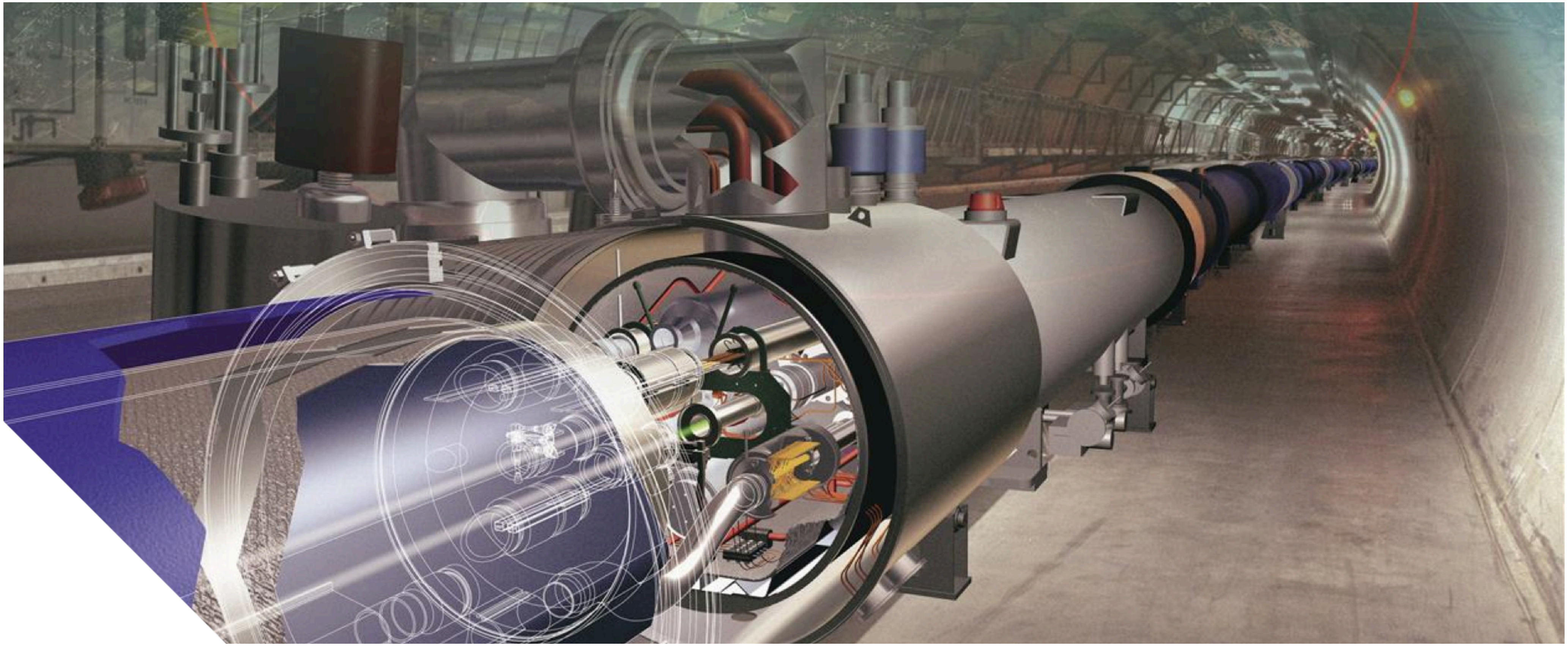


QCD – Lecture 2

QCD: theory and experimental evidence

Claire Gwenlan, Oxford, HT

QCD sum rules

The observation that $\int_0^1 dx\, F_3^{\nu N} = \int_0^1 dx\,(u_v + d_v) = 3$

in early neutrino data was crucial for the parton model.

But there was another more worrying sum rule,

$$\int_0^1 dx\, F_2^{\nu N} = \int_0^1 dx \cdot x\left[u + \bar{u} + d + \bar{d} + s + \bar{s} + c + \bar{c}\right]$$

so QPM predicts,

$$\int_0^1 dx\, F_2^{\nu N} = 1 \quad \text{but} \quad \int_0^1 dx\, F_2^{\nu N} \sim 0.5 \quad \text{was observed.}$$

50% of the momentum is NOT carried by quarks

- Where has the momentum gone? GLUONS
- QPM treats partons as non-interacting.
- Cannot be true, they are bound in hadrons.
- QCD says that quarks interact with gluons

with interaction strength ~ α_S

- $\alpha_S \downarrow$ as $Q^2 \uparrow$ "Asymptotically free"

  $\Rightarrow$ Modify the QPM

So what are gluons?

The force carrier of QCD

QFT and the strong interaction

- in **QFT,** it was thought **no realistic interacting theory predicted scaling**
- success of QPM model and scaling in DIS experiments explained with demonstration of **asymptotic freedom** in **gauge theories**
- **gauge theories** are invariant under local transformation of fields, thus predictions are unchanged by such transformations
- **asymptotic freedom** means that at high momenta gauge theories behave as free, non-interacting field theories

- important steps:
- **1972**: Gross & Coleman – no renormalisable field theory except gauge theories could account for scaling
- **1973:** Politzer and Gross & Wilczek – asymptotic freedom of gauge theories
  (in fact, t'Hooft had already remarked on this in 1972)
- **1973:** Gross & Coleman – no theory without gauge fields could be asymptotically free

- a field theory of the strong interaction must be a **gauge theory → QCD**

QCD, a gauge theory

- **QCD is a locally gauge invariant field theory like QED**

- QED:

$$\psi(x) \rightarrow \psi'(x) = e^{iq\theta(x)}\psi(x)$$

where q is the charge and θ is a space-time dependent phase

- QED Lagrangian density: U(1)

$$\mathcal{L}_{QED} = \sum_f \bar{\psi}_f(i\gamma_\mu D^\mu - m_f)\psi_f - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}$$

electromagnetic tensor $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

covariant derivative $D^\mu = \partial^\mu + iqA^\mu$

the interaction between the fermions and the field is in the covariant derivative

- QCD:

$$\psi(x) \rightarrow \psi'(x) = e^{igt\cdot\theta(x)}\psi(x)$$

where g is the strong charge and t.θ is the product of the colour group generators with a vector of space-time phase functions in colour space

- the group generators t satisfy: SU(3)

$$[t^a, t^b] = if^{abc}t^c$$

where f^{abc} are SU(3) structure constants

$$f_{123} = 1; \quad f_{147} = f_{246} = f_{257} = f_{345} = 1/2;$$
$$f_{156} = f_{367} = -1/2; \quad f_{458} = f_{678} = \sqrt{3}/2.$$

and $t^a = \lambda^a/2$ are 3×3 linearly independent traceless Hermitian matrices, where λ are the Gell-Mann matrices:

Gell-Mann matrices

$$\lambda_1 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_2 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\lambda_4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda_5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix} \qquad \lambda_6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\lambda_7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix} \qquad \lambda_8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

- generators of SU(3)
- also describe the allowed colour configurations of the gluons

QCD Lagrangian

- QCD Lagrangian density

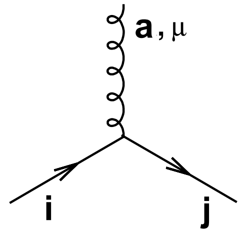
$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}^i_f (i\gamma_\mu D^\mu - m_f)_{ij} \psi^j_f - \frac{1}{4} F^{\mu\nu}_a F^a_{\mu\nu}$$

└── quark part ──┘ └─ gluon ─┘

gluon field strength tensor $F^{\mu\nu}_a = \partial^\mu A^\nu_a - \partial^\nu A^\mu_a + g f^{abc} A^\mu_b A^\nu_c$

covariant derivative $D^\mu_{ij} = \delta_{ij}\partial^\mu + ig(t^a)_{ij} A^\mu_a$

