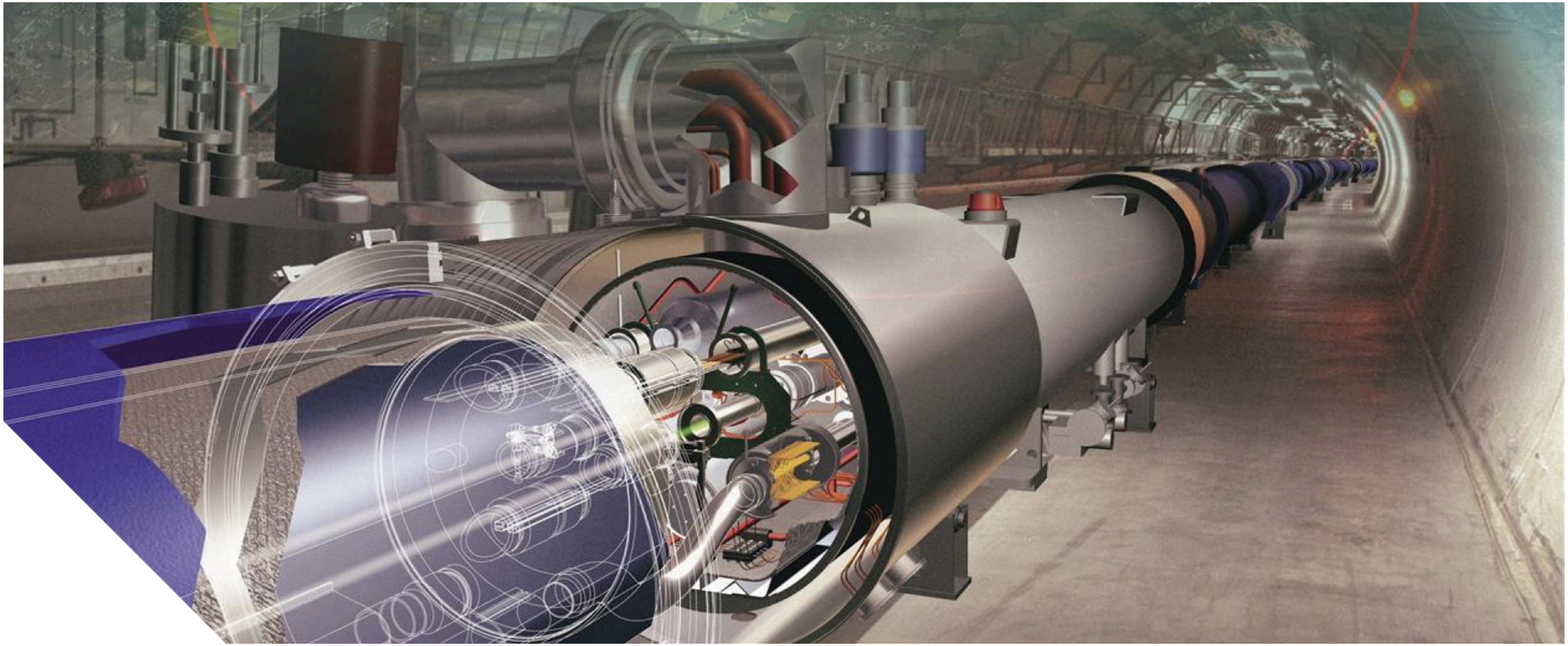


QCD – Lecture 1

Introduction, DIS and the Quark Parton Model

Claire Gwenlan, Oxford, HT

outline

- **Lecture 1:** Introduction, DIS and the Quark Parton Model
- **Lecture 2:** Quantum Chromodynamics: theory and experimental evidence
- **Lecture 3:** QCD-improved Parton Model, and the DGLAP equations
- **Lecture 4:** PDFs and their uncertainties
- **Lecture 5:** PDFs, the Gluon and α_s
- **Lecture 6:** QCD at Low x and Low Q^2
- **Lecture 7:** Implications for and constraints from the LHC, and the Future

Guest Lecture by Gavin Salam:

- **Lecture 8:** QCD, jets and jet substructure

aims of the course and additional resources

This course aims to cover the **basics of QCD**, from the development of the **Quark Parton Model**, though to the establishment of **QCD as a gauge theory of the strong interaction**. It will provide a survey of **experimental measurements of QCD from low to high energies** that have served to establish today's understanding. Methods used to extract the **parton distribution functions (PDFs)** of the proton will be covered, and the **implications for the LHC** will be discussed. The course is NOT designed to go through in-depth QCD calculations in detail, but will provide steps needed to derive some relevant results.

Many good reference works available, the following in particular may be useful:

Deep Inelastic Scattering – Devenish & Cooper-Sarkar

QCD and Collider Physics – Ellis, Stirling & Weber

Quarks and Leptons – Halzen & Martin

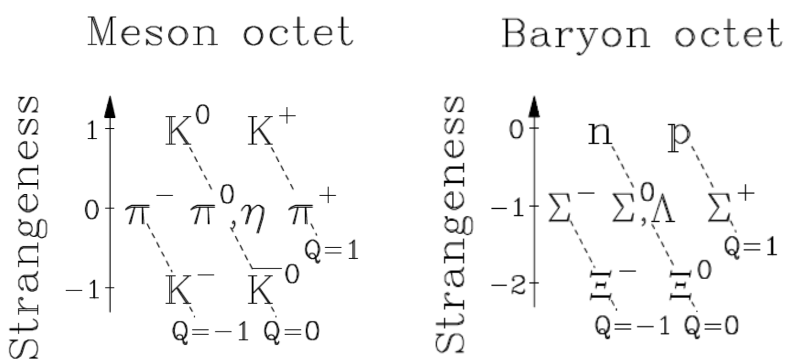
Gauge Theories in Particle Physics – Aitchison & Hey

Handbook of Perturbative QCD – Sterman et al. (CTEQ Coll.), https://www.physics.smu.edu/scalise/cteq/

Lecture Notes and Problem Sheet: http://www-pnp.physics.ox.ac.uk/~gwenlan/teaching/qcd.html

QCD: a quick refresher

- 4 interactions: weak, EM, **strong**, gravity
- 1964: Gell-Mann and Zweig suggested that hadrons were composite, built from 3 basic blocks we now call up, down, strange quarks
- **properties**: spin–1/2; baryon number–1/3, non-integral charge: Qu=+2/3 and Qd,s = –1/3; u and d form isospin doublet with 0 strangeness; s an isospin singlet with unit strangeness

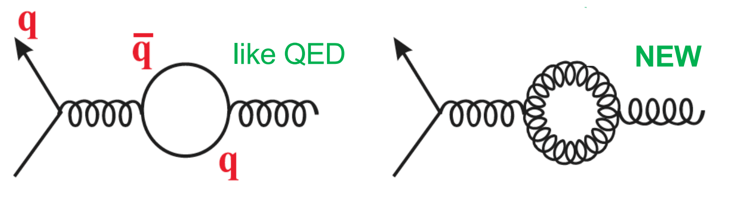


- Quarks carry colour, motivated by existence of EG. $|\Delta^{++}\rangle = |u^{\uparrow}u^{\uparrow}u^{\uparrow}\rangle$
- Must be new quantum number so Ψ antisymmetric (PAULI)

$$\Psi(\Delta^{++}) = \underbrace{\Psi(r)}_{symmetric} \cdot \underbrace{\Psi_{\text{spin}}(J)}_{symmetric} \cdot \underbrace{\Psi_{\text{flavour}}}_{symmetric} \cdot \Psi_{\text{colour}}$$

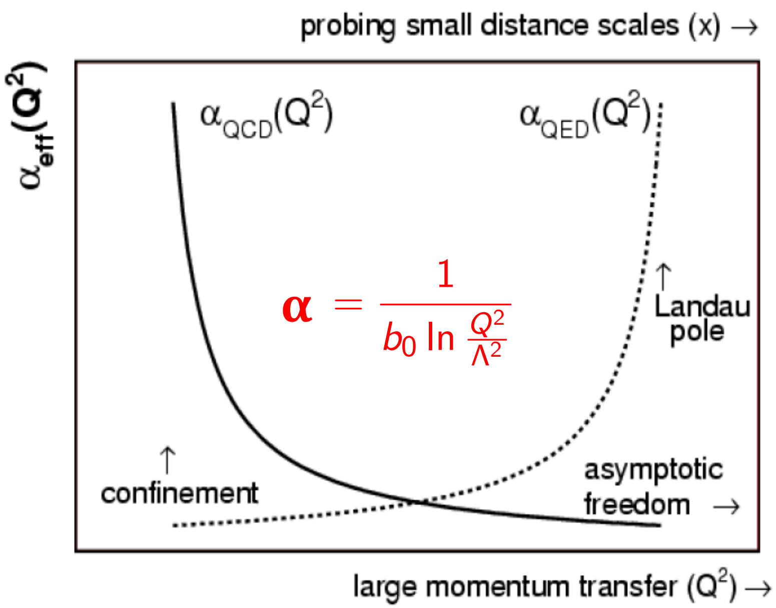
- **QCD:** gluons are the force carriers;
- massless, spin-1 bosons; also carry colour; SU(3) symmetry gives eight gluons

