{\mu_0} = \frac{B_{\parallel\beta}}{\mu}$$

  - For $\mu \gg \mu_0$ $B_{\parallel outside} \ll B_{\parallel inside}$ → magnetic field lines just outside ferromagnet are close to ⊥.



  - Ferromagnetic surfaces used to shape magnetic field lines are called pole pieces.


Dipole Magnet

Guide Fields of Particle Beams

- In the region of the Beam

$$\vec{\nabla}\times\vec{B}=0\Rightarrow\vec{B}=-\vec{\nabla}\Phi$$
$$\vec{\nabla}\bullet\vec{B}=0\Rightarrow\nabla^2\Phi=0$$

  - Magnetic scalar potential is solution to LaPlace's equation

$$\Phi=\sum_{n=1}^{\infty}\Phi_n r^n\sin n\theta$$

  - For idealized field magnetic potential only function of x,y!
  - Multipole expansion gives:

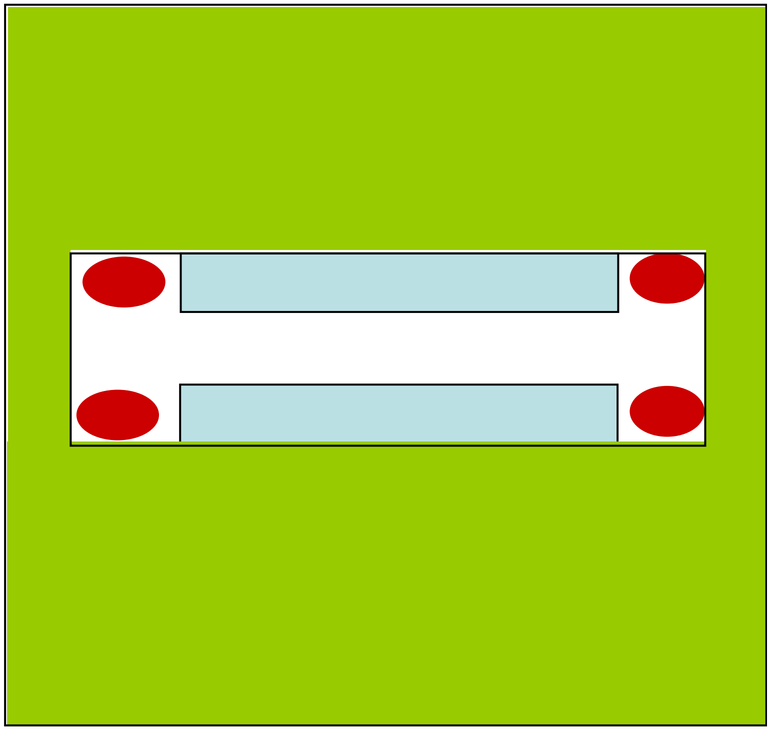
$$B_y=\boldsymbol{B_0}+\boldsymbol{B'x}-\tilde{B}'y+\frac{\boldsymbol{B''}}{\boldsymbol{2}}\boldsymbol{(x^2-y^2)}-\tilde{B}''xy+\dots$$

$$B_x=\tilde{B}_0+\tilde{B}'x+\boldsymbol{B'y}+\frac{\tilde{B}''}{2}(x^2-y^2)+\boldsymbol{B''xy}+\dots$$

dipole terms — quadrupole terms — sextupole terms

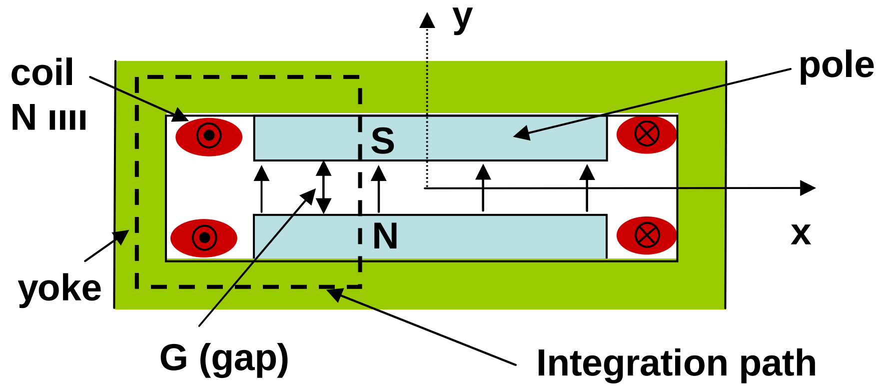
terms without ~: "**normal**" terms

terms with ~: "skew" terms



Guide Fields of Particle Beams

- Dipole Magnet



$$\oint \vec{H} d\vec{l} = I_{enclosed} \text{ around integration path}$$

– Inside iron:

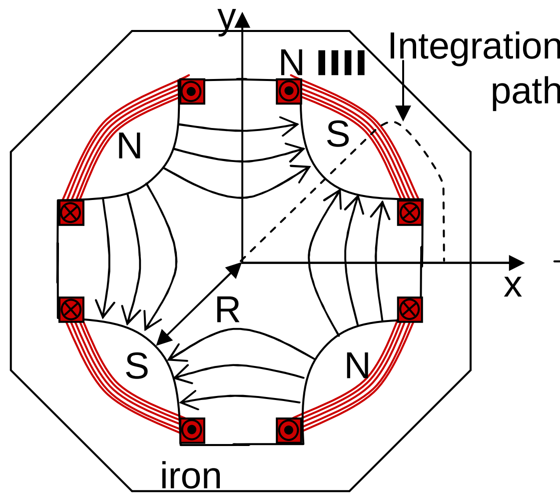
$$\vec{H} = \frac{\vec{B}}{\mu} \rightarrow 0 \text{ for infinite permeability iron}$$

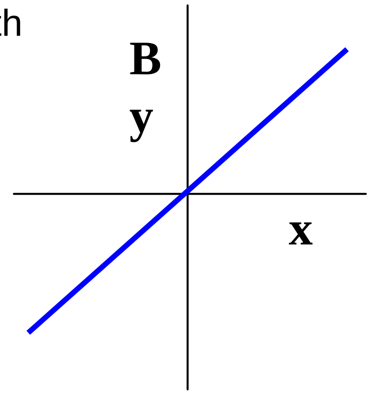
$$H = \frac{2NI}{G} \Rightarrow B[T] = 2.52 \frac{NI[kA \cdot turns]}{G[mm]}$$

Dipole bends pos. particle to left

Rotate by 90º: pure skew dipole

- Quadrupole Magnet





$$\oint \vec{H} d\vec{l} = I_{enclosed} = \frac{1}{\mu_0} \int_0^R B' r dr = \frac{B' R^2}{2\mu_0} = NI$$

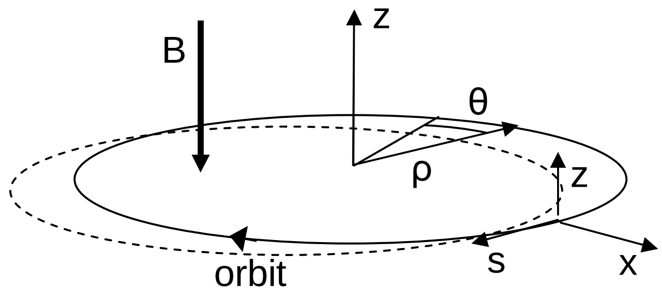
$$\Rightarrow B' = \mu_0 \frac{2NI}{R^2} = \left[\frac{T}{m}\right] = 2.51 \frac{NI[A \cdot turns]}{R^2[mm^2]}$$

$$\text{Focal length: } f = -\frac{p}{eB'L} = \frac{(B\rho)}{B'L}$$

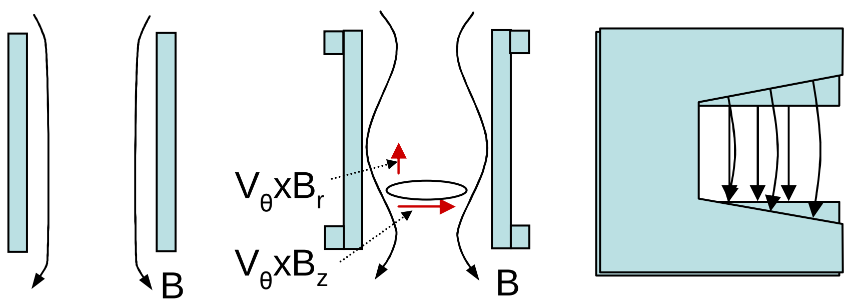
Focusing in x, defocusing in y

Rotate by 45º: pure skew quad.