- quark part of Lagrangian describes the **qqg** interaction

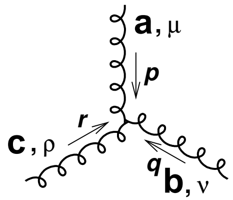


$-ig_s t^a_{ij} \gamma^\mu$

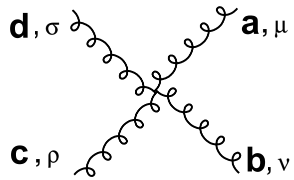
where quarks now have colour indices i=1,2,3 (R,G,B)
as well as flavour indices, f
$t^a = \lambda^a/2$, a=1,2,…,8 where λ^a are the Gell-Mann SU(3) matrices

the gluon fields A have a=1,2,…,8 indices

- second part of QCD Lagrangian is purely **gluonic**



$-g_s f^{abc} [(p-q)^\rho g^{\mu\nu} + (q-r)^\mu g^{\nu\rho} + (r-p)^\nu g^{\rho\mu}]$



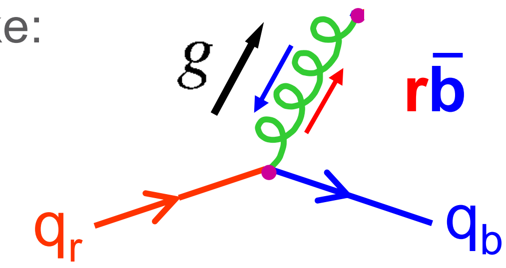
$-ig_s^2 f^{xac} f^{xbd} [g^{\mu\nu} g^{\rho\sigma} - g^{\mu\sigma} g^{\nu\gamma}] + (C,\gamma) \leftrightarrow (D,\rho) + (B,\nu) \leftrightarrow (C,\gamma)$

the difference from QED is the AA term which makes
gluons interact with gluons (**Non-Abelian**), **ggg** and **gggg**

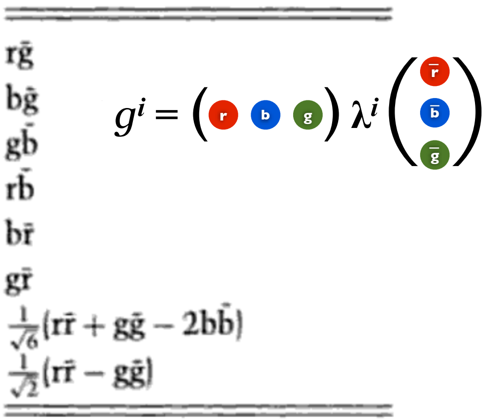
this term is also what makes QCD gauge invariant
under local SU(3) transformation

colour and the number of gluons

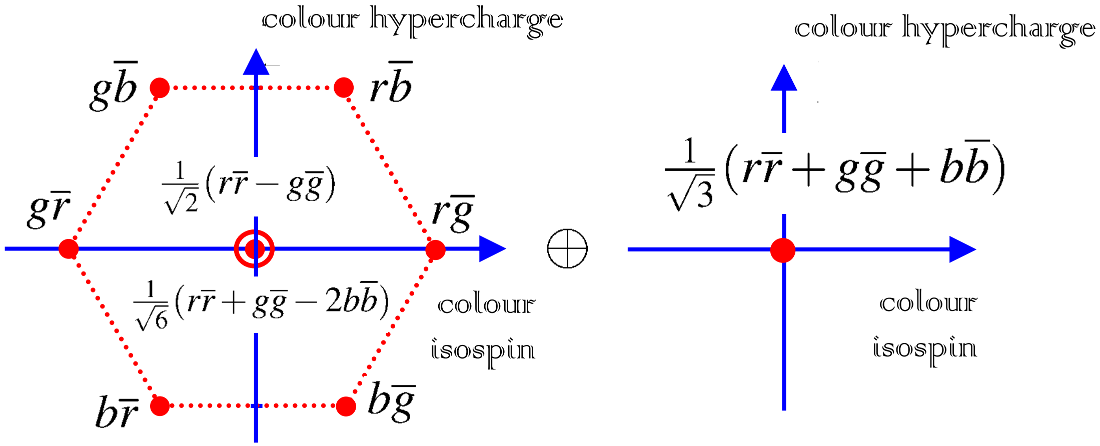
- colour exchange in a **qqg** diagram can be thought of like:
- gluon has colour **red** – **antiblue** in this case: $\mathbf{r\bar{b}}$
- obviously $\mathbf{b\bar{r}}$, $\mathbf{r\bar{g}}$, $\mathbf{g\bar{r}}$, $\mathbf{g\bar{b}}$, $\mathbf{b\bar{g}}$ also possible
- and the combinations:
  $(r\bar{r} - g\bar{g})/\sqrt{2}$, $(r\bar{r} + g\bar{g} - 2b\bar{b})/\sqrt{6}$ AND $(r\bar{r} + g\bar{g} + b\bar{b})/\sqrt{3}$



- in the mathematics of SU(3) this is $3 \otimes \bar{3} = 8 \oplus 1$
  and the last combination is the singlet, which is not coloured at all;
  so, **eight** coloured gluons



$r\bar{g}$
$b\bar{g}$
$g\bar{b}$
$r\bar{b}$
$b\bar{r}$
$g\bar{r}$
$\frac{1}{\sqrt{6}}(r\bar{r} + g\bar{g} - 2b\bar{b})$
$\frac{1}{\sqrt{2}}(r\bar{r} - g\bar{g})$

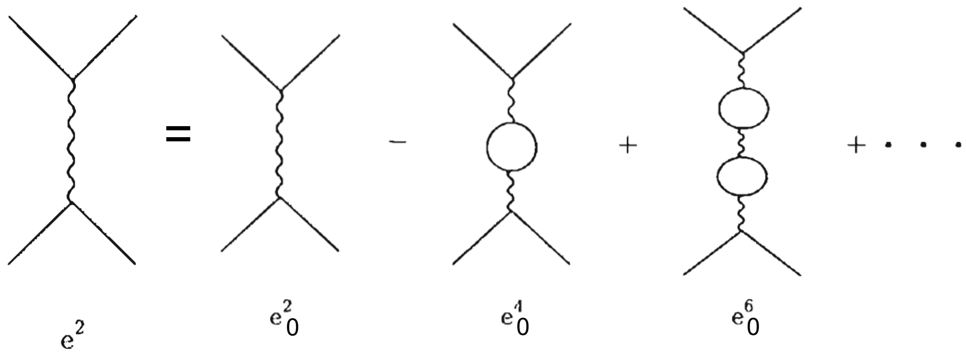


colour singlet does not exist in nature;
can also see this empirically: would be unconfined and so would behave like a strongly interacting photon – infinite range strong force! ✗

ANALOGY: gluon octet exactly mimics the meson flavor octet

renormalisation and running couplings

- contributions to perturbative expansion of scattering amplitudes beyond leading order are often divergent, EG. for **QED**



the loops are divergent due to unrestricted integration over momentum in these loops – theory must be **RENORMALISED**; done by making constants of the theory such as coupling **α** become dependent on the scale of the process – it is successful if it takes care of infinities to **ALL** orders

- for one loop, the fermion propagator becomes:

$$\frac{-ig_{\mu\nu}}{q^2} \rightarrow \frac{-ig_{\mu\nu}}{q^2}\left[1 - \Pi(q^2)\right], \quad \Pi(Q^2) \approx \frac{\alpha_0}{3\pi}\ln\left(\frac{\Lambda^2}{Q^2}\right)$$

where Λ is a high momentum cut-off and $\alpha_0 = e_0^2/4\pi$
where e0 is the bare charge

- for many loops, the effect of summing the "leading logs" (largest corrections), can be accounted for by redefining the coupling:

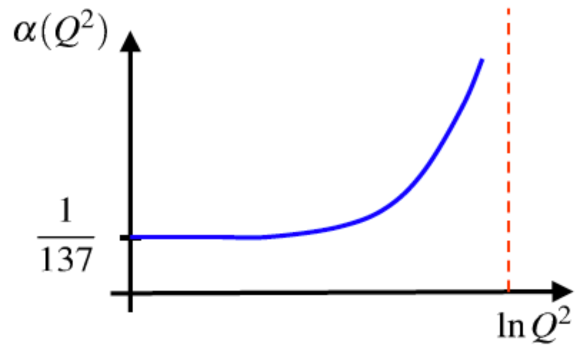
$$\frac{1}{\alpha(Q^2)} = \frac{1}{\alpha_0} + \frac{1}{3\pi}\ln\left(\frac{\Lambda^2}{Q^2}\right)$$

can remove dependence on Λ and **α**0 by defining the **Renormalised Coupling** at some scale μ^2 , and rewriting the above to give:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{Q^2}{\mu^2}\right)}.$$

