CPSC 2S98
XPHC 2S98/2S99
DPHD 2S98/2S99
DPHE 2S98/2S99

FIRST PUBLIC EXAMINATION

Preliminary Examination in Physics

SECOND PUBLIC EXAMINATION

Honour School of Physics,
Parts A and B: 3 and 4 year Courses

SHORT OPTIONS

Tuesday, 9 June 2008 (?)

9:30 a.m. to 11:00 a.m. for candidates offering ONE Short Option
9:30 a.m. to 12:30 p.m. for candidates offering TWO Short Options

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Answer **two** *questions from each option for which you have entered.*

Start the answer to each question on a fresh page.

If you have entered for two Short Options,
keep your answers to the two options in different books
and at the end hand in two bundles, one for each option.

A list of physical constants and conversion factors accompanies this paper.

The numbers in the margin indicate the weight that the Examiners anticipate assigning to each part of the question.

Do NOT turn over until told that you may do so.

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Some sections start with a relevant rubric.

Section S19 PARTICLE ACCELERATOR SCIENCE

Formulae that you may find useful

Energy radiated by synchrotron radiation:

$$E_0 = \frac{4}{3}\pi \frac{r_e}{(m_e c^2)^3} \frac{E^4}{\rho}$$

Where E_0 is the energy radiated per turn due to synchrotron radiation by an electron with an energy E when its orbit has a radius of curvature ρ. $r_e = 2.8 \times 10^{-15}$ m is the classical radius of the electron.

Critical photon energy of synchrotron radiation:

$$\epsilon_c = \frac{3}{2}\frac{\hbar c \gamma^3}{\rho}$$

Where γ is the relativistic factor of the beam and ρ its radius of curvature.

Magnetic rigidity:

$$B\rho = \frac{p}{e}$$

Where B is the magnetic field, p the particles' momentum, e the particle charge and ρ the bending radius.

1. Figure 1 shows the accelerator chain used to produce collisions of 920 GeV protons on 27.6 GeV electrons at DESY.

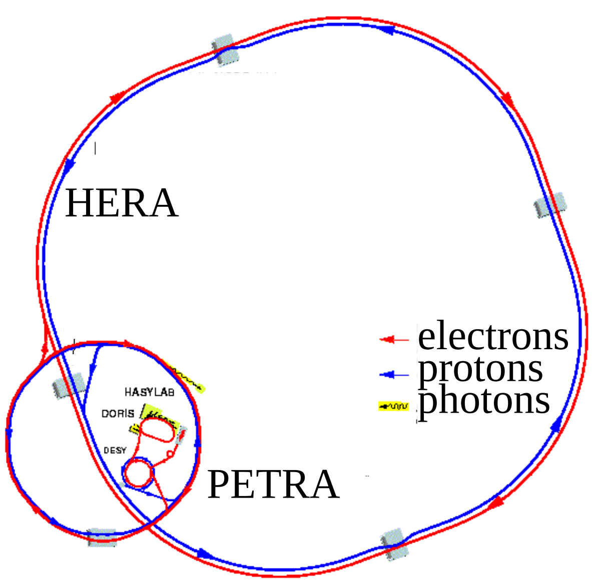

Figure 1: The HERA accelerator chain

a) *Particle sources* Explain briefly how it is possible to produce electrons and protons suitable for acceleration in particle accelerators. [4]

Answer

- Electrons can be produced using a cathode (1). Electrons are released from the cathode when it is heated (thermionic gun) or hit by a laser (photo-cathode). These electrons are then accelerated away from the cathode by an electric field(1). *Mention of the cathode and the electric fields is enough to get full marks*
- Protons can be produced by ionising Hydrogen(1) in an intense electric field (1).
- *Alternative answer* Protons can be produced by sending a H^- (1) beam through a striping (1) device (foil, laser,...).