QED: $\alpha = \frac{e^2}{4\pi}$ **QCD:** $\alpha_S = \frac{g_S^2}{4\pi}$

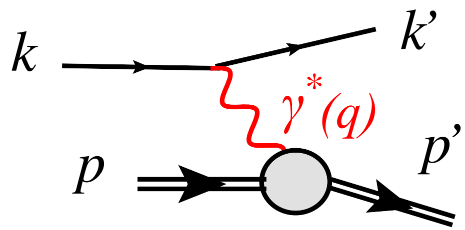


colour charge screening colour charge anti-screening

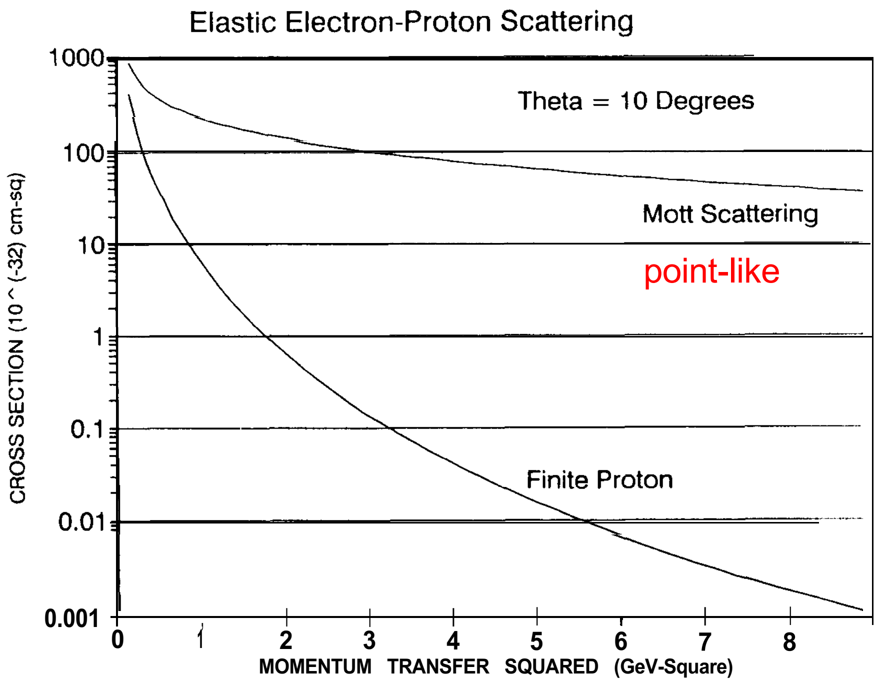
$b_0 = -n_f/6\pi + 11n_c/12\pi$

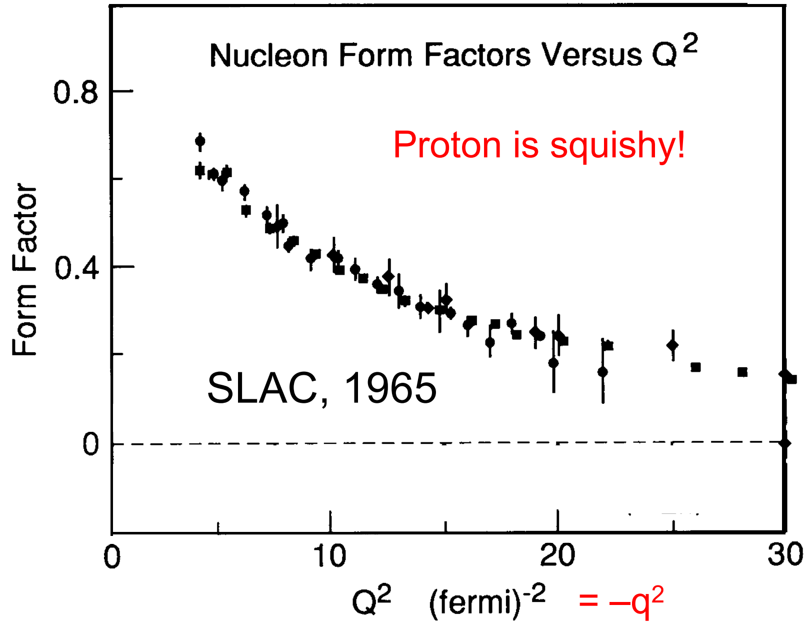


elastic ep scattering



Friedman, Kendall and Taylor, Nobel Lectures, 8 December, 1990





$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \cdot |F(q)|^2$$

Form Factor:
$F(q) = \int d^3r \, e^{iq.r} \rho(r)$

- falling distribution seen in first proton form factor data (McAllister and Hofstader, 1955), and for the next ten years …

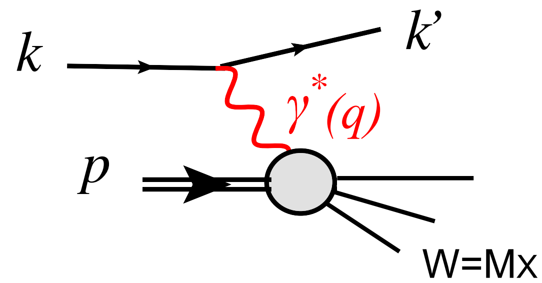
quote from R.E. Taylor:

… The data continued to follow the so-called dipole model to a good approximation. By the Hamburg conference in 1965 there were no dissenters from the view that

$$G_{Ep}(Q^2) \cong \left(\frac{1}{1 + \frac{Q^2}{0.71\ \text{GeV}^2}}\right)^2 \text{ up to } Q^2 \backsim 10\ \text{GeV}^2$$

Form Factor

inelastic ep scattering



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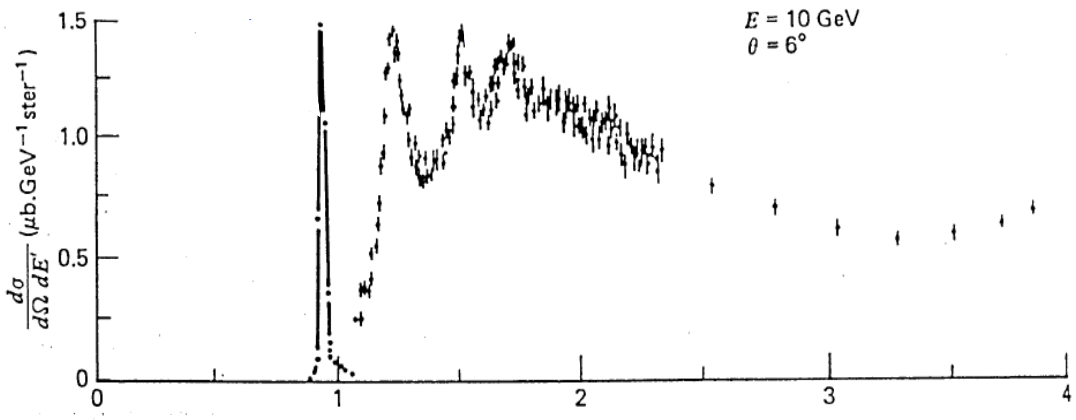
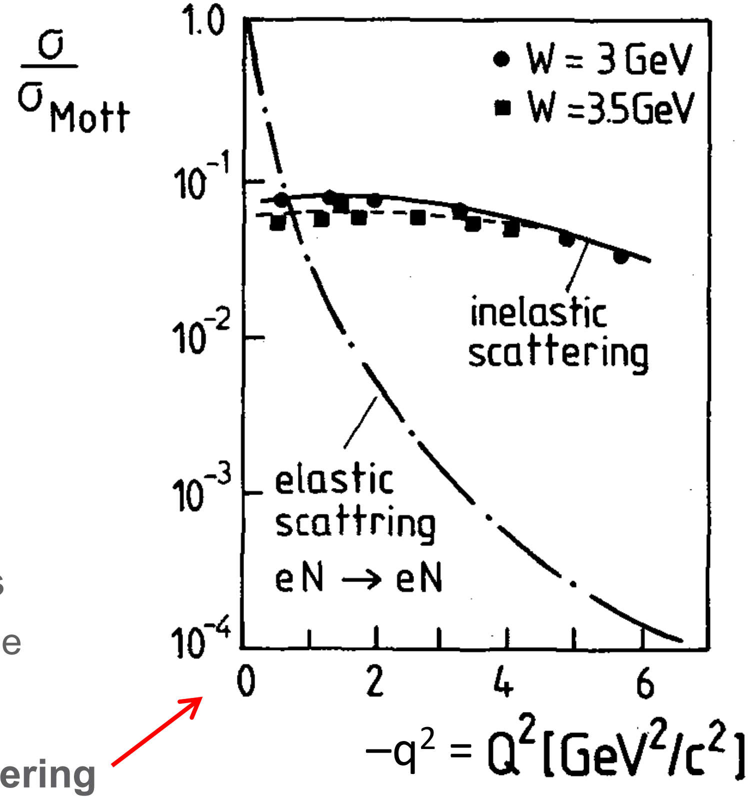
The Structure of Hadrons



W (GeV/c^2) ≡ invariant mass of final state hadronic system

SLAC, 1969



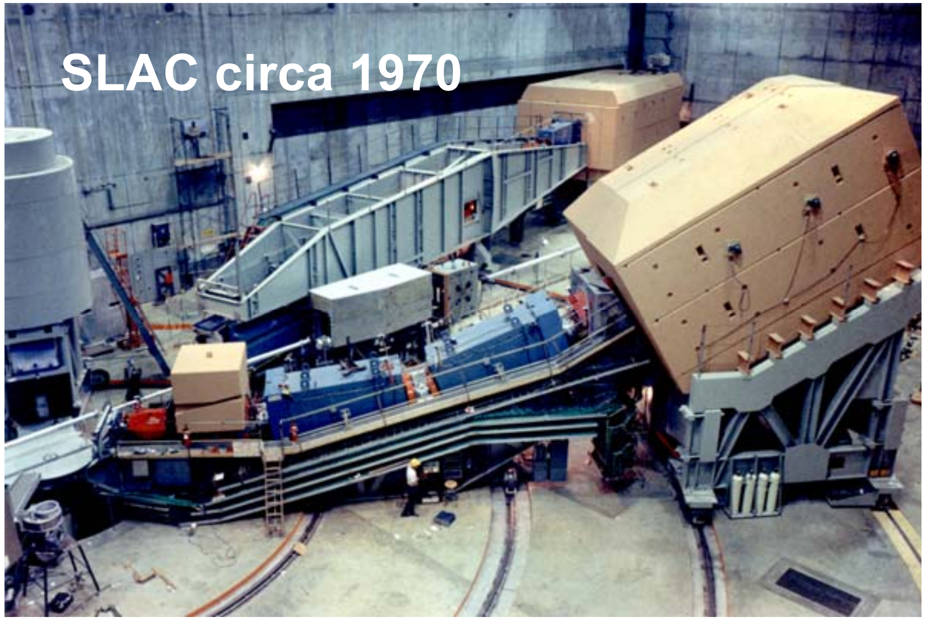
- peaks are from **proton (elastic)** and **baryonic resonances**
- no structure above about W = 1.8 GeV, but there remains a large measured cross section – **this is the region of DIS**
- early example of approximate **scaling** in **inelastic ep scattering**

- **inelastic ep scattering cross section** was behaving as though **proton** contained **hard, point-like scattering centres**