- so, the QED coupling increases for $Q^2 > \mu^2$

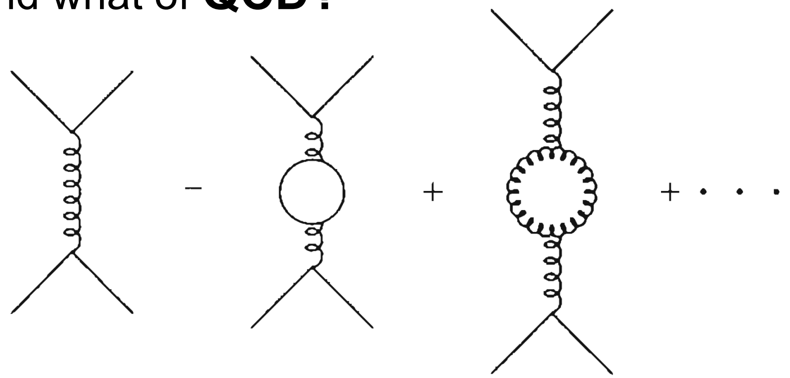
E.G. the **α**=1/137 you are used to at low energies, becomes 1/125 at a scale Q=MZ



QCD running coupling

confinement and asymptotic freedom

- and what of **QCD?**

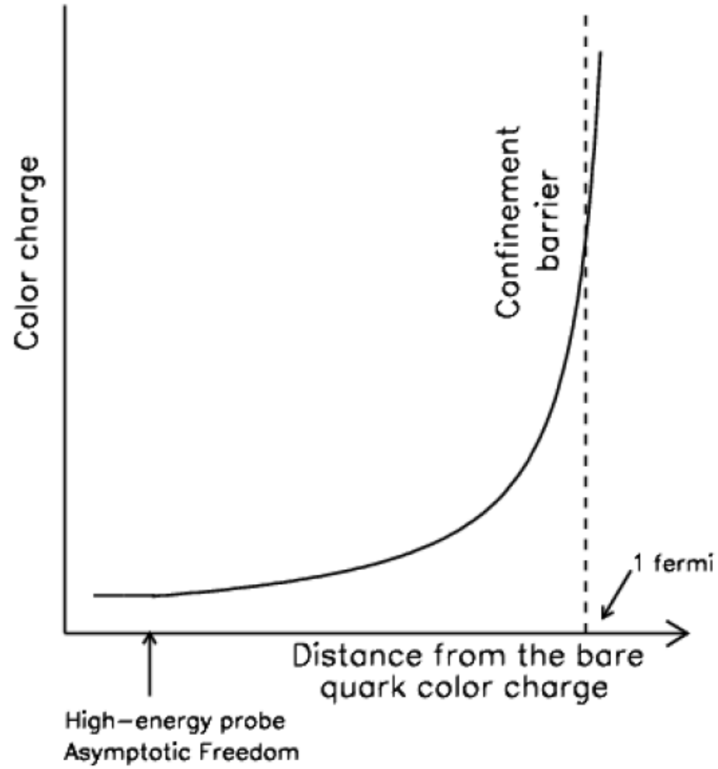


- c.f. QED, there is **another type** of loop diagram
- contributions both diverge logarithmically, but with opposite sign coefficients (can be understood qualitatively in terms of charge screening effects)
- QCD coupling is renormalised as:

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 + \alpha(\mu^2) b_0 \ln\left(\frac{Q^2}{\mu^2}\right)} = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$$

where $b_0 \doteq (33 - 2n_f)/12\pi$ at leading order

NB, the quark loop gives –1/6π for each flavour of fermion (n_f) – this is like the –1/3π of QED except for a conventional factor of 2; the new feature is the 33/12π of the gluon loop which swaps the sign



coupling decreases as energy (1/distance) scale goes up

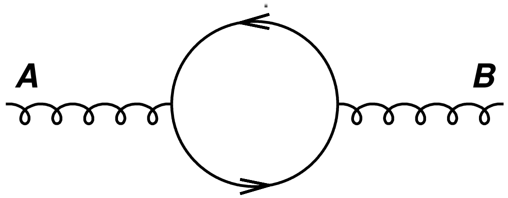
at high energies, we have **ASYMPTOTIC FREEDOM** and may make perturbative calculations;
at low energies, we can not, and we have **CONFINEMENT**

($\Lambda \equiv \Lambda_{QCD} \approx$ 200 MeV can only be determined experimentally)

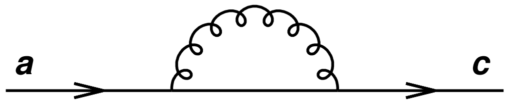
QCD colour factors

- pQCD involves an order-by-order expansion in a small coupling $\alpha_s = g_s^2/4\pi \ll 1$, and calculations are made using Feynman diagrams (rules for vertices have already been shown)
- main complication in comparison to QED is the need for **Colour Factors**; in perturbative calculations the average and sum over all possible colour configurations in the initial and final states leads to combinatoric colour factors **T_F**, **C_F**, **C_A**
- **T_F, C_F** and **C_A** are the physical manifestation of the underlying group structure; **in QCD, they represent the relative strength of the processes:**

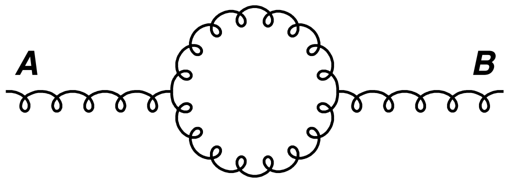
$$\mathrm{Tr}(t^A t^B) = T_F\,\delta^{AB}\,, \quad T_F = \tfrac{1}{2}$$



$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$



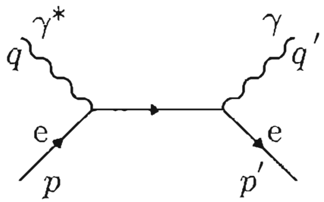
$$\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}\,, \quad C_A = N_c = 3$$

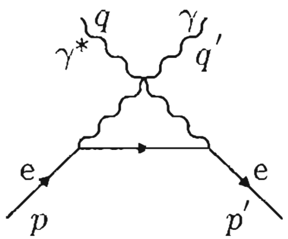


$N_c \equiv$ number of colours $= 3$ for QCD

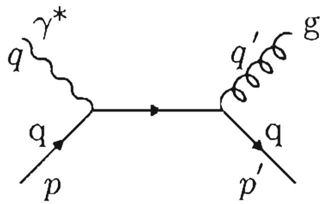
see Devenish & Cooper-Sarkar p38–39 for a simple colour factor calculation

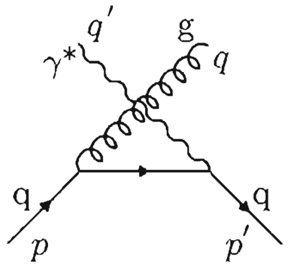
outline of a tree level calculation





- consider **QED Compton** scattering with one virtual photon





- the **QCD** analogue is **QCDC** and the kinematic invariants are:

$$\begin{aligned} s &= (q+p)^2 = (q'+p')^2 = 2q\cdot p - Q^2 \\ t &= (q-q')^2 = (p'-p)^2 = -2p\cdot p' \\ u &= (q-p')^2 = (q'-p)^2 = -2q'\cdot p \end{aligned}$$

- the **amplitudes** for the two **QED** diagrams are:

$$\begin{aligned} \mathcal{M}_a &= -ie^2 \varepsilon_\nu'^* \varepsilon_\mu \bar{u}(p')\gamma^\nu (\not{q} + \not{p})\gamma^\mu u(p)/s \\ \mathcal{M}_b &= -ie^2 \varepsilon_\nu'^* \varepsilon_\mu \bar{u}(p')\gamma^\mu (\not{p} - \not{q}')\gamma^\nu u(p)/u \end{aligned}$$