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b) *Booster ring* Electrons, positrons and protons are then injected in booster rings. The electrons are injected in their booster ring at an energy of 450 MeV and they are ejected at an energy of 6 GeV. The booster uses 8 accelerating cells giving an acceleration of 1.7 MeV each. At the nominal frequency the particles travel 292.8 m in this circular ring. Neglecting the energy losses, estimate how many turns are necessary to reach the ejection energy and how much time such acceleration requires. Give a reason why the electrons loose energy at each turn and estimate what is the energy lost per turn and per electron at the ejection energy? [5]

Answer

The energy gain per turn is $8 \times 1.7\,\mathrm{MeV} = 13.6\,\mathrm{MeV}$. The total energy gain is $6\,\mathrm{GeV} - 450MeV = 5.55\,\mathrm{GeV}$. So $\frac{6\,\mathrm{GeV}-450MeV}{8\times1.7\,\mathrm{MeV}} = 408$ turns are needed. (1)

Each turn takes $\frac{292.8}{c} = 9.8 \times 10^{-7}s$, so 408 turns take 0.4 ms. (1)

Electrons travelling a circular orbit loose energy due to synchrotron radiation. (2)

Using the formula given one electron looses 2.46 MeV per turn. (2)

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c) *Beam emittance* The beam geometric emittance at injection is 10 mm mrad (both vertical and horizontal) and it is 350 nm rad (horizontal) by 35 nm rad (vertical) at ejection. Using your knowledge of storage rings, explain why the emittance has been reduced. Explain why the vertical emittance is 10 times smaller than the horizontal emittance. [6]

Answer

The beam emittance is reduced by a factor $\frac{1}{\gamma}$ due to adiabatic cooling when the electron are accelerated.(2)

The electrons travelling in the storage ring emit synchrotron radiation in the transverse plane. When they emit synchrotron radiation the recoil increases their transverse emittance. This is compensated by further adiabatic cooling to recover the energy lost. As the adiabatic cooling reduces both the vertical and horizontal emittance the vertical emittance will be reduced more than the horizontal one in a storage ring. This is called radiation damping. (4)

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d) *Collider ring* The HERA ring has a circumference of 6.2 km but it is in fact made of straight sections and of curved sections. When the protons reach their maximum energy (920 GeV) the magnets reach a field of 4.7 T. Estimate the radius of curvature of the HERA ring. The electrons ring is similar to the protons ring. Calculate the field required for the bending magnets of the electrons at 27.6 GeV. [4]

Answer

The magnetic rigidity formula given can be rewritten as

$$\rho = \frac{p}{Be} = \frac{E}{cBe} = \frac{920\,\mathrm{GeV}}{3.00 \times 10^{8}\,\mathrm{m\,s} \times 4.7\,\mathrm{T} \times 1 \times e} = 653\,\mathrm{m}$$

(2)

And for the electrons:

$$B = \frac{p}{\rho e} = \frac{E}{c\rho e} = \frac{27.6\,\mathrm{GeV}}{3.00 \times 10^{8}\,\mathrm{m\,s} \times 653\,\mathrm{m} \times 1 \times e} = 141\,\mathrm{mT}$$

(2)

2.

a) Give the formula expressing the force exerted by an electron on another electron situated at a distance d. Give the formula expressing the resulting acceleration. [1]

Assuming that the electrons are 1 um apart, calculate the force exerted by the electrons onto each other and their acceleration. [2]

Answer

Coulomb's law:

$$f = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{d^2} = \frac{e^2}{4\pi\epsilon_0 d^2}$$

(0.5)

$$a = \frac{f}{m_e} = \frac{e^2}{4\pi\epsilon_0 m_e d^2}$$

(0.5)

With $d = 1\,\text{um}$ we get:

$$f = \frac{e^2}{4\pi\epsilon_0 10^{-12}} = 2.307 \times 10^{-16}\,\text{N}$$

(1)

$$a = \frac{e^2}{4\pi\epsilon_0 10^{-12} * m_e} = 2.53 \times 10^{14}\,\text{m/s}^2$$

(1)

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b) Let's consider a spherical bunch of radius R. The bunch contains 10^{10} electrons with a uniform density. Using Gauss law derive the radial force f experienced by an electron situated inside the bunch at a position r_b, θ_b, ϕ_b (in spherical coordinates). [3]

Taking $R = 1\,\text{mm}$, draw $f(r_b)$ between 0.1 mm and 2 mm. [2]

Answer

The charge of the bunch is Q and its volume is $\frac{4}{3}\pi R^3$, so the charge density is $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$.