Betatron Oscillations / Weak Focusing



- Beam bound in x but divergent in z!
  - Need B_r to get force in z direction



- This is called weak focusing
  - in x by dipole field
  - in z by gradiant/quadrupole field

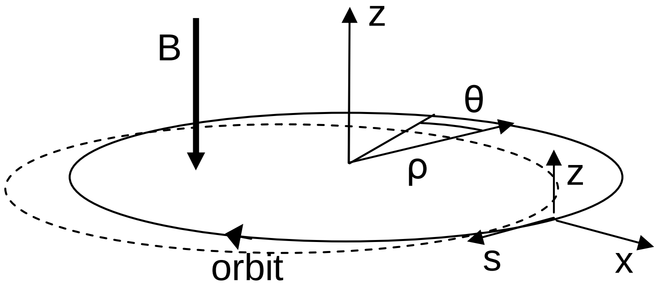
- Motion in a linear focusing force F_x (e.g. array of quad lenses along s)
  - $F_x = F_0(z/z_{beam})$.
  - No acceleration: $$\frac{d^2 z}{dt^2} = \frac{F_0}{\gamma m_0 v_s^2}\frac{z}{z_{beam}}$$
  - Paraxial approx: x'^2, $y'^2 << 1$: $$z(s) = z_0 \cos\left(\frac{2\pi s}{\lambda_z} + \varphi\right),\ \lambda_z = 2\pi\sqrt{\frac{\gamma m_0 v_s^2 z_b}{F_0}}$$

  Same wavelength for all particle orbits (difference: amplitude and phase)
  - Harmonic transverse particle motion: Betatron Oscillations
  - Using field index $$n = -\frac{\rho}{B_z}\frac{\partial B_z}{\partial r}$$

    Number of oscillations per turn:

    $$Q_x = \frac{1}{\rho}\sqrt{1-n},\ Q_z = \frac{1}{\rho}\sqrt{n}$$: betatron Freq.

Betatron Oscillations / Weak Focusing



- Beam bound in x but divergent in z!
  - Need B_r to get force in z direction

horizontal: $$\frac{1}{p_0}\frac{d}{ds}\left(p_0\frac{dx}{ds}\right)+\frac{1}{\rho^2}(1-n)=0$$

vertical: $$\frac{1}{p_0}\frac{d}{ds}\left(p_0\frac{dx}{ds}\right)+\frac{n}{\rho^2}z=0$$

$$\frac{1}{\rho^2}:\ \text{weak focusing strength}$$

- This is called weak focusing
  - in x by dipole field
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Same wavelength for all particle orbits (difference: amplitude and phase)

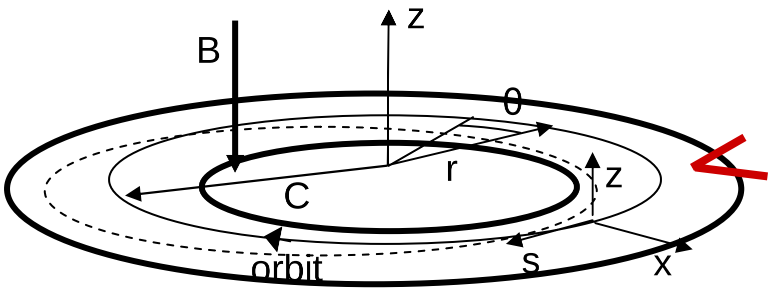
- Harmonic transverse particle motion: Betatron Oscillations
- Using field index $$n=-\frac{\rho}{B_z}\frac{\partial B_z}{\partial r}$$

Number of oscillations per turn:

$$Q_x=\frac{1}{\rho}\sqrt{1-n},\ Q_z=\frac{1}{\rho}\sqrt{n}$$: betatron Freq.

Stable in z if n>0
Stable in x if n<1 } 0<n<1

Strong Focusing



- Acceptance limited by vacuum tube.
  - Large acceptance means: transport of particles with wide spread in angle and momentum
  - Acceptance grows for larger n
    - Requires additional focusing lenses is n>1.

$$\nu_{\text{circular systems}} = \frac{\text{frequency of transverse oscillations}}{\text{frequency of rotation}}$$

  - Short $\lambda_{betatron}$: ν is high
    - or for higher ν: stronger focusing
  - Instability for ν=i (integer)

- Weak focusing treated with global solutions.
- Strong focusing will require us to pay attention to individual elements
- Next lecture

Problems

- Have a look at the magnet exhibits and make a sketch (including relevant sizes):

1. which multipole fields can these magnets generate
2. Assume the quadrupole on display has 30 windings per coil and is operated at 1A. This quadrupole was used in the local Van-de Graff, that can produce about 1MeV/c protons. What is the focal length of this quadupole under these conditions?
3. Calculate the weak focusing strength for the Tevatron:

   B=4.4 T

   p=1 TeV