- adding, squaring and taking care of spins:

$$\frac{1}{4}\sum_{\text{spins}} |\mathcal{M}_a + \mathcal{M}_b|^2 = 2e^4 \left[-\frac{u}{s} - \frac{s}{u} + \frac{2tQ^2}{su} \right]$$

- to go to **QCD** e^4 must be replaced:

$$e^4 \rightarrow e^2 e_i^2 g^2 \rightarrow (4\pi)^2 \alpha\alpha_s e_i^2$$

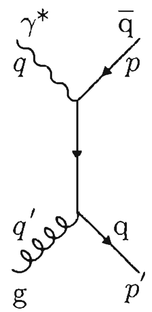
- and the colour factor from the loop insertion **CF=4/3**:

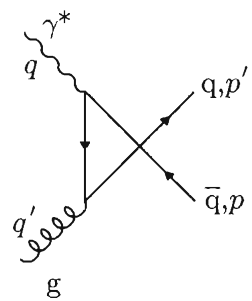
$$\overline{|\mathcal{M}_{\text{QCDC}}|^2} = \frac{1}{4}\sum_{\text{spins}} |\mathcal{M}_a + \mathcal{M}_b|^2 = \frac{8}{3}(4\pi)^2 e_i^2 \alpha\alpha_s \left[-\frac{u}{s} - \frac{s}{u} + \frac{2tQ^2}{su} \right]$$

- and using Fermi's Golden rule to go to the **cross section** via the **phase space factors**:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{QCDC}} = \frac{2}{3}\frac{e_i^2 \alpha\alpha_s}{s} \left[-\frac{u}{s} - \frac{s}{u} + \frac{2tQ^2}{su} \right]$$

- a further important process is **Boson-Gluon Fusion** (**BGF**):





- which similarly has the cross section:

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{BGF}} = \frac{1}{4}\frac{e_i^2 \alpha\alpha_s}{s} \left[\frac{u}{t} + \frac{t}{u} - \frac{2sQ^2}{tu} \right]$$

Devenish & Cooper-Sarkar p40–43

QCD predictions

to test the validity of any theory, its predictions must confront experimental data

important predictions of QCD:

- 3 colour states per quark: measurements should yield evidence that there are 3 colour degrees of freedom for each quark in a final state
- hadronic jets: since quarks and gluons cannot be seen in isolation, we should only directly observe hadronic jets
- gluon jets: some jets should originate from gluons
- gluon self coupling: evidence for a **ggg** vertex should be observed
- scaling violations: structure function $F_2(x,Q^2)$ should increase at small x and decrease at large x, as Q^2 increases
- running of α_s**:** α_s should decrease with increasing Q^2

taken from J Ferrando, SUPA lectures

evidence for quark colour

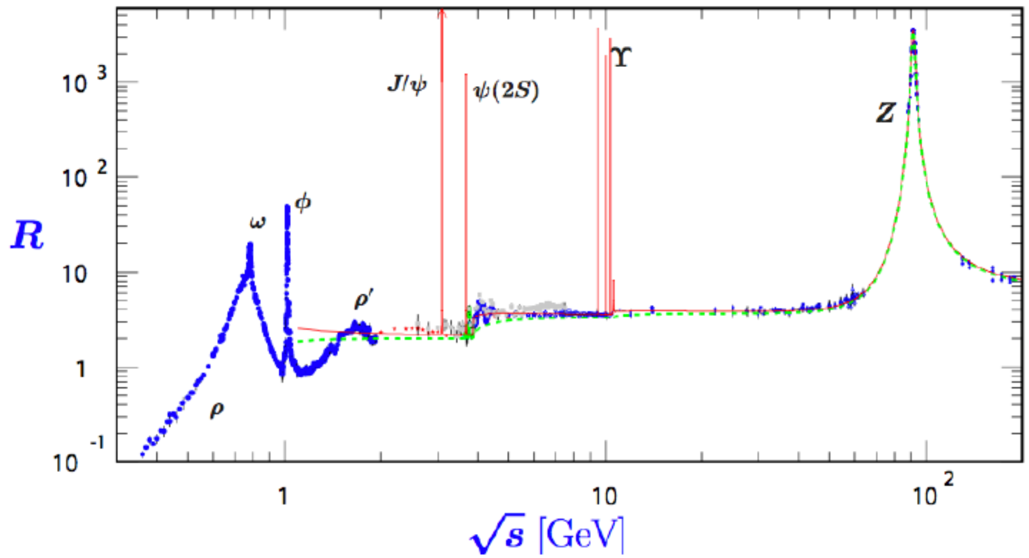
1. Pauli statistics $\Omega^-(s^\uparrow s^\uparrow s^\uparrow)$ Symmetric we can get around this if colour wave-fn is anti symmetric $\Psi = \Psi(r)\cdot\Psi_{\text{spin}}(J)\cdot\Psi_{\text{flavour}}\cdot\Psi_{\text{colour}}$

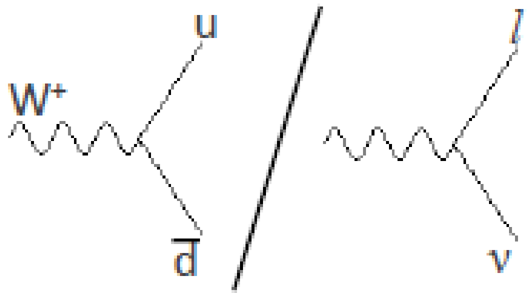
2. $$\frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = nc\sum e_q^2$$

u,d,s: $R_\mu = 3\times(\frac{1}{9}+\frac{4}{9}+\frac{1}{9}) = 2$

u,d,s,c: $R_\mu = \frac{10}{3}$

u,d,s,c,b: $R_\mu = \frac{11}{3}$ $\rightarrow nc=3$

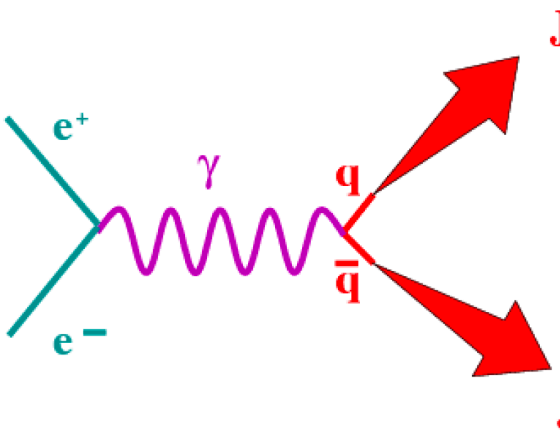


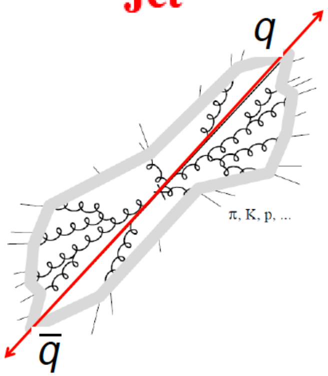
3. 