The charge of a sphere of radius r_b will be $Q' = Q(\frac{r_b}{R})^3$ (1)

Apply Gauss' law:

$$E \times 4\pi r_b^2 = \frac{Q'}{\epsilon_0} = \frac{Q(\frac{r_b}{R})^3}{\epsilon_0}$$

(2)

hence (the field is radial):

$$E_r = E = \frac{Q\frac{r_b}{R^3}}{4\pi\epsilon_0}$$

using $f(r_b) = eE_r$ we get: $f = \frac{Qe\frac{r_b}{R^3}}{4\pi\epsilon_0}$. (2)

Equations for the graph:
For $r_b < R$, $f(r_b) = r_b * 2.307 * 10^{-9}\,\mathrm{N/m}$. (1)
For $r_b > R$, $f(r_b) = \frac{Qe}{4\pi\epsilon_0 r_b^2} = \frac{2.307\times10^{-18}}{r_b^2}\,\mathrm{N}\times\mathrm{m}^2$. (1)

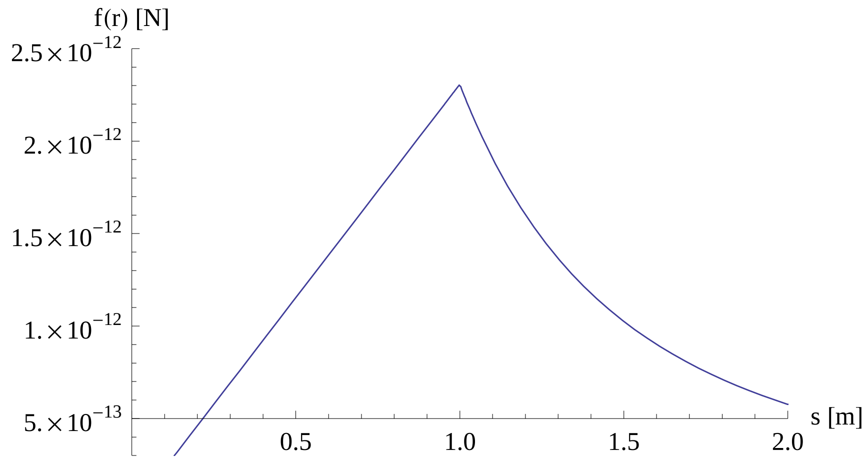

Figure 2: Plot for question 2b (2)

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c) This electron bunch is injected in a 3 GeV accelerator used to produce X-rays. What will be in keV the critical photon energy of the synchrotron radiation produced when the bunch passes through a 2 T dipole magnet? The bunch travels 0.31 m in the magnet. Estimate the total energy of the radiation produced by the bunch. Express the result in Joules. [3]

Answer

This can be calculated using the formula provided.

$$\epsilon_c = \frac{3}{2}\frac{\hbar c \gamma^3}{\rho}$$

with $\gamma = 5871$. Use the magnetic rigity formula to find ρ: $\rho = \frac{p}{Be} = 5.0m$. (1)
Hence $\epsilon_c = \frac{3}{2}\frac{\hbar c\gamma^3}{\rho} = 11.98\,\mathrm{keV}$. (1)
Radiation lost per turn:

$$E_0 = \frac{4}{3}\pi \frac{r_e}{(m_e c^2)^3} \frac{E^4}{\rho} = 1.43\,\mathrm{MeV}$$