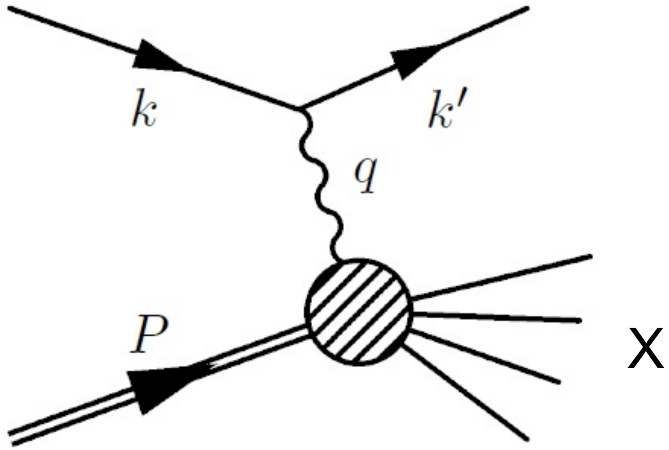
DIS

Neutral Current (NC): exchange of **γ, Z**

Charged Current (CC): exchange of **$W^{\pm}$**



- proton structure first investigated in Deep Inelastic Scattering **(DIS)**
- **DEEP** ≡ high resolving power (high Q^2)
- **INELASTIC** ≡ proton breaks up (high W ≡ Mx)



- Kinematic variables:

$$Q^2 = -q^2 = -(k-k')^2$$

Virtuality of the exchanged boson

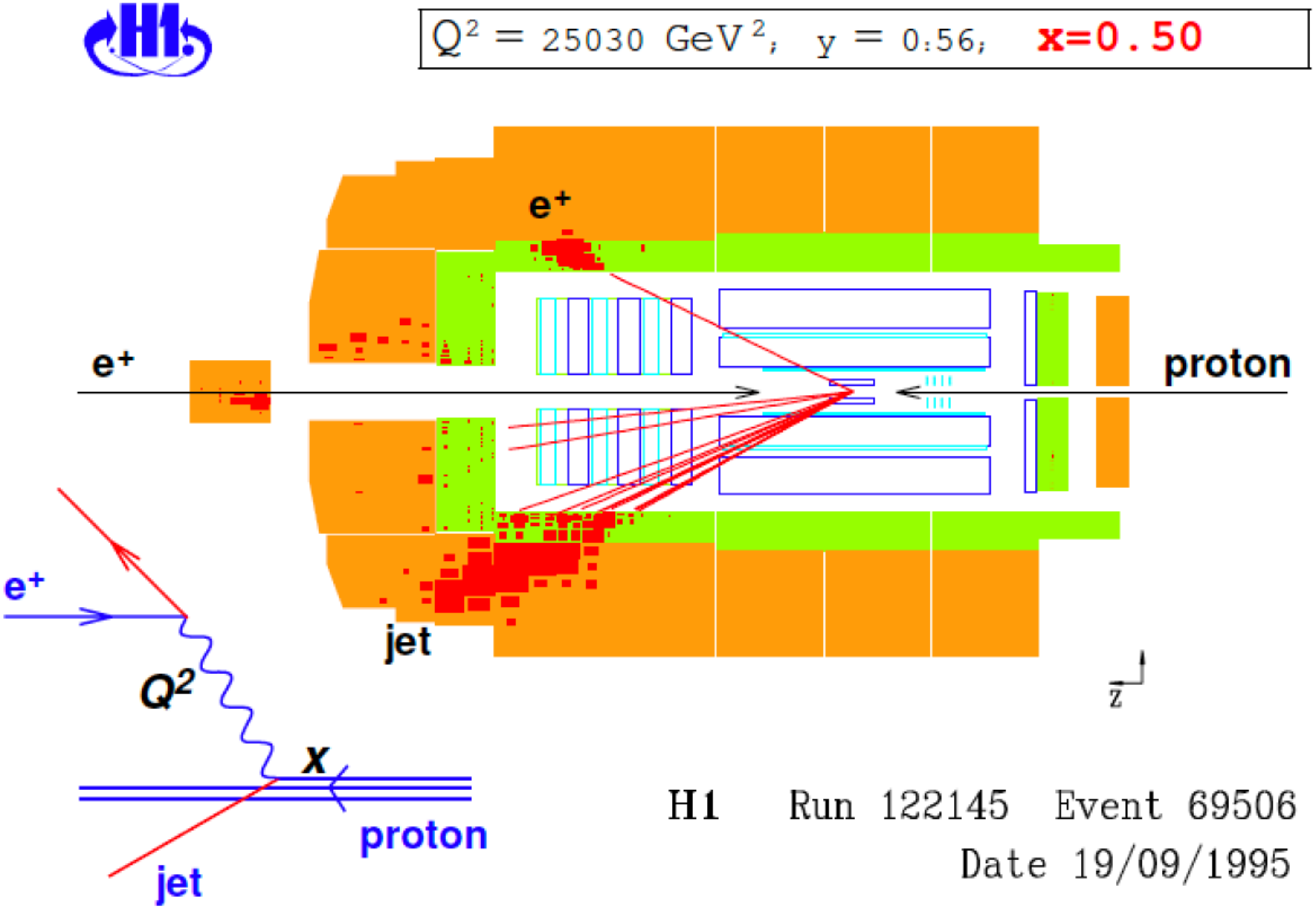
$$x = \frac{Q^2}{2p \cdot q}$$

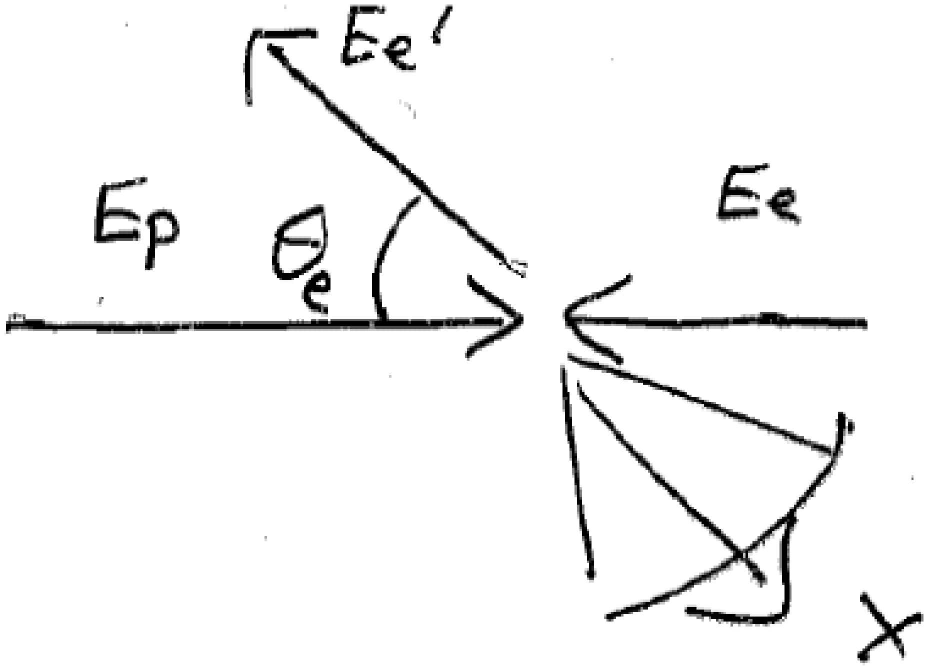
$$y = \frac{p \cdot q}{p \cdot k}$$

$$s = (k+p)^2 = \frac{Q^2}{xy}$$ Invariant c.o.m.

- these are all **4-vector invariants** AND are **experimentally measurable**

NC DIS event at HERA





Recoil Hadrons

(Mass W)

The kinematic variables are measureable both from electron E', θ_e (NC)
AND from hadron variables (CC)

$$s = 4\,E_e\,E_p$$

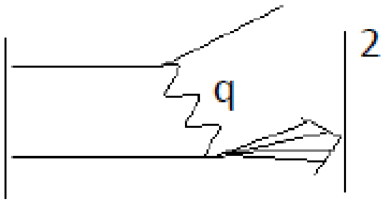
$$Q^2 = 4\,E_e\,E'\,\sin^2\frac{\theta_e}{2}$$

$$y = \left(1 - \frac{E'}{E_e}\sin^2\frac{\theta_e}{2}\right)$$

$$x = Q^2 / s\,y$$

Lorentz Invariant form of the cross section

Schematically,

$d\sigma \sim \sum_x \Big|$  $\Big|^2 \sim L^{\mu\nu} \cdot W_{\mu\nu}$

$L^{\mu\nu}$: Leptonic tensor, calculable ELECTROWEAK

$W_{\mu\nu}$: Hadronic tensor, constrained by LORENTZ INVARIANCE

⇒ Charged lepton-neutral current γ, Z

$$\frac{d^2\sigma(l^\pm)}{dxdQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[Y_+ F_2^{\rm NC}(x,Q^2) - y^2 F_{\rm L}^{\rm NC}(x,Q^2) \mp Y_- xF_3^{\rm NC}(x,Q^2)\right]$$

$$Y_\pm = 1 \pm (1-y)^2$$

(NB, cross sections sometimes written instead in terms of F1, F2, xF3 where: $F_{\rm L} \equiv F_2 - 2xF_1$)

Charged lepton- charged current $W^\pm$

$$\frac{d^2\sigma(l^\pm)}{dxdQ^2} = \frac{G_F^2}{4\pi x}\frac{M_W^4}{(Q^2+M_W^2)^2}\left[Y_+ F_2^{\rm CC}(x,Q^2) - y^2 F_{\rm L}^{\rm CC}(x,Q^2) \mp Y_- xF_3^{\rm CC}(x,Q^2)\right]$$

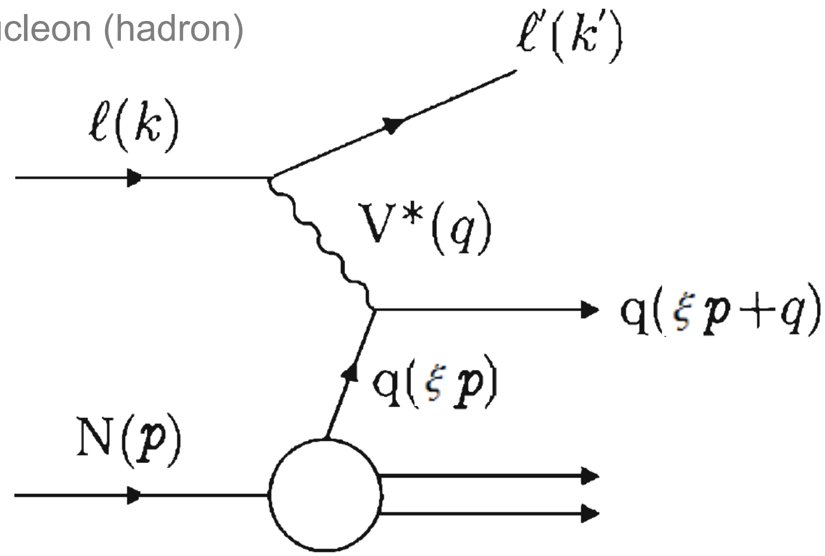
F_2 , F_L , xF_3 are STRUCTURE FUNCTIONS

which parameterise our ignorance of the hadronic sector

⇒ MEASUREABLE as functions of x, Q^2

Quark Parton Model (QPM)