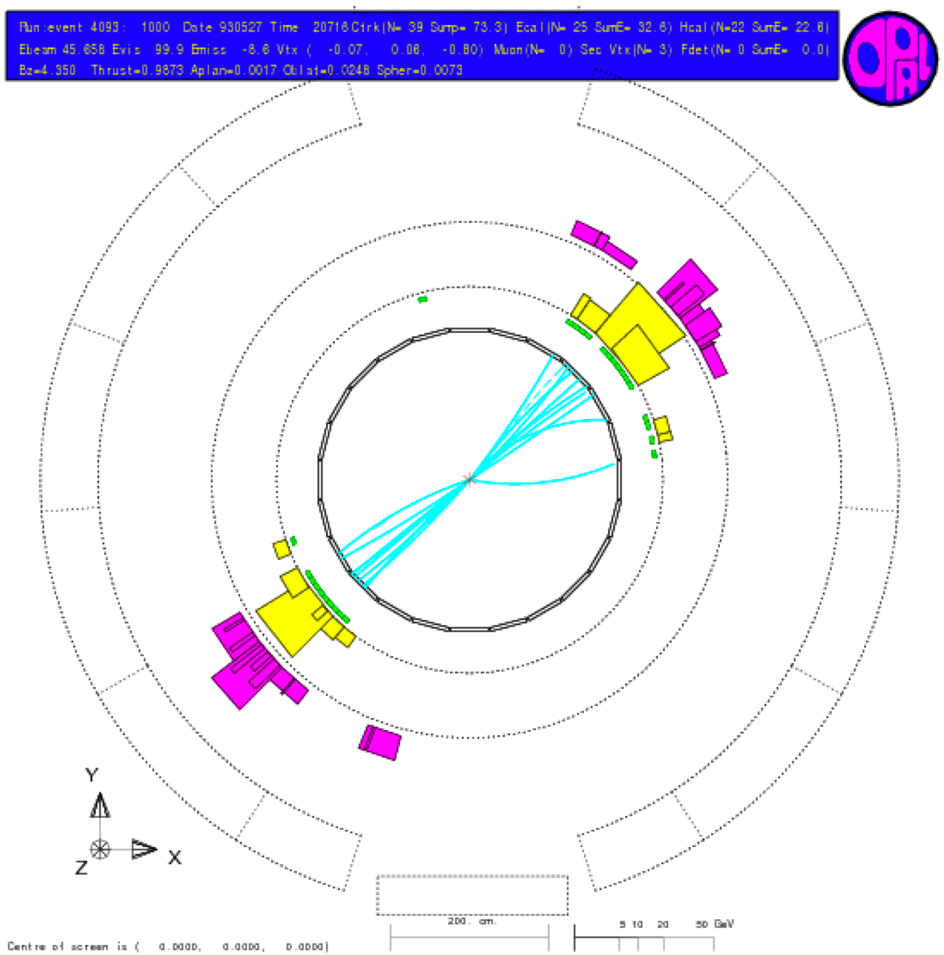
In W decay quark channel branching fractions are a factor of 3 times larger than lepton channels

W data: $e\,\nu = \mu\,\nu = \tau\,\nu \simeq 11\,\%\ (10.8 \pm 0.1\,\%)$
$c\bar{s} \simeq u\bar{d} = 32.8 \pm 0.3\,\%$
$\Big\} \approx 3$

evidence for hadronic jets





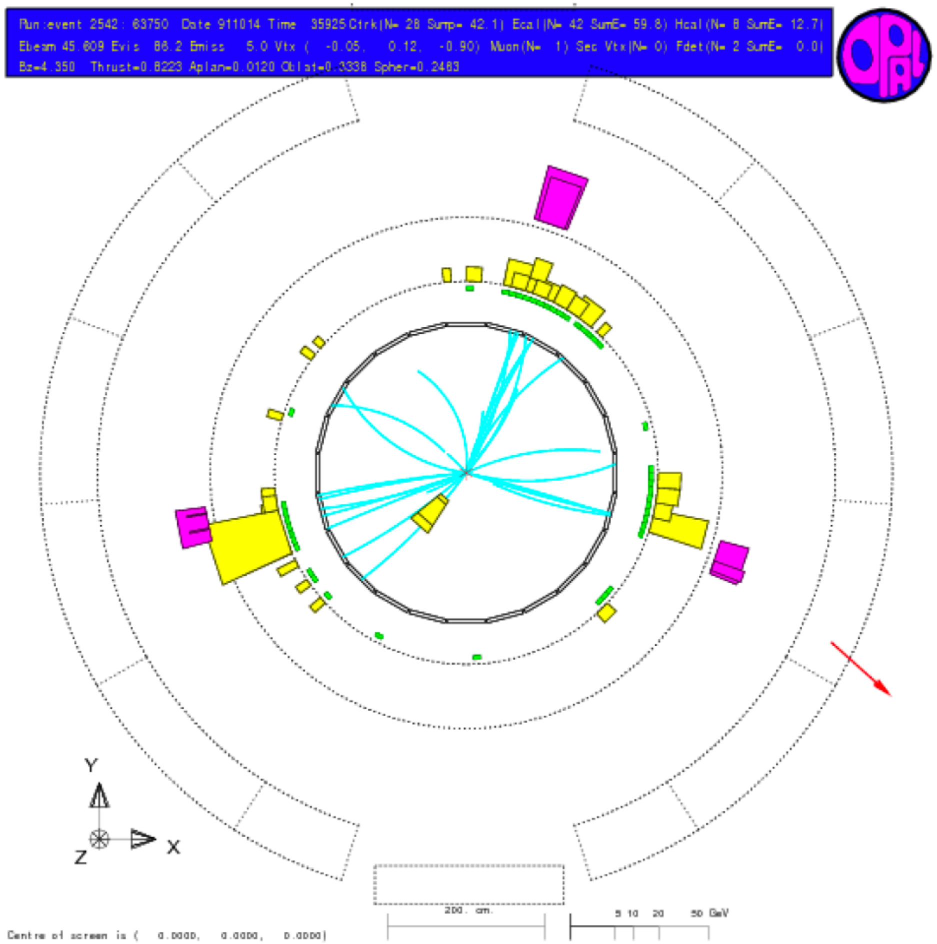


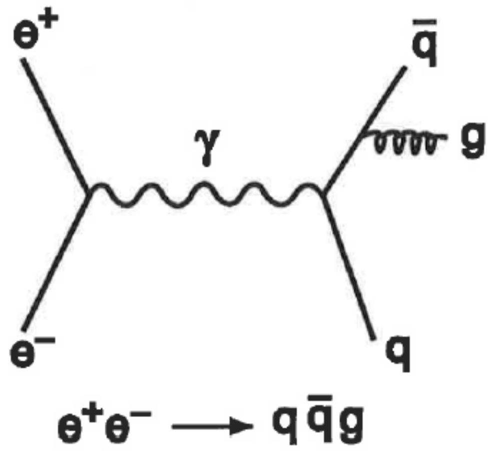
- quarks and gluons cannot be observed directly due to **confinement**; observe collimated **jets**

- 1st observation of back-to-back **dijet events** in $e^+e^- \to q\bar{q}$ at SPEAR in 1975

- back-to-back collimated bunches of tracks from charged hadrons seen in central tracking detector, and back-to-back hadronic clusters in calorimeter

3-jet events and the gluon



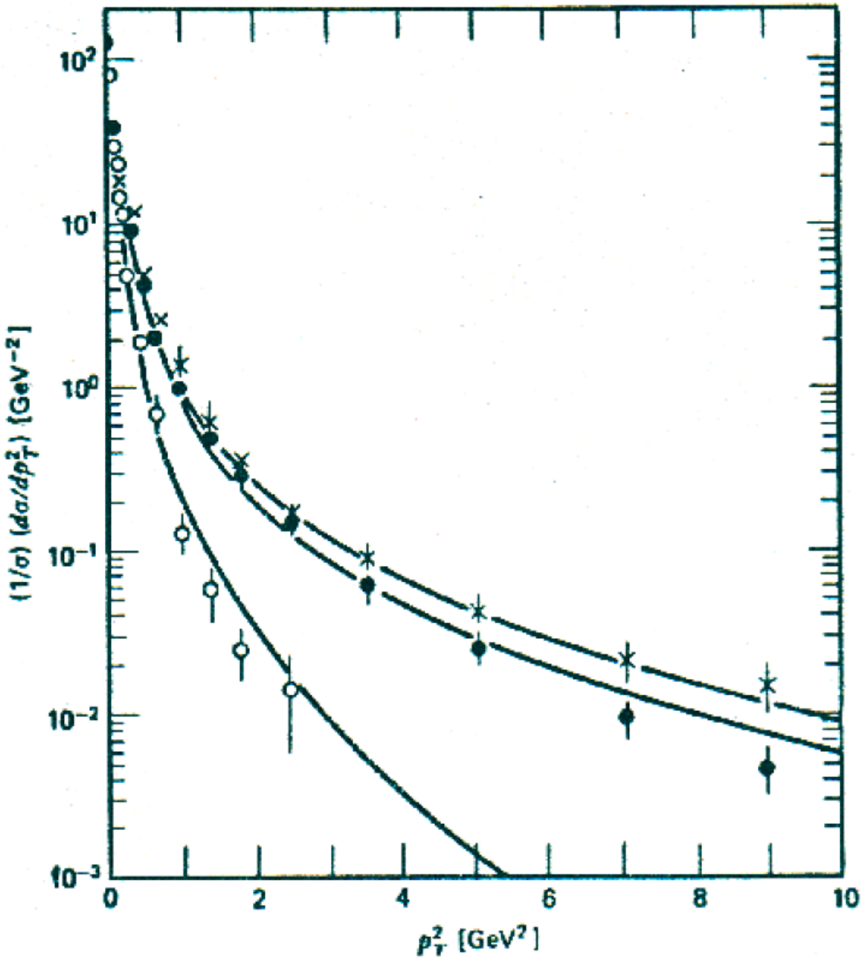


- $\mathcal{O}(\alpha_S)$ correction to $e^+e^- \to q\bar{q}$ gives events with 3 jets in the final state
- jets are coplanar to conserve momentum
- first direct evidence for **gluons** by observation of 3 jet events at PETRA in 1979