The electron travels 0.31 m in the dipole, that is $(\pi/10)$ m, so it travels $\frac{1}{20}$ turns and radiates 71.7 keV $= 1.14\times 10^{-14}$ J. So 10^{10} electrons will radiate 1.14×10^{-4} J $= 11.4$ mJ. CHECK !!! (2)

Suggest and describe a device that could be used to enhance the radiation flux. [2]

Answer

A wiggler can be used to enhance the amount of synchrotron radiation produced. (1) *The word undulator will also be accepted.*

A wiggler is an array of magnets with alternating polarity. When an electron goes through a wiggler, it wiggles (or undulate) and thus emits synchrotron radiation. (2)

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d) In an accelerator it is often important to control accurately the position of the beam. Name a device that can measure the beam charge and a device that can measure the beam position and explain briefly how they operate. [4]

Answer

Acceptable answers (1) mark each for the name and (2) marks for describing how it operates.

Beam position

- A beam position monitor measure the position of a beam by comparing the signal of 4 electromagnetic pickups.
- a scintillating screen emits radiation at the position at which it is traversed by the beam. This can be observed by a screen.

Beam charge

- a Faraday cup captures all the charged particule in a beam. The total charge going out of the cup to the ground can then be measured.
- A toroid (or a wall current monitor or an integrating current transformer): the beam induces a current propoprtional to the charge in the toroid, the current can then be measured to get the charge (with appropriate calibration).

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e) Explain what is the emittance of a beam. Describe briefly a method used to measure the transverse emittance of a beam in a single shot. [2]

In a transfer line a commonly used method of transverse emittance measurement involves the measurement of the beam size at several locations. Describe this method in details, explaining how many measurements are needed and how the transverse emittance is deduced from these measurements. [2]

Answer

The emittance of a beam is the volume it occupies in the six-dimensions momentum-position space. The emittance of a beam should be conserved when no acceleration is applied.

The emittance of a beam can be measured in a single shot by using the pepper-pot method: by using an array of holes the beam is split in several beamlets which are measured for example on a screen. For each beamlet the position is known (it is the position at which the beam went through the hole) and the divergence can be measured by looking at the size of the beam on the screen (compared to the hole aperture).

It is also possible to compute the emittance of a beam by measuring its size at 3 or more properly spaced locations. The Courant-Snyder parameters of the beam can then be calculated by making a parabolic fit. The emittance can be deduced from the β parameter combined with the beam size.

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f) A linac accelerates pulses of 10^{10} charged electrons at a repetition rate of 1 Hz up to an energy of 160 MeV over 33 m using 4 klystrons each delivering 1 μs, 12 MW RF pulses. What would be the length of the linac needed to reach 920 GeV and what would be the wall-plug power consumption of such linac assuming a klystron efficiency of 20%? Would this power consumption change if the single electron pulses were replaced by a 0.8 ms-long train of electron pulses 5 μs apart and containing 10^{10} electrons each (assuming that the klystrons fire for the same duration and at the same repeition rate)? How much of this energy is transferred to the electron beam (in the case of single pulses and in the case of pulses trains)? Comment on this results. [4]

Answer

The linac needs 33 m to reach 160 MeV. To reach 920 MeV it will need

$$\frac{33\,\text{m}920\,\text{GeV}}{160\,\text{MeV}} = 189750\,\text{m} = 189.750\,\text{km}$$

(1)

The klystrons would require $4 \times 12\,\text{MW} \times 1\,\mu\text{s} \times \frac{920\,\text{GeV}}{160\,\text{MeV}} * \frac{1}{20\%} = 1.38\,\text{MW}$. (1) Accelerating one bunch takes 1.38 MJ. The same amount of power is needed to accelerate a train of pulses as long as the train is shorter than 1 ms (1). At 920 GeV the 10^{10} electrons have a total energy of 1.47 kJ. So the transfer effeciency is 10.7×10^{-4}. (1)

A 0.8 ms train of pulses that are 5 μs apart contains approximately 160 pulses. As the power consumption is the same, the effeciency would be 160 times better, that is 0.17. In fact such high effeciency would not be reached because the accelerating cavities would be depleted as the electrons bunches extract energy, so it would be necessary to increase the RF power injected by the klystrons. (1)

3.

An electron transport line (shown on figure 3) is made of the following elements (the beam line starts at $s = 0$):

- A bending magnet BEND1, at $s = 1.0\,\text{m}$;
- A sextupole sf, at $s = 2.0\,\text{m}$;
- A sextupole sd, at $s = 2.5\,\text{m}$;
- A quadrupole QF.0, at $s = 3.0\,\text{m}$;
- A dipole giving a vertical deflection vkick, at $s = 3.5\,\text{m}$;
- A quadrupole QD.1, at $s = 4.0\,\text{m}$;
- A quadrupole QF.1, at $s = 5.0\,\text{m}$;
- A dipole giving a horizontal deflection hkick, at $s = 5.5\,\text{m}$;
- A quadrupole QD.2, at $s = 6.0\,\text{m}$;
- A quadrupole QF.2, at $s = 7.0\,\text{m}$;
- A bending magnet BEND2, at $s = 8.0\,\text{m}$;

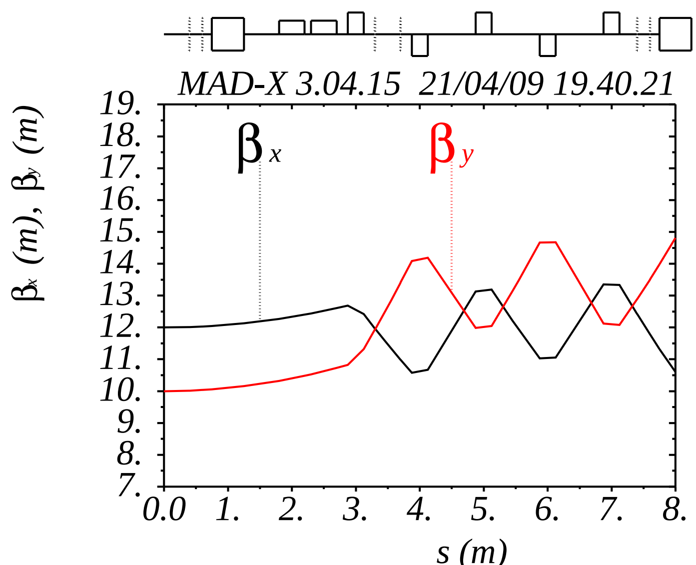

Figure 3: Beam line

a) Explain briefly how each type of magnet mentioned above affects the beam and what purpose it serves in a typical beamline. [4]

Answer

- Dipole: Bends the trajectory of the beam. It is used to steer the beam. (1)
- Bending magnet: a type of dipole.
- Quadrupole: Focusses the beam. (1)
- Sextupole: corrects higher order effects such as chromatic aberrations. (1)

b) At $s = 0\,\text{m}$, the beam horizontal β-function is $\beta_x = 10\,\text{m}$ and the vertical β-function is $\beta_y = 12\,\text{m}$. The beam geometric R.M.S. emittance is 1 mm mrad. Estimate the horizontal R.M.S. beam size at $s = 0$. Can the beam pipe have the same size than the beam? Describe briefly two phenomena than can affect the beam quality with a too small beam pipe. Explain what is the acceptance of an accelerator. [5]

Answer

The R.M.S beam size at s=0 is $\sigma = \sqrt{\epsilon \times \beta_y} = 3.16\,\text{mm}$. (1)

No, the beam pipe must be wider than the beam.

A too small beam affects the beam in several ways *2 answers expected, (0.5) marks each*:

- Clipping (some particles are lost because they hit the beam pipe)
- Generation of secondary "halo" particles when the beam pipe is hit by other particules
- Wakefield effects: the beam is kicked by its own wakefield
- Impedance coupling: the beam induces a strong current in the beam pipe and looses a significant amount of power.
- Material damage: when the beam hits the beam pipe the energy deposited damages the metal of the pipe

The acceptance of an accelerator is the size of the hole of the vacuum chamber, transformed into the beam phase space. A particle outside the acceptance of the accelerator is likley to hit the beam pipe at some point. (1)

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c) The quadrupoles have a focal length of 2 m. QF-type quadrupoles are focusing in the vertical plane and QD-type quadrupoles are focusing in the horizontal plane. Assume that all other elements are set so that they have no effect on the particles' vertical trajectory. The quadrupoles can be considered to have a negligible length (zero-length thin lens approximation).

Consider a particle located at $y = 1\,\text{mm}$ above the beam axis at $s = 2.5\,\text{m}$ and with no transverse momentum. Use matrix multiplications to estimate the particle's position with respect to the beam axis at $s = 4.5\,\text{m}$. [7]

Answer

The matrix for a quadrupole with a focal length f can be written as follow:

$$\begin{pmatrix} 1 & 0 \\ \frac{1}{f} & 1 \end{pmatrix}$$

where f is positive for a defocussing quadrupole and negative for a focussing quadrupole.

The space between the quadrupoles is a drift space described by the following matrix:

$$\begin{pmatrix} 1 & le \\ 0 & 1 \end{pmatrix}$$

where le is the length of the drift space.

So the beam line from $s = 2.5\,\mathrm{m}$ to $s = 4.5\,\mathrm{m}$ can be written as follow:

drift(0.5) × quad(-f) × drift(1) × quad(f) × drift(0.5)

This leads to the following multiplication:

$$\begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{-2} & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{pmatrix} \begin{pmatrix} 1 & 0.5 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1.375 \\ -0.25 \end{pmatrix}$$

The particle position at $s = 5.5\,\mathrm{m}$ is $1.4\,\mathrm{mm}$.

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d) Assuming that the beam studied in (b) has a low intensity and an energy of 1 GeV. What diagnostics would you use to measure the beam size? Explain the basic principle of this diagnostic. [4]

Answer

The following answers are acceptable:

- The beam size can be measured using a scintillating screen that emits light where it is traversed by a high energy beam.
- The beam size can be measured using a wire-scanner or a laser-wire. In these devices a wire crosses the beam generating X-rays. A measurement of the X-ray emission as a function of the wire position gives the beam profile and hence the beam size.

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e) In the beam line described above, one of the horizontally focussing quadrupoles has in fact been positioned improperly. Its magnetic axis is parallel to the beam axis but it is displaced by $x = +1\,\mathrm{mm}$ horizontally with respect to the axis of the beam line. Vertically the two axis are still properly aligned. A bunch travelling along the beam axis ($x = 0\,\mathrm{mm}; y = 0\,\mathrm{mm}$) traverses this quadrupole. Describe and calculate the effect that the quadrupole misalignement will have on the bunch (both horizontally and vertically)? A scintillating screen is installed 500 mm after this quadrupole. How the image observed on this screen compares with the one that would be observed if the quadrupole was properly aligned? [5]

Answer

A misaligned quadrupole can be decomposed in the combinaison of a dipole and a quadrupole, both aligned with the beam axis. Thus the misalignment will result in a bending effect. (1)

Vertically the two axis are still aligned and thus there will be no effect.

Horizontally a particle crossing the quadrupole 1 mm away from the quadrupole's axis will have its trajectory bent so that it crosses the quadrupole axis on the focal point (located 2 m further down the line). Thus the beam will be deflected by 0.5 mrad in the $-x$ direction. This particle will then arrive off the magnetic axis of the quadrupoles further down the line. This is likely to increase the fraction of the beam lost in the transfer line. (1)

Because of the kick introduced by the misaligned quadrupole, the image on the screen will be offset by 0.25 mm in the $-x$ direction in comparison with what would be observed if the quadrupole was properly aligned. This illustrate why both active and passive alignement systems are very important in a particle accelerator. (1)