F_2, F_L and xF_3 are structure functions, which express the dependence of the cross section on the structure of the nucleon (hadron)



Quark Parton Model (QPM)

proposed to explain scaling (Feynman, 1969)

interprets the structure functions as related to the momentum distributions of point-like quarks, or **partons**, within the nucleon

the measurable kinematic variable **x = $Q^2/(2p.q)$** is interpreted as the FRACTIONAL momentum of the incoming nucleon taken by struck quark

we can extract all 3 structure functions **experimentally** by looking at the x, y, Q^2 dependence of the double differential cross section – thus, we can **check out the parton model predictions**

$$(\xi p + q)^2 = \xi^2 p^2 + q^2 + 2\xi p \cdot q = 0$$

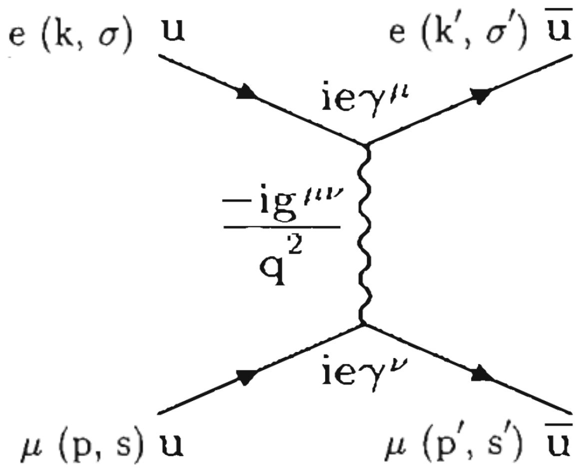
At large Q^2 ignore terms of order M^2 and put quarks on mass shell
$$\rightarrow \xi = \frac{Q^2}{2p \cdot q} = x$$

in proton infinite momentum frame:

FRACTIONAL momentum of the incoming nucleon taken by the struck quark is the MEASUREABLE quantity **x**

outline of cross section calculation

Consider elastic electron-muon scattering in γ exchange — Devenish & Cooper-Sarkar, p17



$$j_e^\nu = ie\bar{u}(k',\sigma')\gamma^\nu u(k,\sigma)$$

$$j_{\text{muon}}^\mu = ie'\bar{u}(p',s')\gamma^\mu u(p,s)$$

Matrix element is
Current . propagator. current

$$\mathcal{M} = i\frac{ee'}{q^2}\left[\bar{u}(k',\sigma')\gamma_\mu u(k,\sigma)\right]\left[\bar{u}(p',s')\gamma^\mu u(p,s)\right]$$

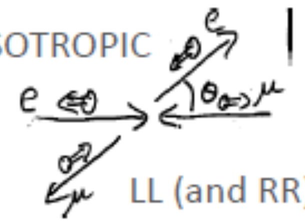
sum + average over spins σ', σ & s, s'

$$\frac{1}{4}\sum_{\text{spins}}|\mathcal{M}|^2 = \frac{e^2e'^2}{q^4}L_e^{\mu\nu}L_{\mu\nu}^{\text{muon}} = \frac{e^2e'^2}{Q^4}2s^2[1+(1-y)^2] \qquad Q^2 = sy$$

$$\Rightarrow \frac{d\sigma}{dy} = \frac{2\pi\alpha^2}{Q^4}\left(\frac{e'}{e}\right)^2\left[1+(1-y)^2\right]s$$

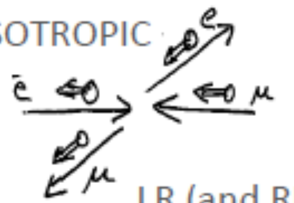
$$y = \frac{(1-\cos\theta)}{2}$$

ISOTROPIC



LL (and RR)

NON-ISOTROPIC



LR (and RL)

As
y → 1, θ → π

(θ = angle in electron – muon ZMF)

σDIS and Parton Distribution Functions (PDFs)

Scattering from ANY fermion (e.g. quarks) similar → depends on fermion charge e'

Consider a collection of quarks and antiquarks (in a hadron)
→ any one can be struck
→ let this one have *x* of the protons momentum, so *s* → *xs*

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4}\left(e^i\right)^2\left[1 + (1-y)^2\right] x\,s$$

for a quark of charge ($e^i e$)

so for the HADRON

why antiquarks?
see later!

$$\frac{d^2\sigma}{dx\,dy} = \frac{2\pi\alpha^2}{Q^4}\left[1 + (1-y)^2\right] s \sum_i (e_i)^2 [x\,q_i(x) + x\,\overline{q}_i(x)]$$

where q(x) is the PROBABILITY of the quark having the momentum fraction *x* of the momentum distribution and *xq(x)* is called a PARTON DISTRIBUTION FUNCTION (PDF)

Now compare the above quark parton model $\frac{d^2\sigma}{dx\,dy}$ to the general predictions

Bjorken scaling and the Callan-Gross relation

Rewrite the double differential cross section for the QPM using $Q^2 = s.x.y$

QPM
$$\frac{d^2\sigma}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[1 + (1-y)^2\right]\sum_i (e_i)^2 [x\,q_i(x) + x\,\bar{q}_i(x)]$$

General
$$\frac{d^2\sigma(l^\pm)}{dx\,dQ^2} = \frac{2\pi\alpha^2}{Q^4 x}\left[Y_+ F_2(x, Q^2) - y^2 F_L(x, Q^2) \mp Y_- \, x F_3(x, Q^2)\right]$$

remember $Y_+ = \left[1 + (1-y)^2\right]$

⇒ QPM predictions

$$F_2(x, Q^2) = \sum_i e_i^2 \, x \,[q_i(x) + \bar{q}_i(x)]$$

i.e. F_2 is only a function of **x** → BJORKEN SCALING

contrast elastic scattering $F(q^2) \sim \dfrac{1}{(1 + q^2/m_0^2)^2}$

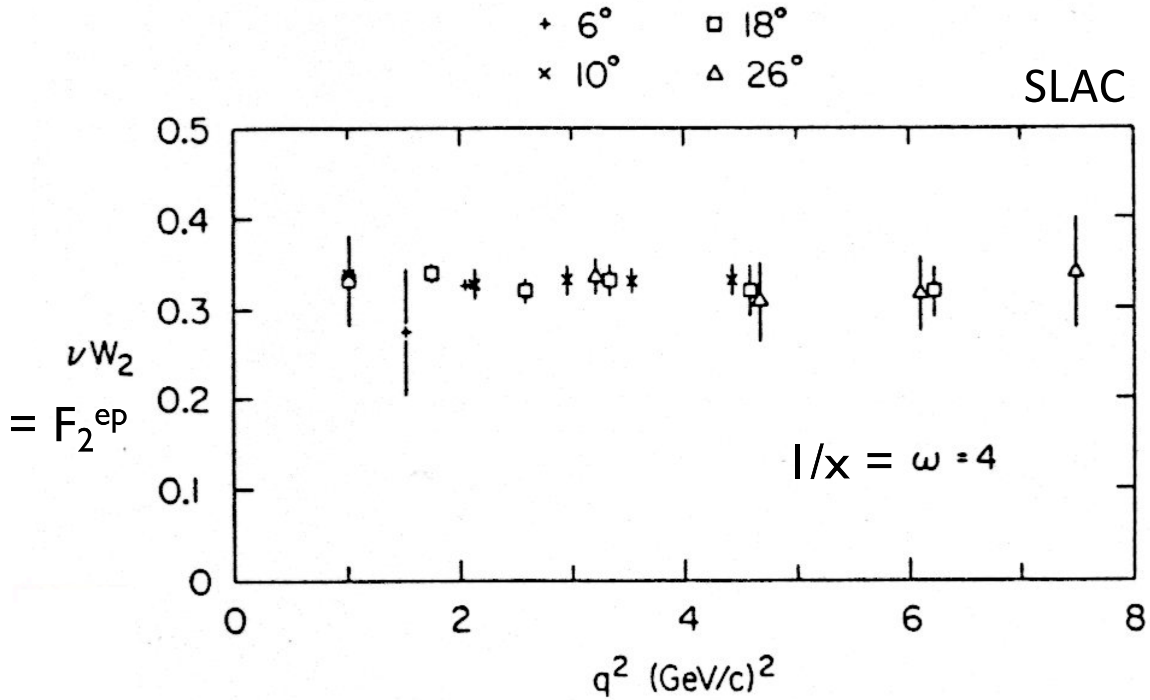
the **QPM** predicts both **BJORKEN SCALING** and the **CALLAN-GROSS** relation

$F_L(x, Q^2) = 0$ because quarks are spin ½ fermions

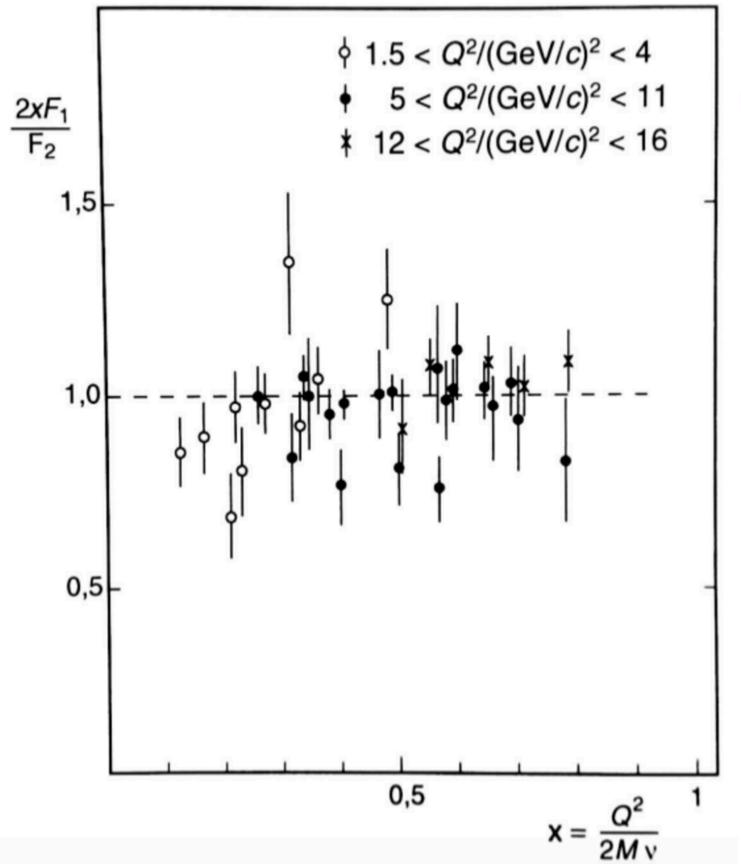
$x F_3(x, Q^2) = 0$ because only γ exchange considered

The results are for charged-lepton/hadron scattering via γ exchange and are independent of lepton charge

Early observations that F_2 is independent of Q^2 → point-like scattering centres "partons" in the nucleon (otherwise $F_2 \sim 1/Q^2$)



Let's see the early data on Bjorken scaling, and on $F_L = 0$



$$F_L \equiv F_2 - 2xF_1 \quad \equiv 0 \text{ for spin } 1/2$$

Callan-Gross relation:

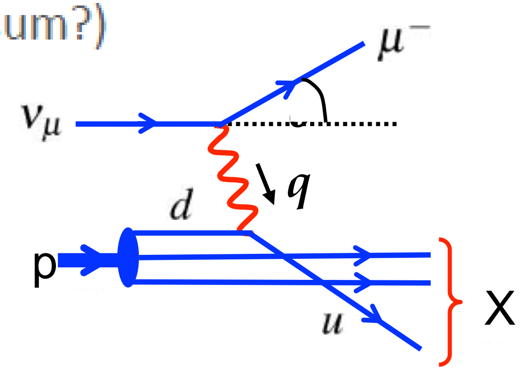
$$2xF_1(x) = F_2(x)$$

Establish spin $^1/_2$ nature of partons

… early DIS measurements supported the **QPM**

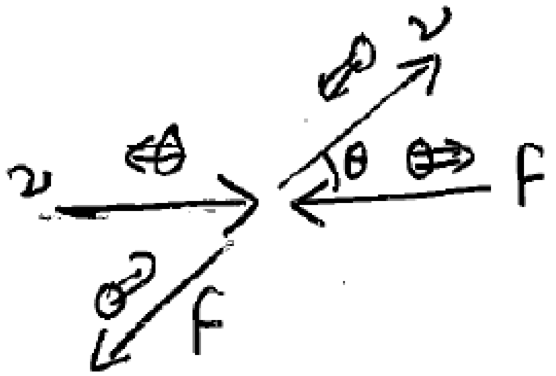
neutrino-quark scattering

⇒ BUT there's more! (what made me include antiquarks in this sum?)



FIRST evidence came from neutrino-induced DIS

Consider $\nu, \bar{\nu}$ which are left/right handed, and at low energy $M_W >> Q$



νf scattering - both left handed, NO net spin along beam direction → ISOTROPIC

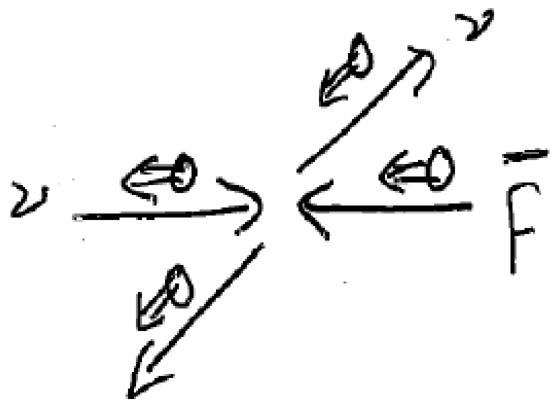
$$\frac{d\sigma}{dy} = \frac{G_F^2 s}{\pi} \sim \text{ independent of } y = \left(\frac{1-\cos\theta}{2}\right)$$

Similarly, $\bar{\nu}\bar{f}$ both right handed

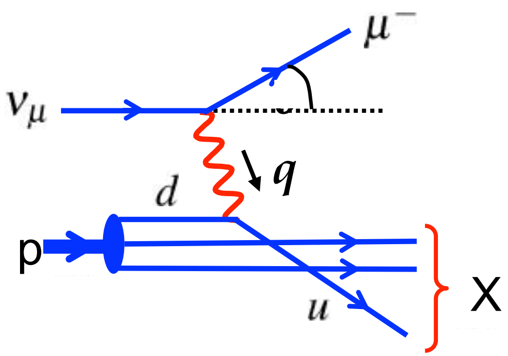
$\nu \bar{f}$ left-right → net spin along the beam direction
→ NOT ISOTROPIC

$$\frac{d\sigma}{dy} = \frac{G_F^2 s}{\pi}(1-y)^2$$

Similarly, $\bar{\nu} f$ (right-left)



neutrino-quark scattering



Consider $\nu, \bar{\nu}$ scattering via the $W^{\pm}(cc^{\nu})$

AND at low enough energy (Q^2) that $\dfrac{M_W^4}{(Q^2 + M_W^2)^2} = 1$

The equivalent result for νq scattering is $\dfrac{d\sigma(\nu q)}{dy} = \dfrac{G_F^2}{\pi} xs$ ISOTROPIC

and for $\bar{\nu} q$ scattering $\dfrac{d\sigma(\bar{\nu} q)}{dy} = \dfrac{G_F^2}{\pi}(1-y)^2 xs$ NON-ISOTROPIC

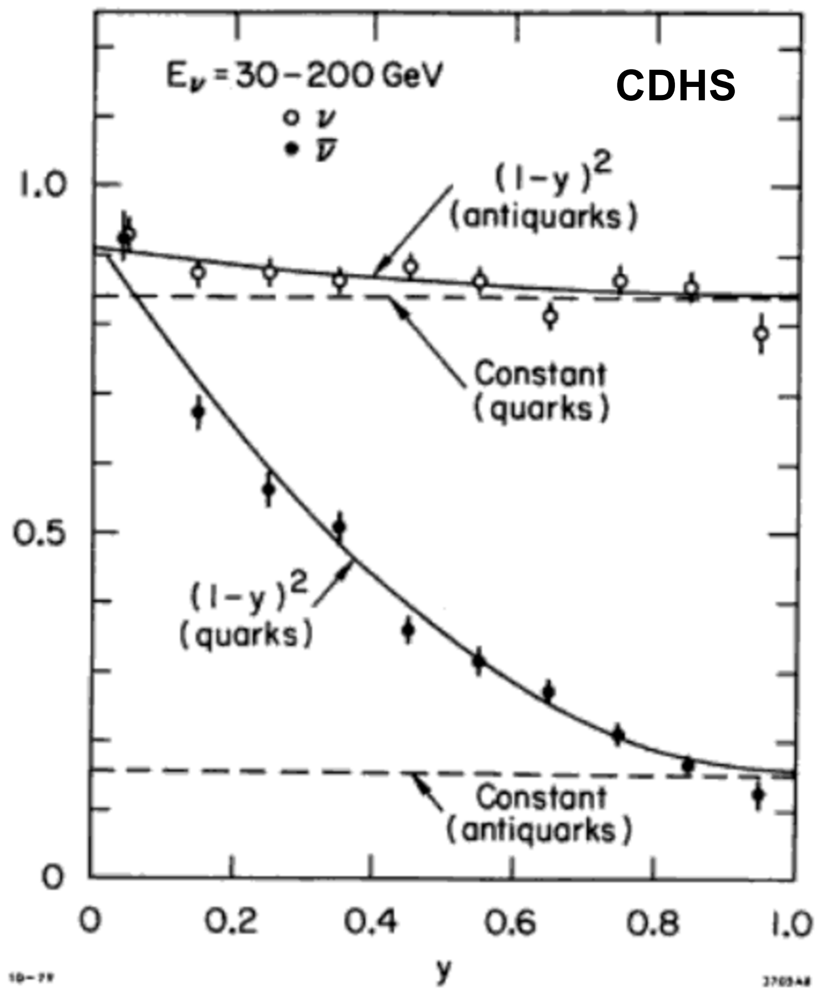
because $\nu, \bar{\nu}$ are LEFT, RIGHT handed.

For $\nu \bar{q}$, $\bar{\nu} \bar{q}$ these results are oppositely handed.
So for a HADRON,

$$\frac{d^2\sigma(\nu h)}{dx\,dy} = \frac{G_F^2 s}{\pi} \Sigma_i \left[x q_i(x) + (1-y)^2 x \bar{q}_i(x) \right]$$

$$\frac{d^2\sigma(\bar{\nu} h)}{dx\,dy} = \frac{G_F^2 s}{\pi} \Sigma_i \left[(1-y)^2 x q_i(x) + x \bar{q}_i(x) \right]$$

y distribution measurement



adapted from Z Phys C1 (1979) 143

There is clearly a need for the qbar term

This leads to the idea of 3-valence quarks PLUS a $q\,\overline{q}$ Sea

$$\begin{cases} xq(x) = xq_v(x) + xq_{\text{sea}}(x) \\ x\bar{q}(x) = x\bar{q}_{\text{sea}}(x) \end{cases}$$

neutrino DIS structure functions

For $\nu\bar{q},\ \bar{\nu}q$ these results are oppositely handed. (using also $Q^2=s.x.y$)

So for a HADRON,

$$\frac{d^2\sigma^{(\nu)}}{dx\,dQ^2} = \frac{G_F^2}{\pi}\left[\sum_i q_i(x) + (1-y)^2\sum_i \bar{q}_i(x)\right]$$

$$\frac{d^2\sigma^{(\bar{\nu})}}{dx\,dQ^2} = \frac{G_F^2}{\pi}\left[\sum_i \bar{q}_i(x) + (1-y)^2\sum_i q_i(x)\right]$$

Now compare these to the general formula

$$\frac{d^2\sigma^{(\nu,\bar{\nu})}}{dx\,dQ^2} = \frac{G_F^2}{4\pi x}\left[Y_+F_2(x,Q^2) - y^2F_L(x,Q^2) \pm Y_- xF_3(x,Q^2)\right]$$

$$Y_+ = \left[1+(1-y)^2\right] \qquad Y_- = \left[1-(1-y)^2\right]$$

$$F_L(x,Q^2) = 0$$

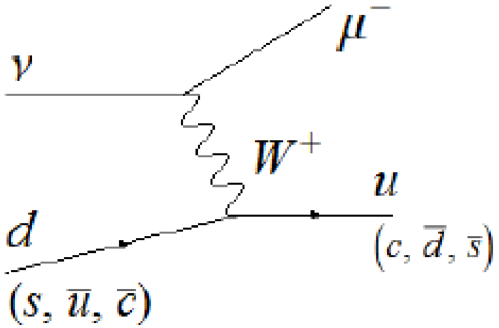
Bjorken scaling as before

$$F_2(x,Q^2) = 2\sum_i x\left[q_i(x) + \bar{q}_i(x)\right] \qquad xF_3(x,Q^2) = 2\sum_i x\left[q_i(x) - \bar{q}_i(x)\right]$$

- Clearly a relationship between F_2's for $\nu,\ \bar{\nu}$ and charged lepton scattering
- More information from xF_3

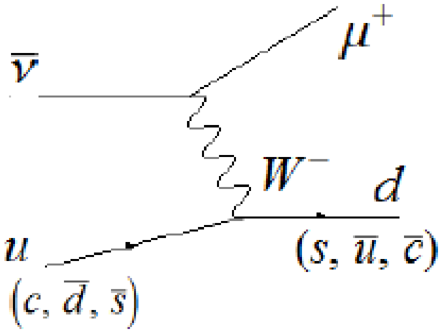
neutrino DIS and flavour

- ν, $\overline{\nu}$ scattering is FLAVOUR SENSITIVE



W^+ MUST hit:
q charge -$^1/_3$e or
$\overline{q}$ charge -$^2/_3$e

$$\frac{d^2\sigma^{(\nu)}}{dxdy} = \frac{G_F^2 sx}{\pi}\left[(d(x)+s(x))+(1-y)^2(\bar{u}(x)+\bar{c}(x))\right]$$



W^- MUST hit:
q charge $^2/_3$e or
$\overline{q}$ charge $^1/_3$e

$$\frac{d^2\sigma^{(\bar{\nu})}}{dxdy} = \frac{G_F^2 sx}{\pi}\left[(u(x)+c(x))+(1-y)^2(\bar{d}(x)+\bar{s}(x))\right]$$

NB, FLAVOUR separation can also come from charged-lepton CC DIS (HERA; see later)

neutrino DIS structure functions

The flavours were written down assuming a proton target,

$$F_2^{(\nu p)} = 2\,x(d + s + \bar{u} + \bar{c})$$

$$x\,F_3^{(\nu p)} = 2\,x(d + s - \bar{u} - \bar{c})$$

For a neutron target, SWAP d $\rightarrow$ u and $\bar{u} \rightarrow \bar{d}$ STRONG ISOSPIN

$$F_2^{\nu n} = 2\,x\left(u + s + \bar{d} + \bar{c}\right)$$

$$xF_3^{\nu n} = 2\,x\left(u + s - \bar{d} - \bar{c}\right)$$

Finally MOST $\nu, \bar{\nu}$ data are taken on ISOSCALAR targets $\frac{n+p}{2}$

$$F_2^{(\nu N)} = x\left(u + d + \bar{u} + \bar{d} + 2\,s + 2\,\bar{c}\right)$$

$$x\,F_3^{(\nu N)} = x\left(u + d - \bar{u} - \bar{d} + 2\,s - 2\,\bar{c}\right)$$

Similarly for $\bar{\nu}$

$$F_2^{(\bar{\nu}\,N)} = x\left(u + d + \bar{u} + \bar{d} + 2\,s + 2\,\bar{c}\right)$$

$$x\,F_3^{(\bar{\nu}\,N)} = x\left(u + d - \bar{u} - \bar{d} - 2\,\bar{s} + 2\,\bar{c}\right)$$

Since the contribution of s, c is small and $s = \bar{s}$, $c = \bar{c}$

$$F_2^{(\nu N)} = F_2^{(\bar{\nu}\,N)} \qquad x\,F_3^{(\nu N)} \approx x\,F_3^{(\bar{\nu}\,N)}$$

← since neutrino cross sections are so small, need massive detectors, usually made of IRON; so experimentally, measure **combination of proton/neutron scattering** cross sections

QPM tests

Now go further,

$$u = u_{\text{valence}} + u_{\text{sea}} = u_v + u_{\text{sea}}$$

$$\bar{u} = \bar{u}_{\text{sea}} \quad \text{and} \quad \bar{q}_{\text{sea}} = q_{\text{sea}}$$

Similarly $d = d_v + d_{\text{sea}}$

So,

$$x F_3^{(\nu,\bar{\nu}\, N)} = x\left(u - \bar{u} + d - \bar{d}\right) = x(u_v + d_v) = x(\text{valence})$$

$$F_2^{(\nu,\bar{\nu}\, N)} = x\left(u_v + d_v + u_{\text{sea}} + d_{\text{sea}} + \bar{u} + \bar{d} + s + \bar{s} + c + \bar{c}\right)$$

$$= x(\text{valence} + \text{sea})$$

Measuring F_2 and xF_3 in $\nu, \bar{\nu}$ scattering separates valence and sea.

Compare $F_2^{(\nu,\bar{\nu})}$ to $F_2^{(e^{\pm})}$

$$F_2^{(\text{ep})} = x\left[\frac{4}{9}(u + \bar{u}) + \frac{1}{9}\left(d + \bar{d}\right) + \frac{1}{9}(s + \bar{s}) + \frac{4}{9}(c + \bar{c})\right]$$

for isoscalar targets,

$$F_2^{(\text{eN})} = \frac{5}{18}\, x\left[(u + \bar{u}) + \left(d + \bar{d}\right) + \frac{2}{5}(s + \bar{s}) + \frac{8}{5}(c + \bar{c})\right]$$

$F_2^{(\text{eN})} = \frac{5}{18} F_2^{(\nu,\bar{\nu}\, N)}$ again assuming *s*, *c* small ⟶ OBSERVATIONS

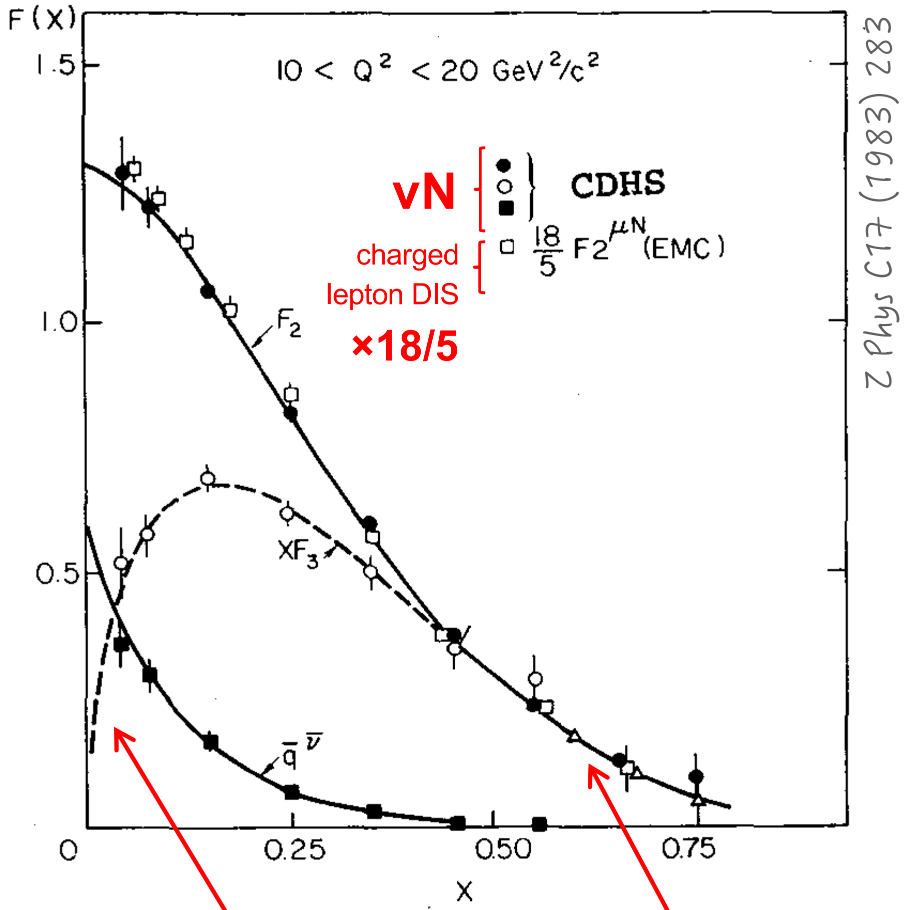
$F_2(x)$ and $xF_3(x)$ measurements

$$F_2^{(eN)} = \frac{5}{18} F_2^{(\nu,\bar{\nu}\ N)}$$

ALSO:

$$F_2^{\nu N} = x[u(x) + d(x) + \bar{u}(x) + \bar{d}(x)]$$
$$xF_3^{\nu N} = x[u(x) + d(x) - \bar{u}(x) - \bar{d}(x)]$$

➡ $F_2^{\nu N} - xF_3^{\nu N} = 2x[\bar{u} + \bar{d}]$



sea dominates so expect **xF_3** to go to zero as q(x) =qbar(x)

sea contribution goes to zero

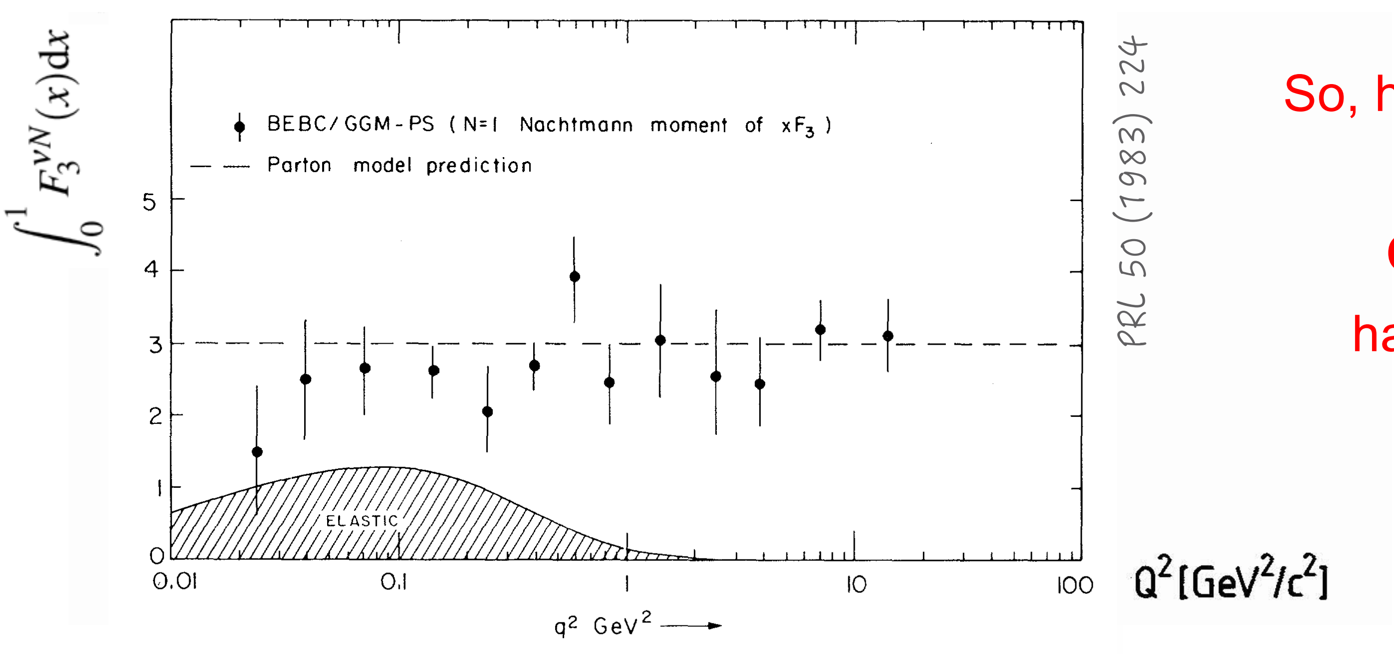
QPM tests: valence contribution

➡ $$F_3^{\nu N} = [u(x) + d(x) - \overline{u}(x) - \overline{d}(x)] = u_V(x) + d_V(x)$$

➡ $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = \int_0^1 (u_V(x) + d_V(x))\,\mathrm{d}x = N_u^V + N_d^V$$

expect $$\int_0^1 F_3^{\nu N}(x)\mathrm{d}x = 3$$

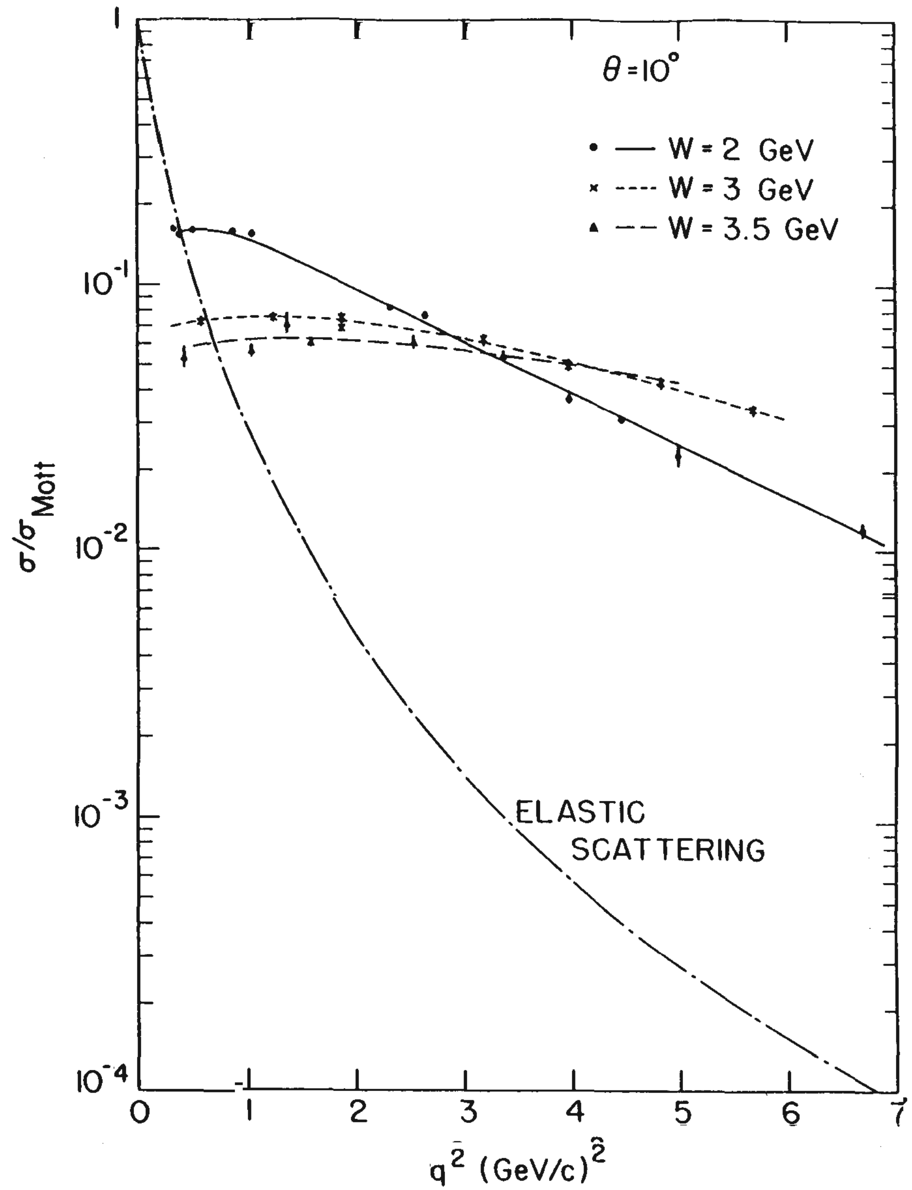
Gross-Llewellyn-Smith Sum Rule



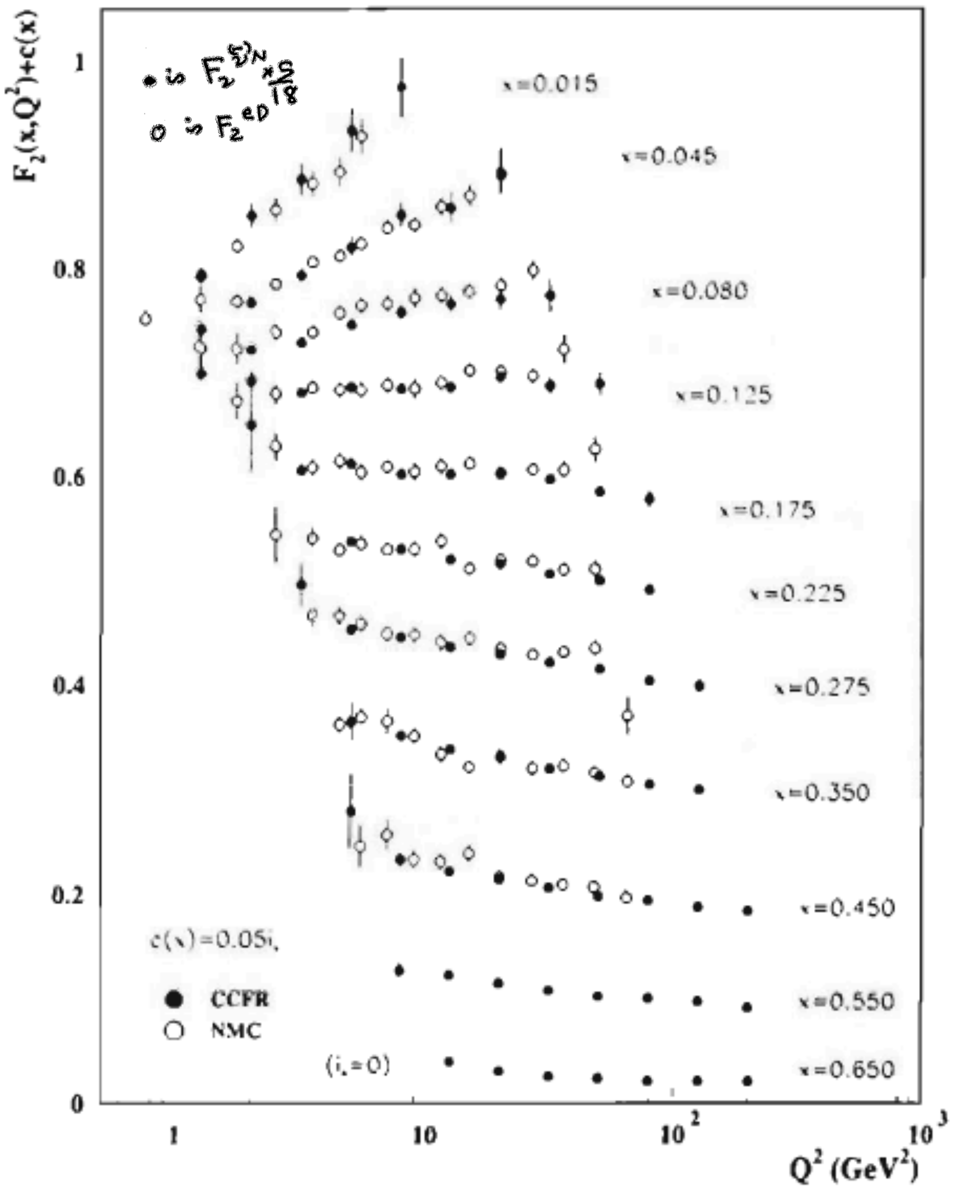
So, hopefully you are now convinced that the **Quark Parton Model** has some basis in fact!

extras

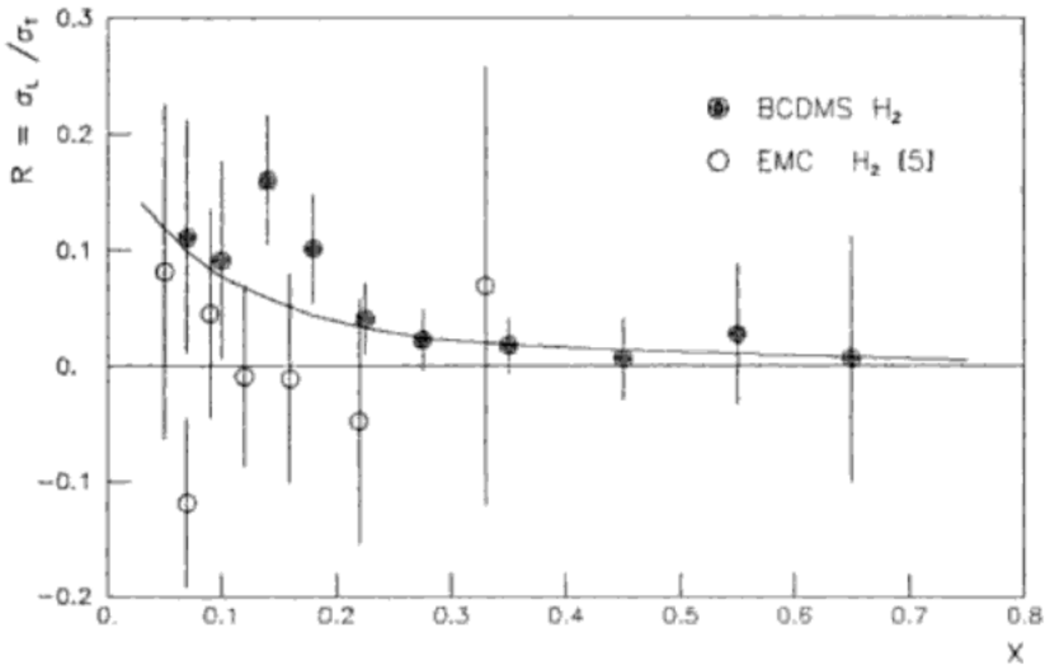
elastic scattering



$F_2(x)$ measurements



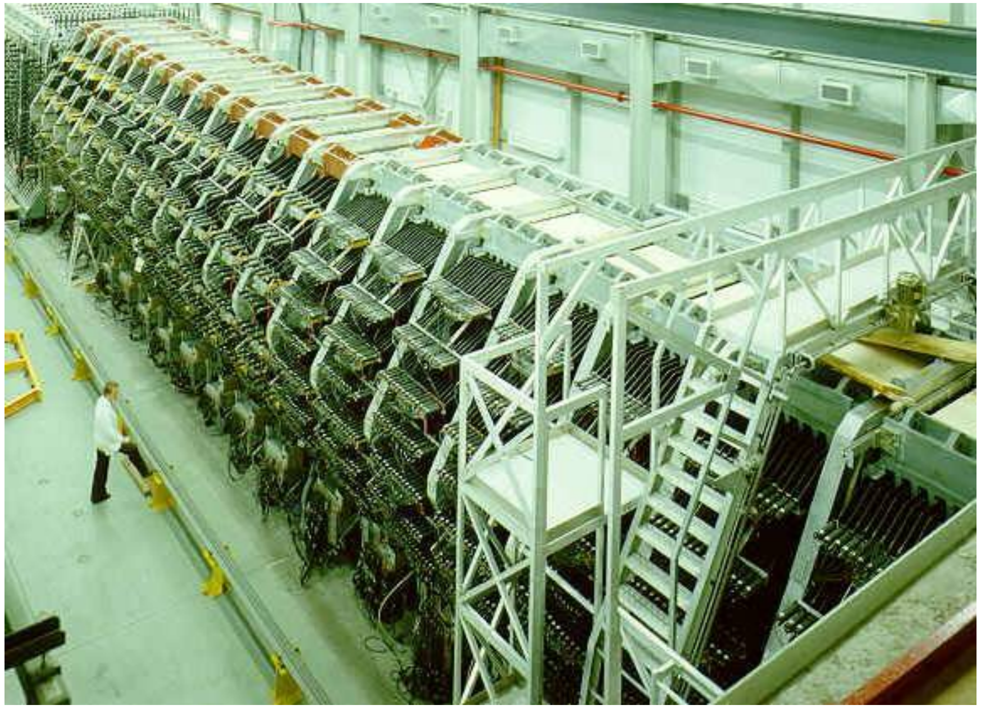
early evidence for FL=0



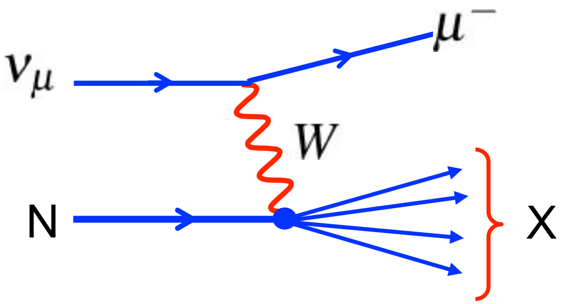
$$\rightarrow R = \frac{F_L}{F_2 - F_L} = \frac{F_L}{2\,x\,F_1} \approx 0$$

e.g. CDHS Experiment (CERN 1976-1984)

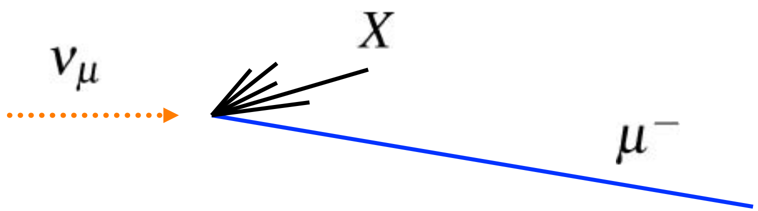
- 1250 tonnes
- magnetised iron modules
- separated by drift chambers



study Neutrino DIS



Experimental Signature:



M Thomson

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Etymology [edit]

For some time, Gell-Mann was undecided on an actual spelling for the term he intended to coin, until he found the word *quark* in James Joyce's book *Finnegans Wake*:[49]

> – Three quarks for Muster Mark!
> Sure he hasn't got much of a bark
> And sure any he has it's all beside the mark.

The word *quark* itself is a Slavic borrowing in German and denotes a dairy product,[50] but is also a colloquial term for ″rubbish″.[51][52] Gell-Mann went into further detail regarding the name of the quark in his book *The Quark and the Jaguar*:[53]

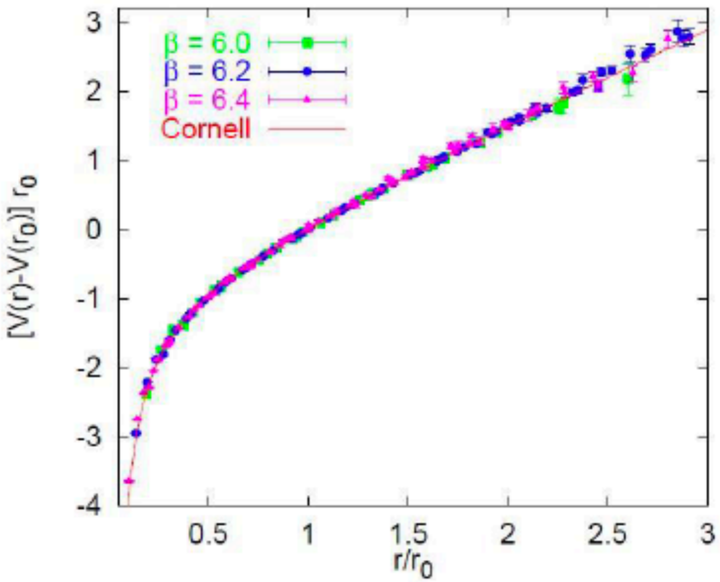
> In 1963, when I assigned the name "quark" to the fundamental constituents of the nucleon, I had the sound first, without the spelling, which could have been "kwork". Then, in one of my occasional perusals of *Finnegans Wake*, by James Joyce, I came across the word "quark" in the phrase "Three quarks for Muster Mark". Since "quark" (meaning, for one thing, the cry of the gull) was clearly intended to rhyme with "Mark", as well as "bark" and other such words, I had to find an excuse to pronounce it as "kwork". But the book represents the dream of a publican named Humphrey Chimpden Earwicker. Words in the text are typically drawn from several sources at once, like the "portmanteau" words in *Through the Looking-Glass*. From time to time, phrases occur in the book that are partially determined by calls for drinks at the bar. I argued, therefore, that perhaps one of the multiple sources of the cry "Three quarks for Muster Mark" might be "Three quarts for Mister Mark", in which case the pronunciation "kwork" would not be totally unjustified. In any case, the number three fitted perfectly the way quarks occur in nature.

Color force and quark potential

2 quarks at distance $r \sim O(1)\mathrm{fm})$ define a *string* of *tension* k, potential $V(r) = kr$.

Stored energy/unit length is constant: separation of quarks requires infinite energy.

QCD Potential: QED-like at $r \leq 0.1\mathrm{fm}$ but increases linearly at $r \geq 1fm$.



Force:
$|\frac{dV}{dr}| = k = 1.6 \times 10^{-10}\mathrm{J}/10^{-15}\mathrm{m} = 16000\mathrm{N}$
weight of a car!

This stored energy gives the proton most of its mass (and not the Higgs as you sometimes hear)! Recall $m_u + m_u + m_d \sim 9\mathrm{MeV}$ *but* $m_{\mathrm{proton}} = 938\mathrm{MeV}$

Calculating σDIS I – Further Details

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^2 e'^2}{q^4} L_e^{\lambda\nu} L^{\mu}_{\lambda\nu}$$

taken from
J Ferrando, SUPA lectures

Where: $L_e^{\lambda\nu} = 2(k'^{\lambda}k^{\nu} + k'^{\nu}k^{\lambda} - (k'.k)g^{\lambda\nu})$

Contract the leptonic tensors

$$L_e^{\lambda\nu} = 2(k'^{\lambda}k^{\nu} + k'^{\nu}k^{\lambda} - (k'.k)g^{\lambda\nu})$$
$$L^{\mu}_{\lambda\nu} = 2(p'_{\lambda}p_{\nu} + p'^{\nu}p_{\lambda} - (p'.p)g_{\lambda\nu})$$
$$L_e.L^{\mu} = 8[(k'.p')(k.p) + (k'.p)(k'.k)]$$

Rewrite in terms of the Mandelstam variables

$$s = (k+p)^2 = (k'+p')^2, t = (k-k')^2 = (p'-p)^2, u = (k-p')^2 = (k'-p)^2$$
$$L_e.L^{\mu} = 2(s^2 + u^2).$$

substitute $y = \frac{(p.q)}{(p.k)} = \frac{u}{s} + 1$

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}|^2 = \frac{e^2 e'^2}{Q^4} 2s^2[1 + (1-y)^2]$$

Insert phase space and flux factor

$$\frac{\mathrm{d}\sigma}{\mathrm{dy}} = \frac{e^2 e'^2}{8\pi Q^4}[1 + (1-y)^2]s \rightarrow \frac{\mathrm{d}\sigma}{\mathrm{dy}} = \frac{2\pi\alpha^2}{Q^4}[1 + (1-y)^2]s$$

One isotropic contribution from same handed spin directions