3-jet events and the gluon

- cross section up to $\mathcal{O}(\alpha_S)$: $\sigma(e^+e^- \to q\bar{q}) + \sigma(e^+e^- \to q\bar{q}g) = \sigma_0(1 + \frac{\alpha_S(s)}{\pi})$

  where σ_0 is: $\sigma_0(e^+e^- \to q\bar{q}) = 3e_q^2 \frac{4\pi\alpha^2}{3s}$



- can rewrite in terms of transverse momentum p_T between q and qbar

$$\frac{1}{\sigma}\frac{d\sigma}{dp_T} \sim \alpha_S \frac{1}{p_T^2} \ln\left(\frac{s}{4p_T^2}\right)$$

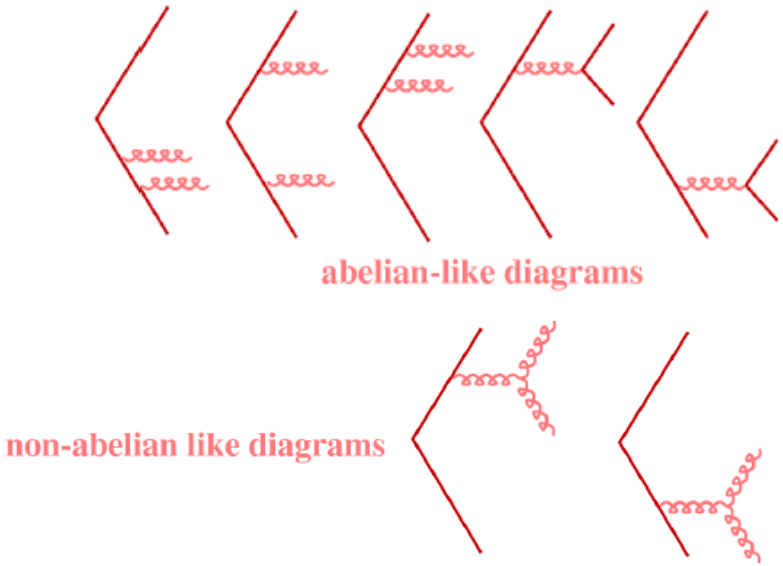
- **p_T** is **non-zero** only when there is **gluon** emission

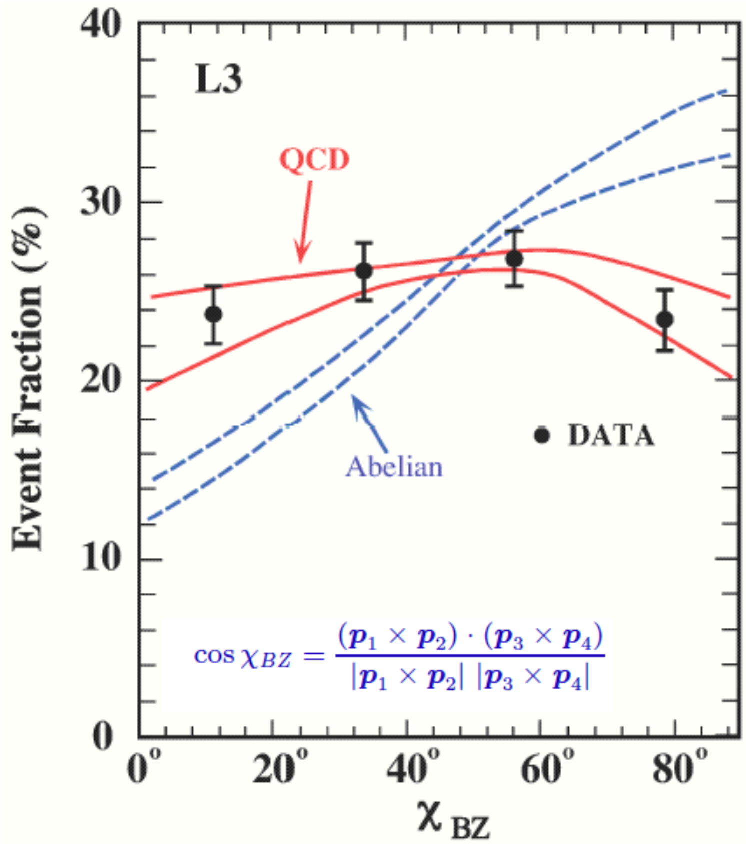
- measurement shown is of the p_T distribution w.r.t. the thrust axis of hadrons at PETRA at different $\sqrt{s}$

  ($\sqrt{s}$ increases from lowest to highest curves)

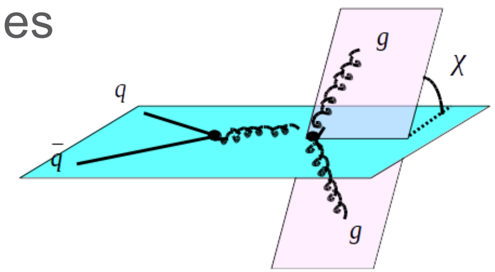
4-jet events and the triple gluon vertex

- **ggg** vertex should be observable in events containing at least 4 jets



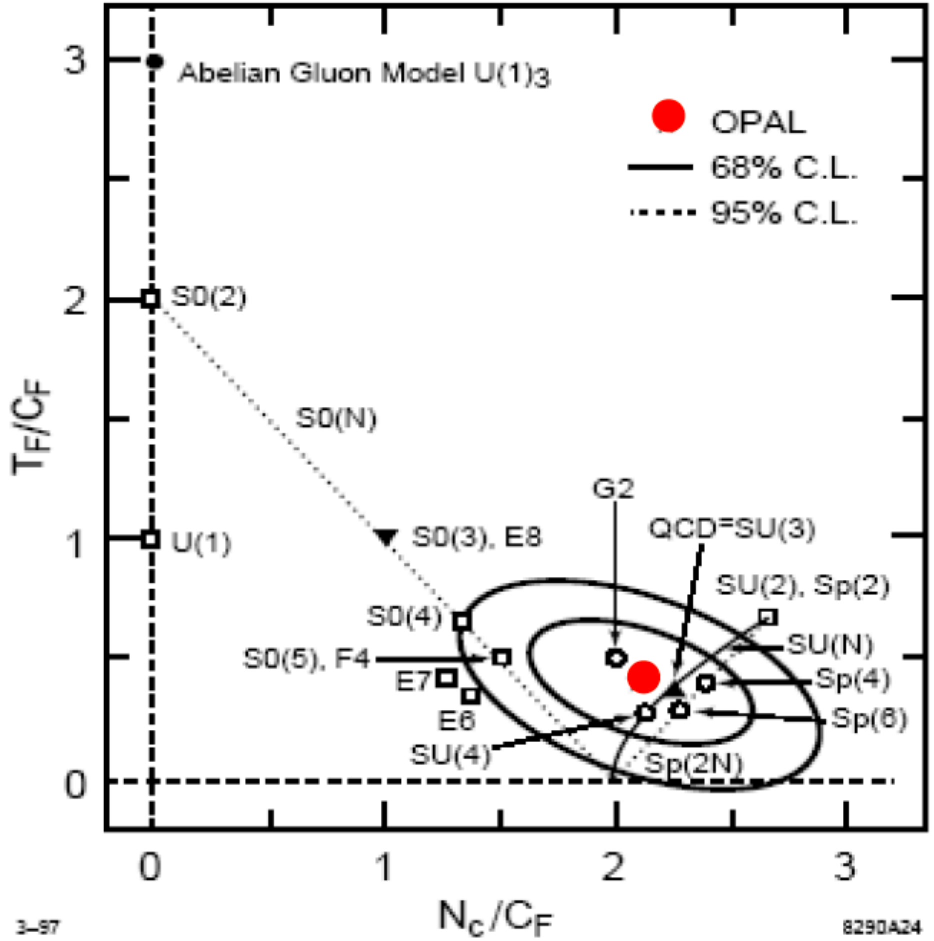


- **LEP** measured 4 jet events as a function of observables designed to highlight non-abelian nature of **QCD**
- EG. angle between planes of two lowest and two highest energy jets
- **non-Abelian** theory clearly favoured by data



QCD colour factor measurement

- simultaneous measurements of CA/CF and TF/CF in e+e- collisions have been made



Predictions:

Group	C_A/C_F	T_F/C_F
$SU(3)$	$9/4$	$3/8$
$[U(1)]^3$	0	3
$SO(3)$	1	1

Measurements:

$$C_A/C_F = 2.11 \pm 0.16(\text{St.}) \pm 0.28\ (\text{Sy.})$$

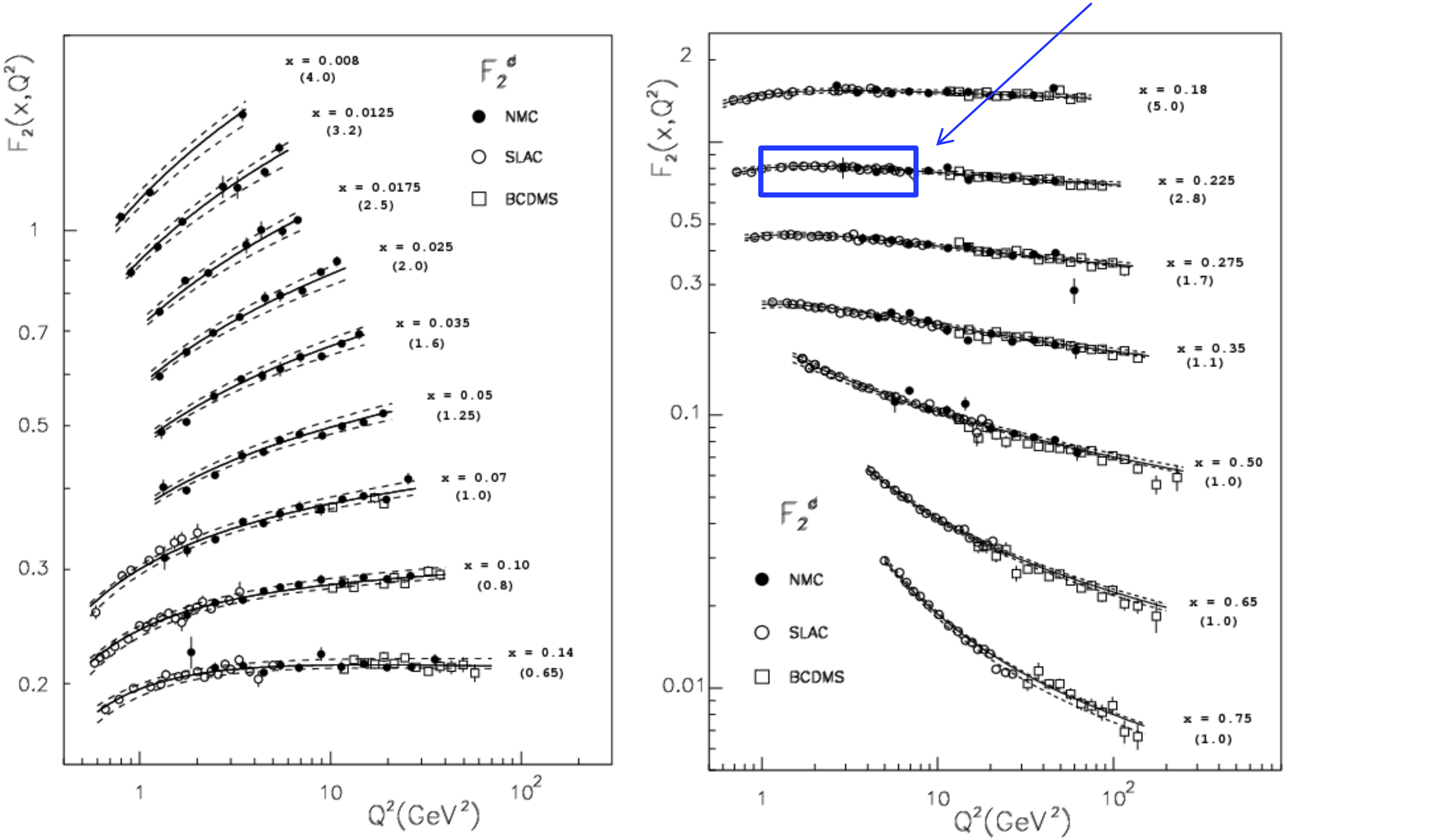
$$T_F/C_F = 0.40 \pm 0.11(\text{St.}) \pm 0.14\ (\text{St.})$$

SU(3) is clearly favoured

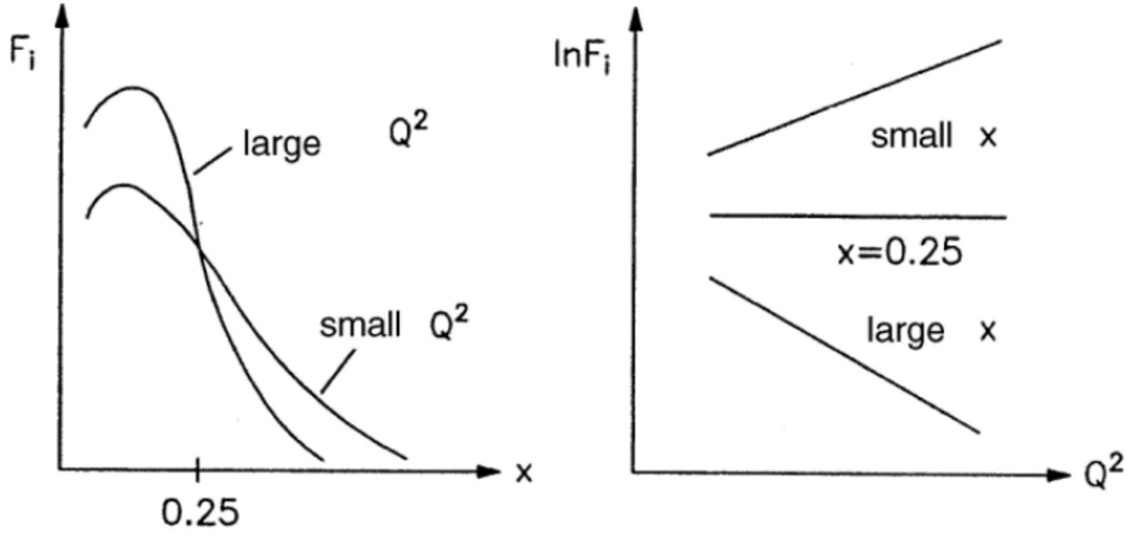
observation of scaling violation

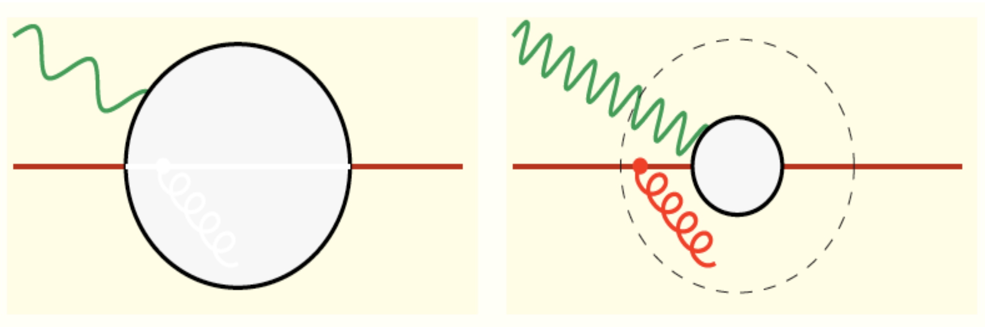
LO QCD: $F_2(x,Q^2) = \Sigma_i e_i^2(xq(x,Q^2) + xqbar(x,Q^2))$

Region of 1st SLAC measurement (1972)



origin of scaling violation



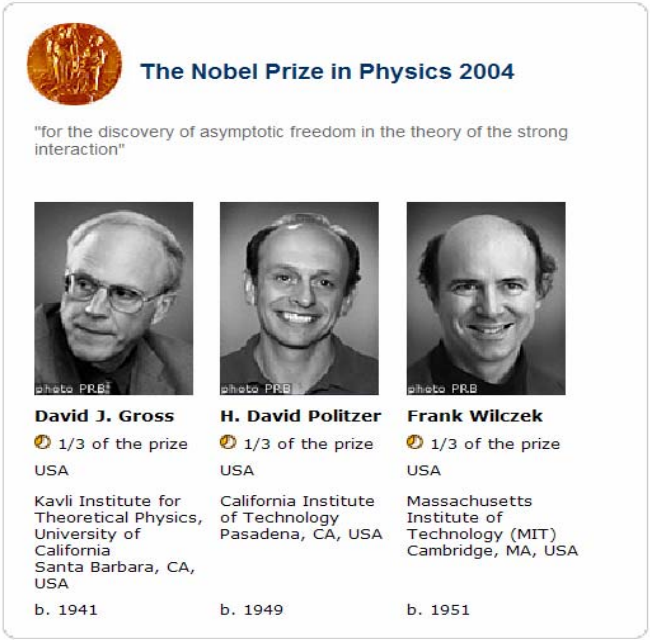


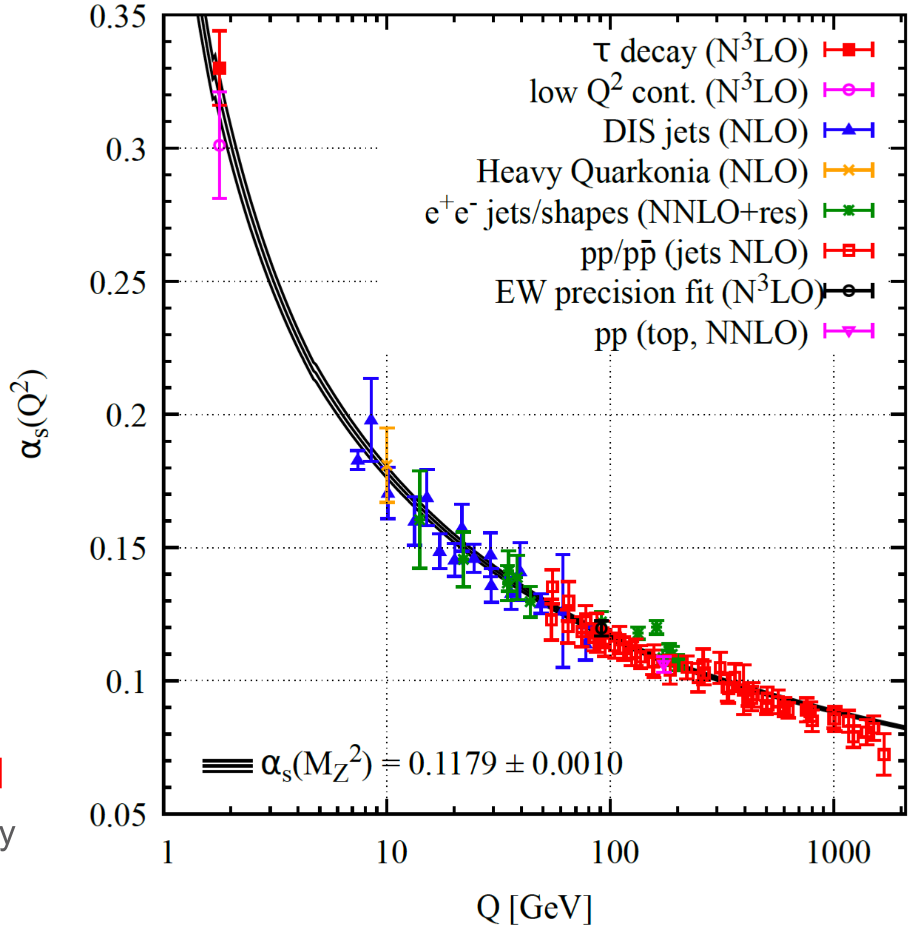
- observe "small" deviations from **exact Bjorken scaling** $F_2(x) \rightarrow F_2(x,Q^2)$

- **intuitive picture:**
- At high Q^2 observe more low x quarks
- **EXPLANATION**: at high Q^2 (shorter wavelength) resolve finer structure i.e. reveal quark is sharing momentum with gluons
- At higher Q^2 expect to "see" more low x quarks (and therefore fewer at high x)

As we shall shortly see, QCD **can** predict the **Q^2** dependence of F_2 BUT **cannot** predict the **x** dependence

QCD running coupling





- at low Q^2, **αs** is large: at Q^2=1 GeV^2, **αs ~1**
- **cannot** use perturbation theory; this is the reason why QCD calculations at low energy are so difficult, EG. properties of hadrons, hadronisation of quarks to jets, ….
- at high Q^2, **αs** is rather small, EG. at Q^2=Mz^2, **αs** =0.12 – **ASYMPTOTIC FREEDOM**
- **can** use perturbation theory, and this is the reason why in DIS at high Q^2, quarks behave as if quasi-free
- experimentally confirmed using many different kinds of measurements

extras

colour singlets – analogy with spin

★ It is important to understand what is meant by a singlet state

★ Consider spin states obtained from two spin 1/2 particles.

- Four spin combinations: $\uparrow\uparrow,\ \uparrow\downarrow,\ \downarrow\uparrow,\ \downarrow\downarrow$
- Gives four eigenstates of $\hat{S}^2,\ \hat{S}_z$ $\qquad (2\otimes 2 = 3\oplus 1)$

$$|1,+1\rangle = \uparrow\uparrow$$
$$|1,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow)$$
$$|1,-1\rangle = \downarrow\downarrow$$

spin-1 triplet

$$\oplus \quad |0,0\rangle = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow)$$

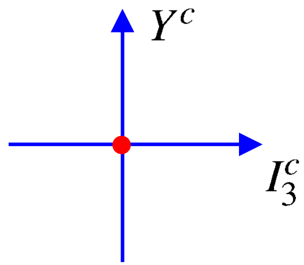
spin-0 singlet

★ The singlet state is “spinless”: it has zero angular momentum, is invariant under SU(2) spin transformations and spin ladder operators yield zero

$$S_{\pm}|0,0\rangle = 0$$

★ In the same way COLOUR SINGLETS are “colourless” combinations:

- they have zero colour quantum numbers $I_3^c = 0,\ Y^c = 0$
- invariant under SU(3) colour transformations
- ladder operators $T_{\pm},\ U_{\pm},\ V_{\pm}$ all yield zero



★ NOT sufficient to have $I_3^c = 0,\ Y^c = 0$: does not mean that state is a singlet

M Thomson

colour hypercharge and colour isospin

- colour hypercharge (Y^c) and colour isospin charge (I_3^C) can be used to define the **three colour** and **three anticolour** states that the **quarks** can be in

	Y^c	I_3^c		Y^c	I_3^c
r	1/3	1/2	$\bar{r}$	-1/3	-1/2
g	1/3	-1/2	$\bar{g}$	-1/3	1/2
b	-2/3	0	$\bar{b}$	2/3	0

gluon combinations: