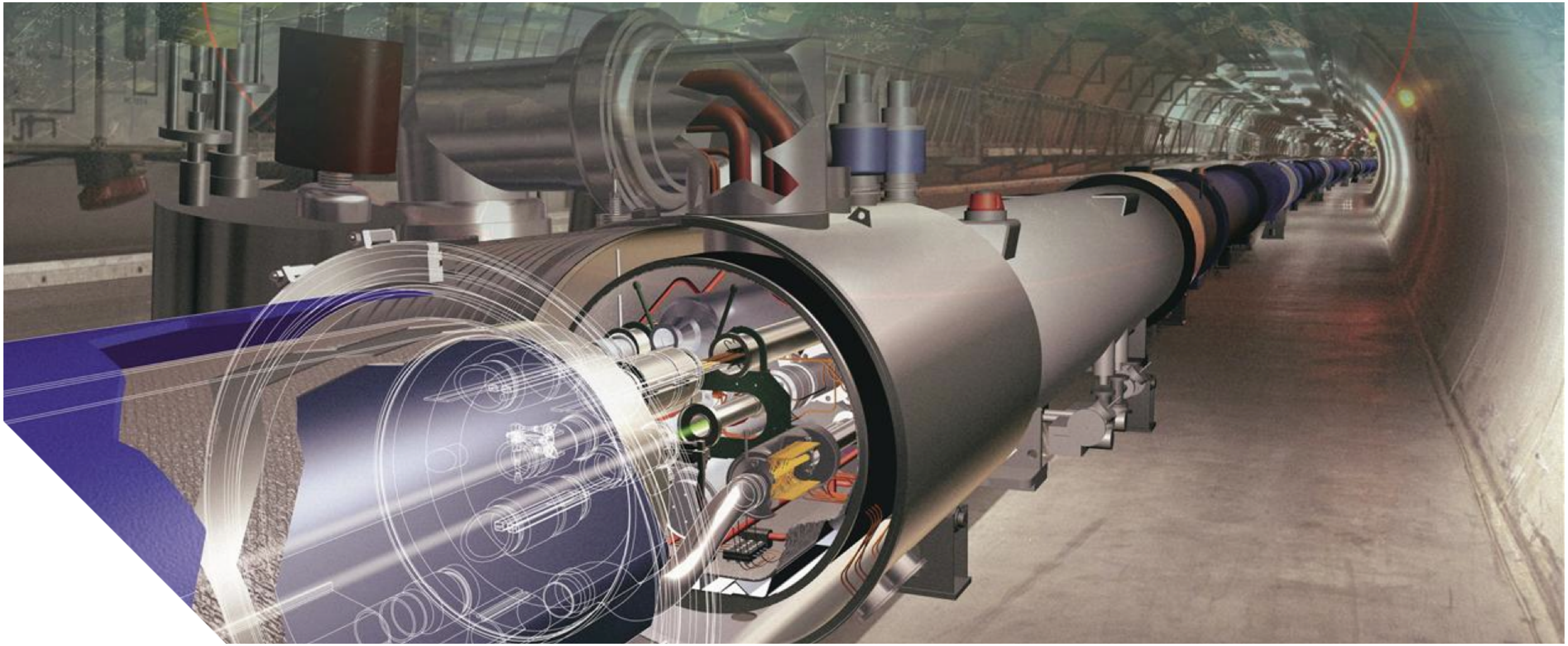


QCD – Lecture 5

PDF uncertainties; the Gluon & α_s

Claire Gwenlan, Oxford, HT

Heavy Quark treatment

illustrate with charm

Massive quarks introduce another scale into the process, the approximation $m_q^2 \sim 0$ cannot be used

Zero Mass Variable Flavour Number Schemes (ZMVFNs) traditional

c=0 until $Q^2 \sim 4m_c^2$, then charm quark is generated by $g \rightarrow$ c cbar splitting and treated as massless-- disadvantage incorrect to ignore m_c near threshold

Fixed Flavour Number Schemes (FFNs)

If $W^2 > 4m_c^2$ then c cbar can be produced by boson-gluon fusion and this can be properly calculated - disadvantage $\ln(Q^2/m_c^2)$ terms in the cross-section can become large- charm is never considered part of the proton however high the scale is.

General Mass variable Flavour Schemes (GMVFNs)

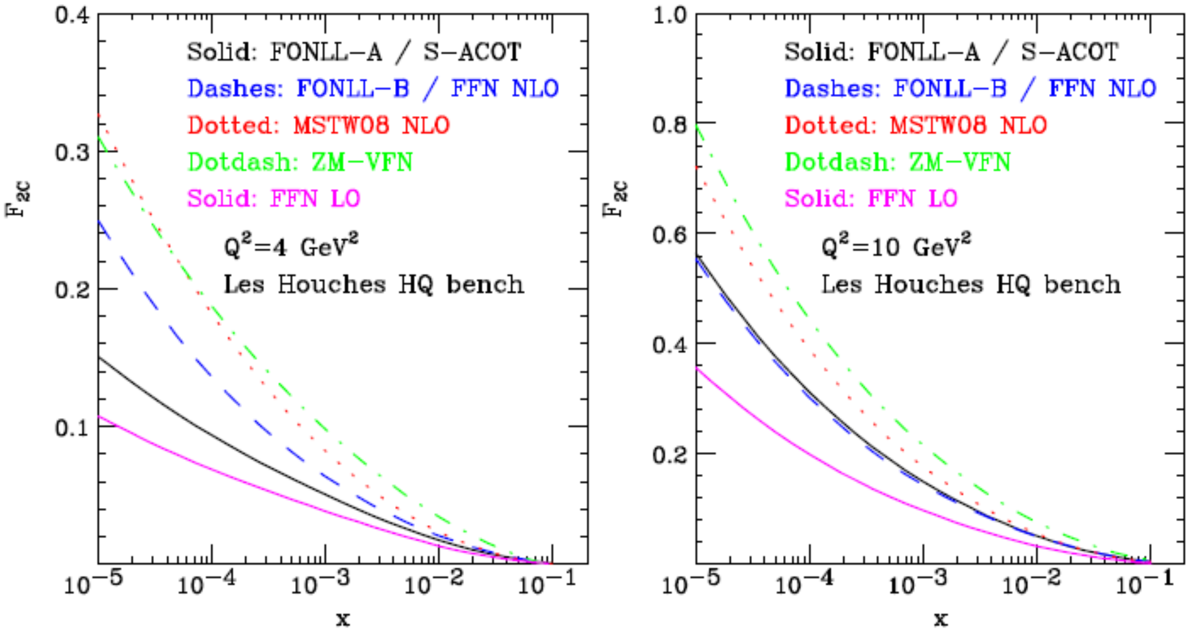
Combine correct threshold treatment with resummation of $\ln(Q^2/m_c^2)$ terms into the definition of a charm quark density at large Q^2

Arguments as to correct implementation but should look like FFN at low scale and like ZMVFN at high scale.

Additional complications for W exchange $s \rightarrow c$ threshold.

Heavy Quarks

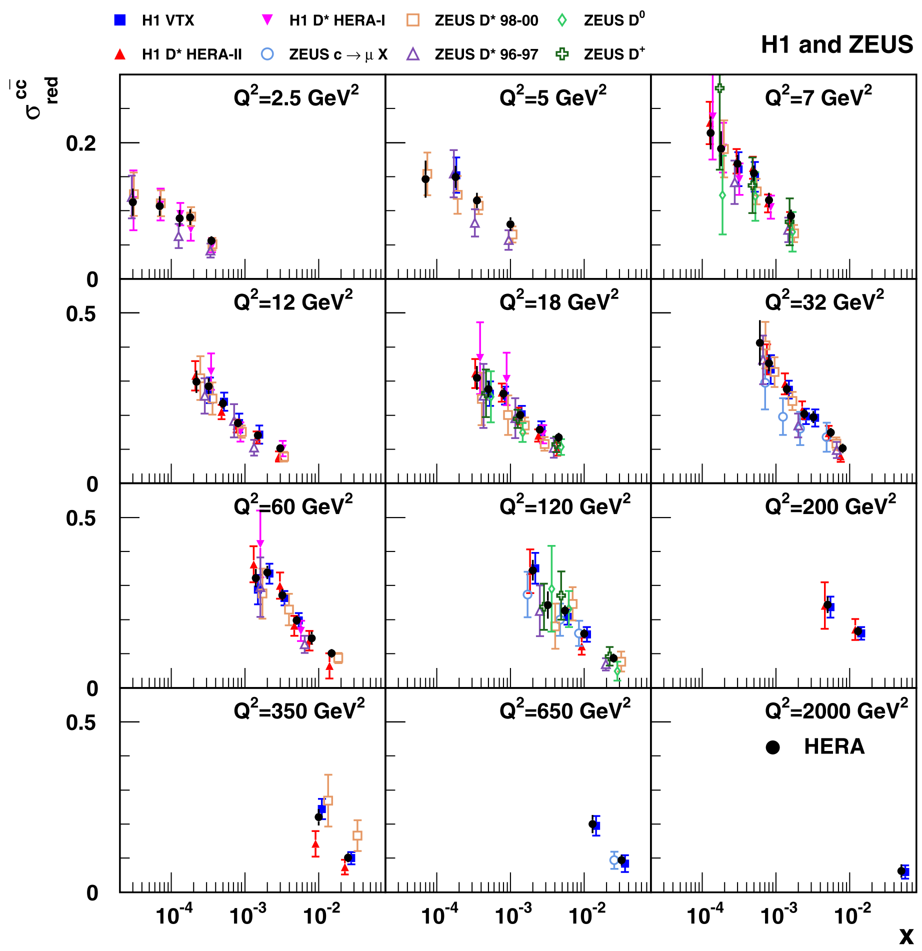
We must account properly for heavy quark production.: there are two extremes-
Use only 3 massless parton flavours and calculate exact ME's for heavy quark production (FFN method)- wrong at high scale since $\ln(Q^2/m_c^2)$ terms not resummed
Consider all partons as massless except that charm and beauty turn on abruptly at their kinematic thresholds (ZMVFN) – WRONG at low scale near these thresholds
A GMVFN (General-mass Variable Flavour Number Scheme) is supposed to give us the best of both worlds..
BUT there are different ways to do this...ACOT, Thorne, FO-NLL



plot from J Rojo

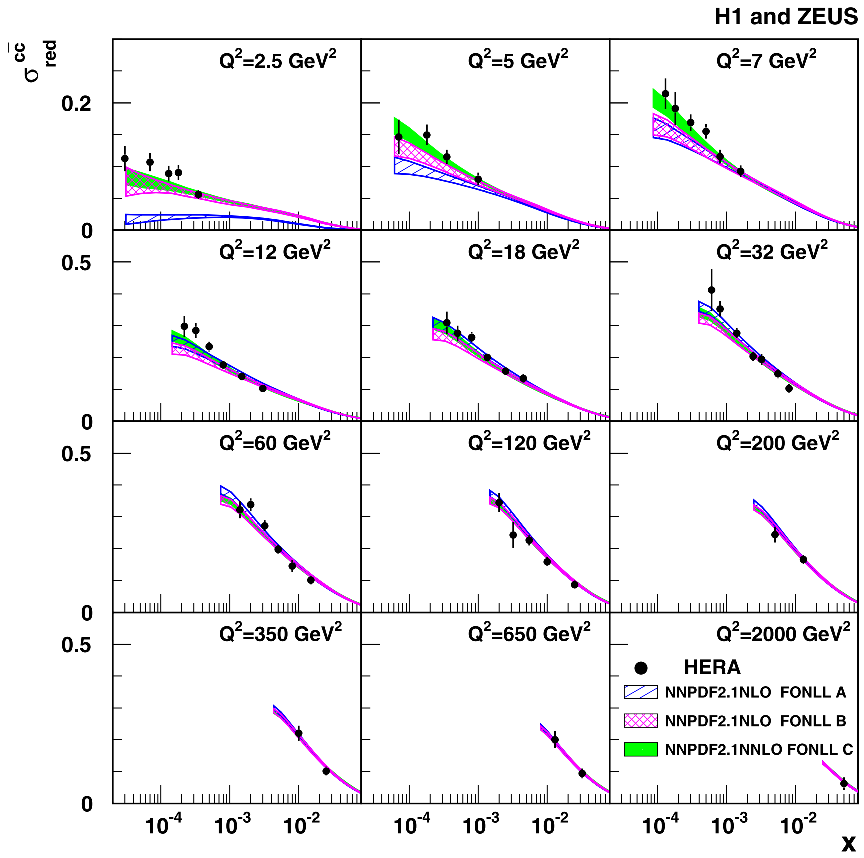
ZMVFN gives the largest c-cbar contribution to F2 and FFN the smallest

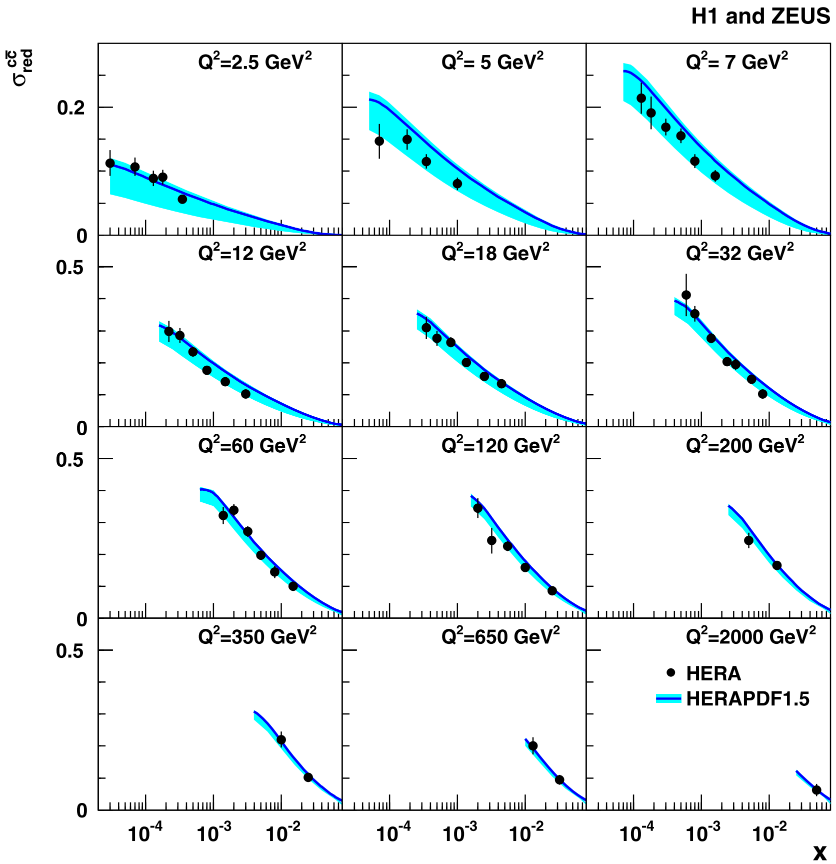
Heavy Quarks



- H1 and ZEUS have also **combined charm data** (arXiv:1211.1182)
- these data have sensitivity to the **heavy quark mass scheme** and **heavy quark mass**

Heavy Quarks





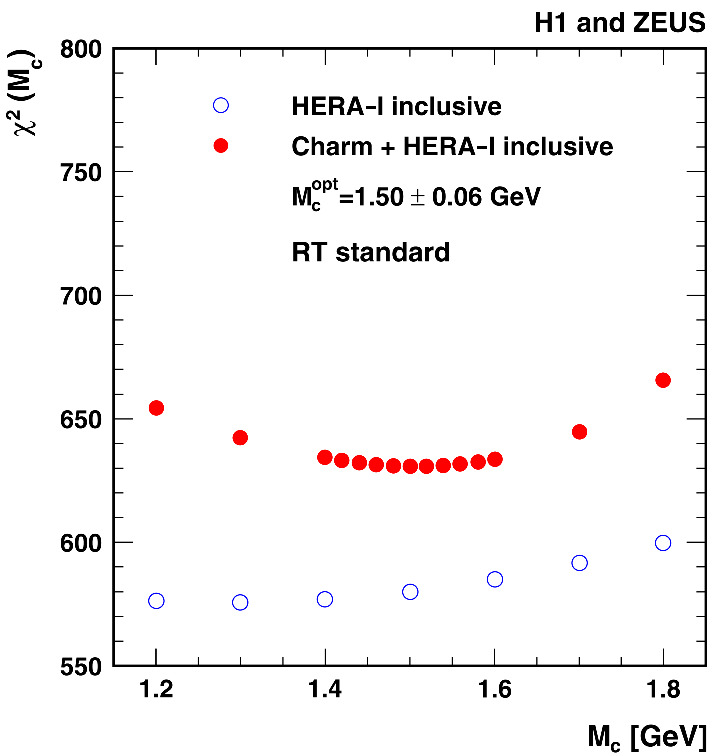
- **GM-VFNS:** some show better agreement than others

(FONLL details in: arXiv:1001.2312)

- **HERAPDF** gives good description – within error band
- error band dominated by choice of charm mass, spanning mc=1.35 (high) to mc=1.65 (low) GeV
- measurement shows some preference for higher charm mass than standard choice mc=1.4 GeV

sensitivity to charm mass

If we input the charm data to the PDF fit it does not change the PDFs significantly BUT

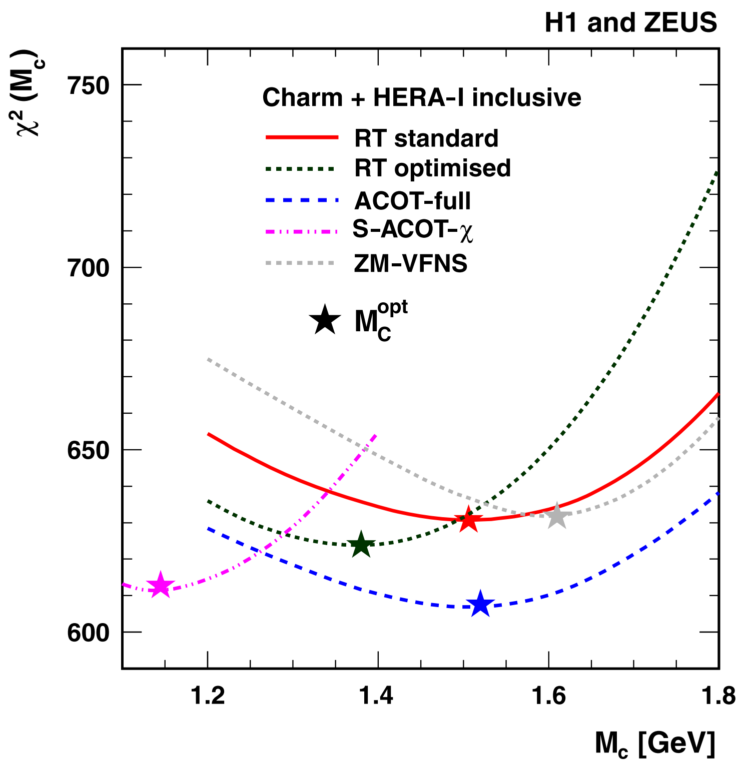


Before charm is input the χ2 profile vs the charm mass parameter is shallow..

After charm is input the χ2 profile vs the charm mass parameter gives

m_c = 1.50 ± 0.06 GeV

sensitivity to charm mass



BUT the HERAPDF uses the Thorne General Variable Flavour Number Scheme for heavy quarks as used by MSTW08

This is not the only GM-VFNS

CTEQ uses ACOT

NNPDF2.0 used ZM-VFN/2.1 and later use FONLL

These all have different preferred charm mass parameters, and all fit the data well when used with their own best fit charm mass while we've got a long way in agreeing on reasonable model assumptions…

…there is still room for choice:

- values of heavy quark masses – and even the heavy quark scheme
- value of $\alpha_s(M_Z^2)$ – or determine it in the fit
- value of Q^2_0
- value of Q^2_{min} of the data – include low Q^2, low W^2 data or not
- the choice of datasets included
- form of the parameterisation

AND there is also the matter of how you treat the experimental uncertainties …

PDF experimental uncertainties

So we have a QCD prediction for F_2 for a particular x,Q2

F_2^{lp} made up for evolved singlet + non-singlet densities

and we have a measurement F_2^{meas}

We perform χ^2 fitting

Traditionally,

$$\chi^2 = \sum_i \left[\frac{F_{2,i}^{\mathrm{QCD}} - F_{2,i}^{\mathrm{meas}}}{\sigma_i^2} \right]^2$$

i sums over x,Q^2 points

$$\sigma_i^2 = \left(\text{statistical error on } i^{\text{th}} \text{ measurement}\right)^2 + \left(\text{systematic error on } i^{\text{th}} \text{ measurement}\right)^2$$

Good $\chi 2$ → theoretical picture is *valid*

→ determines ~ 15 parameters (note α_s may also be a parameter)

→ errors on these parameters can also be propagated back to give errors on parton distributions + predictions of structure functions, cross sections etc. not yet measured.

Not good enough!
What about correlated *systematic* errors?

PDF experimental uncertainties

Correlated errors

- Normalisations → all points move up or down together
- More subtle. e.g. calorimeter energy scale moves events between x, Q^2 bins
  → correlations change the *shape* of the **function**

$$\chi^2 = \sum_i \sum_j \left(F_i^{\mathrm{QCD}} - F_i^{\mathrm{meas}}\right) V_{\mathrm{ij}}^{-1} \left(F_j^{\mathrm{QCD}} - F_j^{\mathrm{meas}}\right)$$

$$V_{ij} = \delta_{ij}\sigma_i^2 + \sum_\lambda \Delta_{i\lambda}^{sys}\Delta_{j\lambda}^{sys}$$

correlation matrix

$\Delta_{i\lambda}^{sys}$ is the correlated systematic error on point *i* due to source λ

$$\Rightarrow \chi^2 = \sum_i \left(\frac{F_i^{\mathrm{QCD}}(p,s) - F_i^{\mathrm{meas}}}{\sigma_i^2}\right)^2 + \sum_\lambda s_\lambda^2$$

$$\text{where } F_i^{\mathrm{QCD}}(p,s) = F_i^{\mathrm{QCD}}(p) + \sum_\lambda s_\lambda \Delta_{\mathrm{i}\lambda}^{\mathrm{sys}}$$

i.e. the prediction is modified by each source of systematic uncertainty.
$s\lambda$ are fit parameters which have zero mean and unit variance if all systematics have been estimated correctly.

PDF experimental uncertainties

The PDF fit results in a set of parameters ***p*** with errors.

Now let's talk about the experimental errors.
The PDF shapes are functions ***F*** of these parameters so the errors on the PDFs:

$$\langle \sigma^2_{\mathrm{PDF}} \rangle = T^2 \sum_j \sum_k \frac{\partial F}{\partial p_j} V_{\mathrm{jk}} \frac{\partial F}{\partial p_k}$$

The cross sections / structure functions are more complex functions of the PDFs and their errors can be similarly evaluated.
Two points:

- PDF groups diagonalise V_{jk} and refer to PDF eigenvectors – which are just suitable combinations of parameters,

$$\langle \sigma^2_F \rangle = T^2 \sum_j \left[\frac{F(p_j^+) - F(p_j^-)}{2} \right]^2$$

Or you can use the asymmetric version adding up +ve and –ve deviations from the central predictions in quadrature separately – better when errors are non-Gaussian

- For 68% CL error bands you would think that the tolerance, $T^2=1$, and similarly for 90%, $T^2=2.71$, but this is NOT so for MMHT or CT (and their precursors)

(where: $T = \sqrt{\Delta\chi^2_{\mathrm{global}}}$)

choice of tolerance I

MSTW example

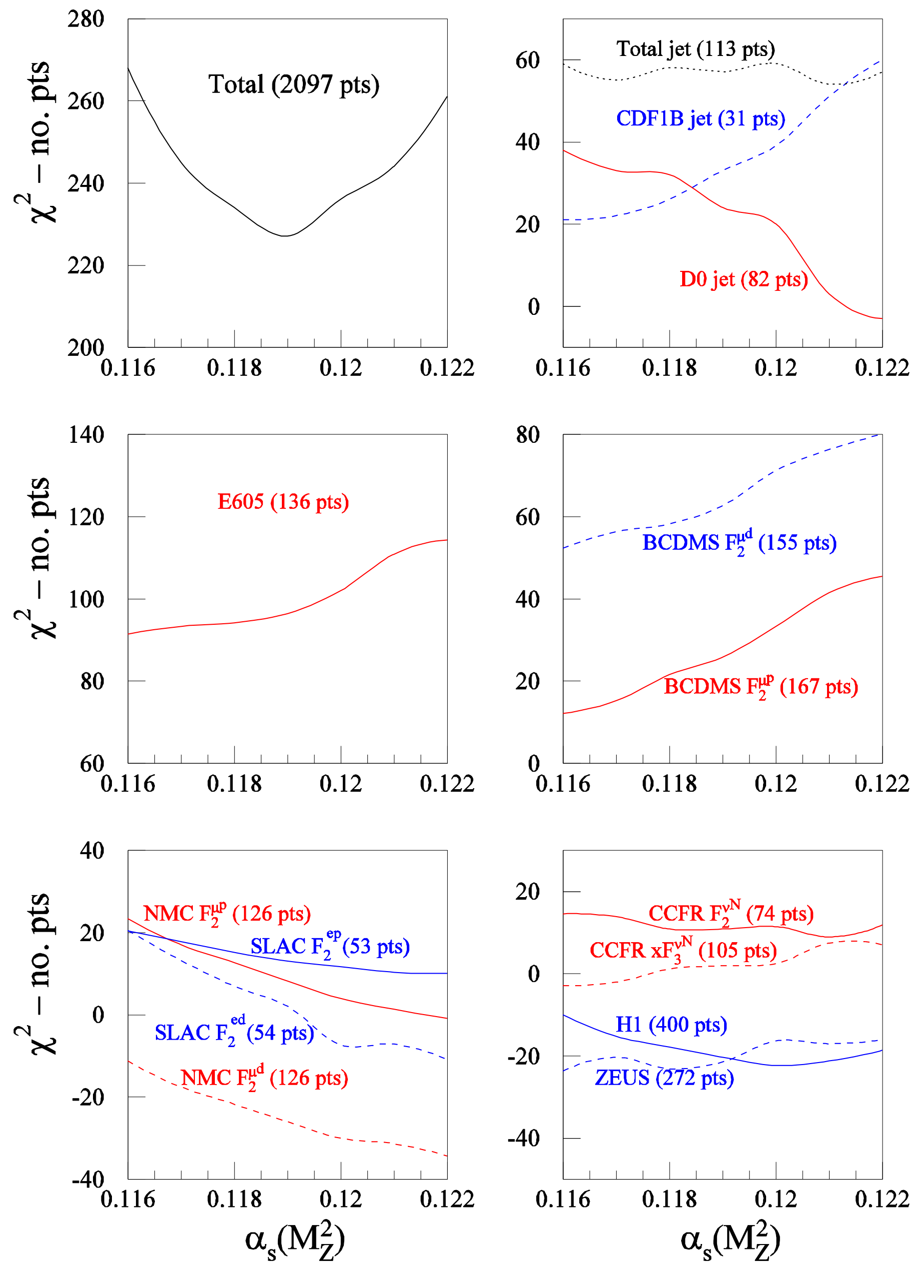
some data sets incompatible or only marginally compatible?

to illustrate: X2 for the MSTW global fit is plotted versus the variation of a particular parameter (αs in this case)

individual X2 for each experiment also plotted versus this parameter in the neighbourhood of the global minimum → each experiment favours a different value of αs

PDF fitting is a compromise;

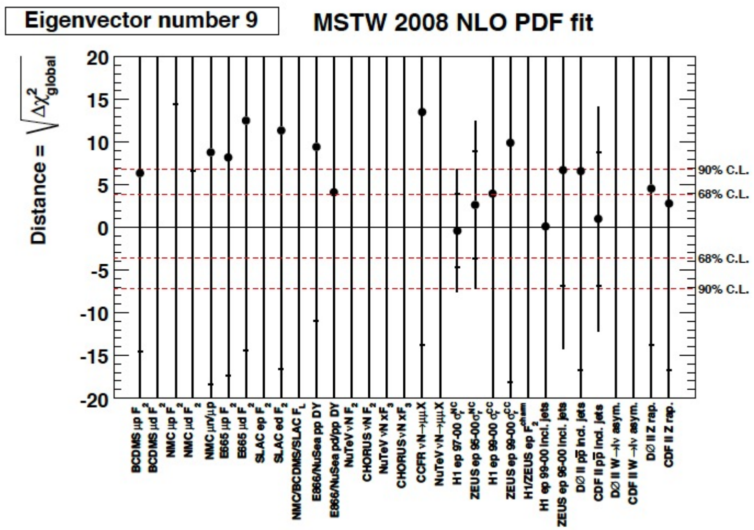
can one evaluate acceptable ranges of the parameter values with respect to the individual experiments?



choice of tolerance II

MSTW example

Tolerance



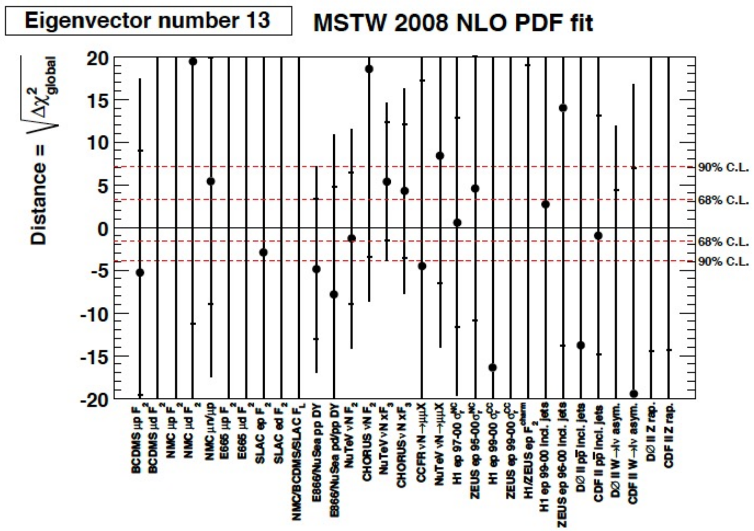


Figure 9: Ranges of $(\Delta\chi^2_{\rm global})^{1/2}$ along (a) eigenvector 9 and (b) eigenvector 13 for which data sets are satisfied within their 90% C.L. limit (outer error bars) or 68% C.L. limit (inner error bars). The points (•) indicate the minimum with respect to each particular data set. The tolerance, indicated by the horizontal dashed lines, is chosen to ensure that all data sets are described within their 68% or 90% C.L.

HOW far away from the central fit can you go and still fit each data set within 90% (or 68%) CL?

no further than this

HERA data determine the limit on this eigenvector

E866/NuTeV data determine the limit on this one

choice of tolerance III

MSTW example

summary of which eigenvectors are determined by which datasets

in MSTW example:
68% CL has T=4–5
90% CL has T=7–8

which means:
ΔX^2 ≈ 20 (68% CL)
≈ 50 (90% CL)

CT typically use even larger tolerances

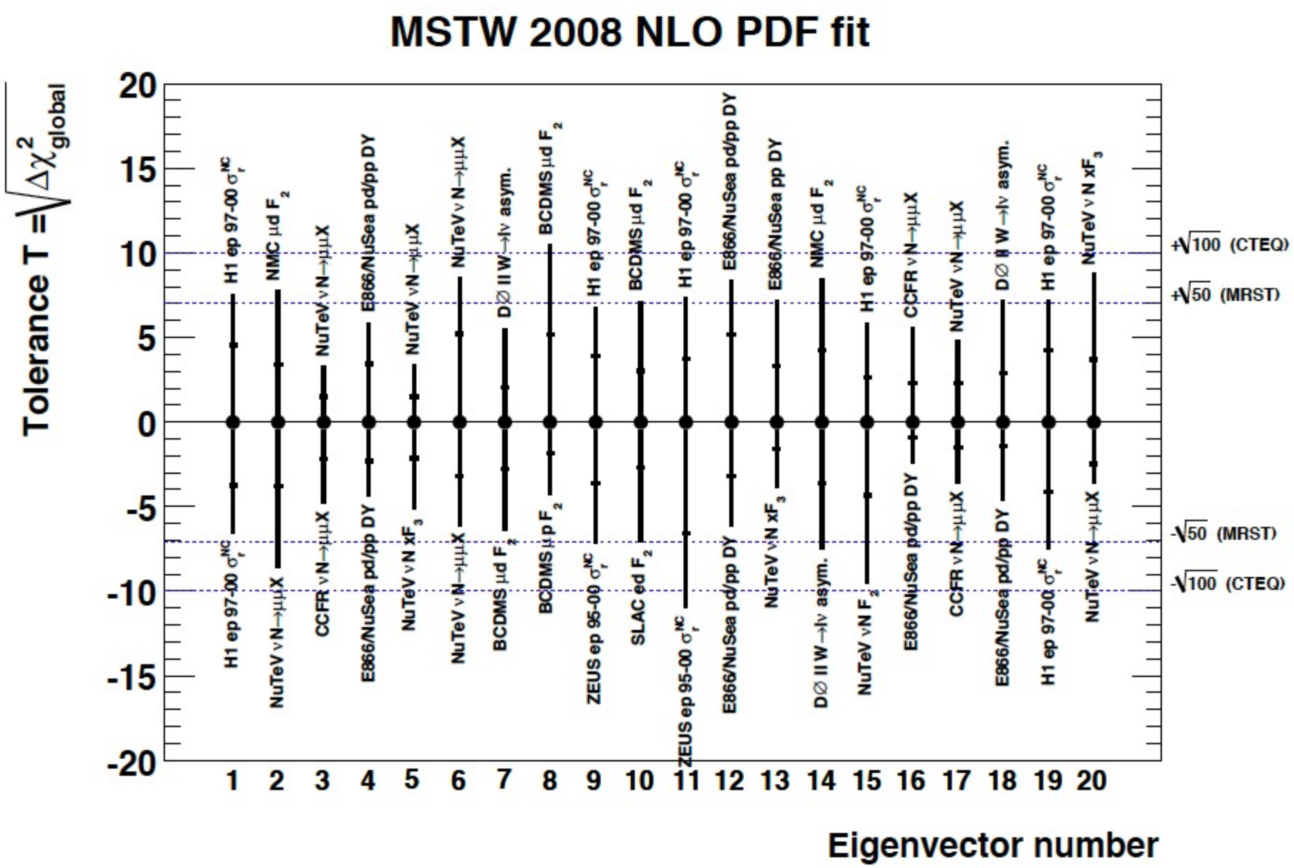


Figure 10: Tolerance for each eigenvector direction determined dynamically from the criteria that each data set must be described within its 90% C.L. (Eq. (58)) (outer error bars) or 68% C.L. limit (inner error bars). The labels give the name of the data set which sets the 90% C.L. tolerance for each eigenvector direction.

FEAR NOT: you can use the PDF sets in LHAPDF as a "black box" – it is all done for you

PDF uncertainties

All public PDFs available at:
https://lhapdf.hepforge.org/

All YOU have to do is this:

symmetric case:

$$\Delta F = \frac{1}{2}\sqrt{\sum_{k=1}^{n}\left[F(S_k^+) - F(S_k^-)\right]^2}$$

OR asymmetric case:

$$(\Delta F)_+ = \sqrt{\sum_{k=1}^{n}\left\{\max\left[\, F(S_k^+) - F(S_0),\ F(S_k^-) - F(S_0),\ 0\right]\right\}^2}$$

$$(\Delta F)_- = \sqrt{\sum_{k=1}^{n}\left\{\max\left[\, F(S_0) - F(S_k^+),\ F(S_0) - F(S_k^-),\ 0\right]\right\}^2}$$

measuring the gluon and α_S

Now let's consider the measurement of $\alpha_S(M_Z^2)$ and the gluon PDF in DIS

Ways to measure α_S:

- For non-singlet (valence) quark distributions,

$$\frac{\partial q^{\mathrm{NS}}}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes q^{\mathrm{NS}}$$

There is no contribution to evolution from the gluon

Thus the evolution of a *non-singlet* structure function,

- Like xF_3 in $\nu, \bar{\nu}$ N
- or xF_3 in $e^{\pm}p$ at high Q^2 via Z_0 exchange

Can directly measure α_S with the smallest number of assumptions.
Unfortunately it also has the largest experimental difficulty

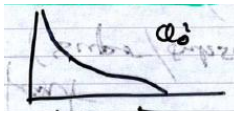
measuring the gluon and α_S

- More usually the scaling violations of the *singlet* structure function have to be used so that the determination of α_S is *coupled* to the *gluon* shape determination.

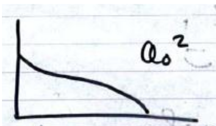
$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix}\Sigma \\ g\end{pmatrix} = \frac{\alpha_s}{2\pi}\begin{pmatrix}P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg}\end{pmatrix} \otimes \begin{pmatrix}\Sigma \\ g\end{pmatrix}$$

Increasing α_S increases the *negative* contribution from P_{qq} term, but this can be compensated by the *positive* contribution from P_{qg} term if the *gluon* is made harder.

α_S increases → gluon harder

So, α_S = 0.115 and 

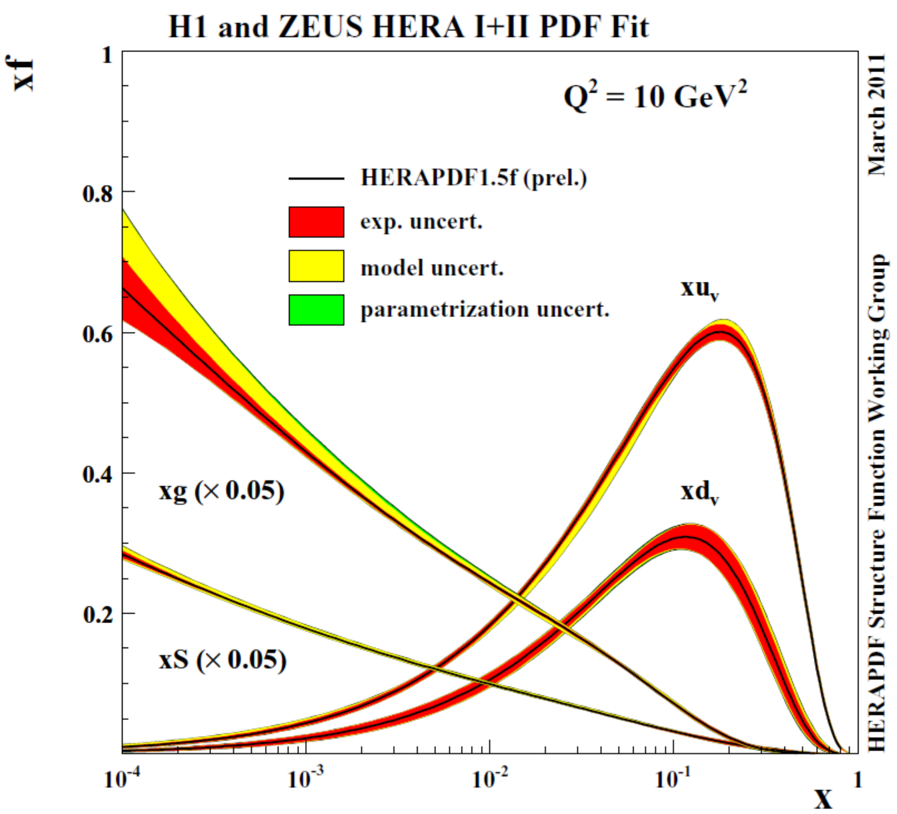
may give a similar χ^2 to

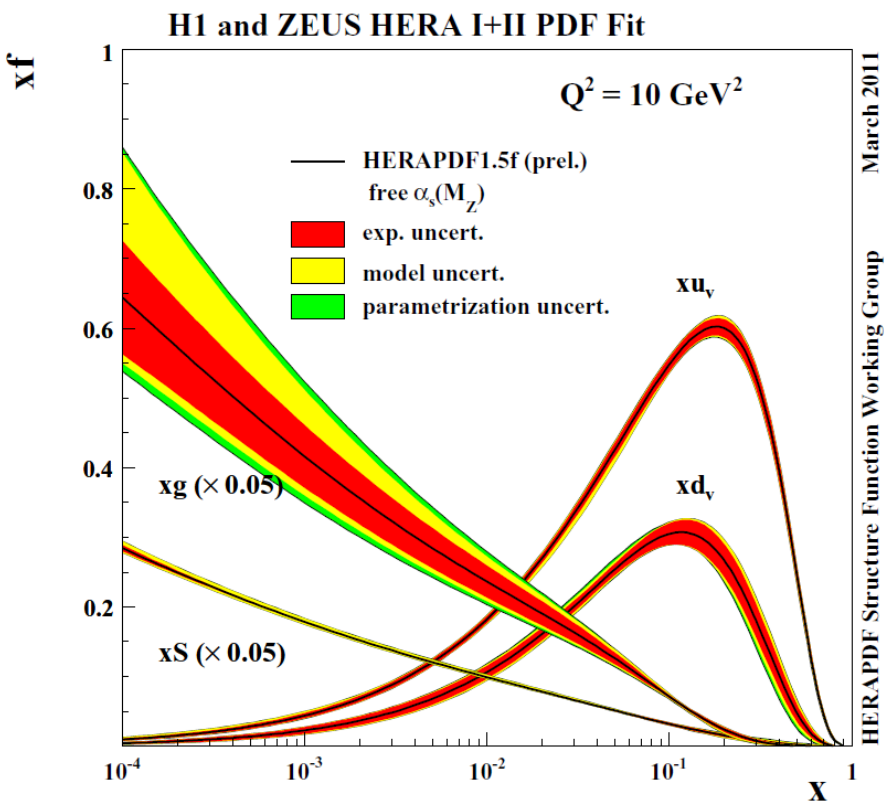
α_S = 0.118 and 

α_S is determined in the same *global* fits which determine PDF parameters

→ Fortunately there are now so many data points now that there are limits to this freedom.

PDF+αs fits





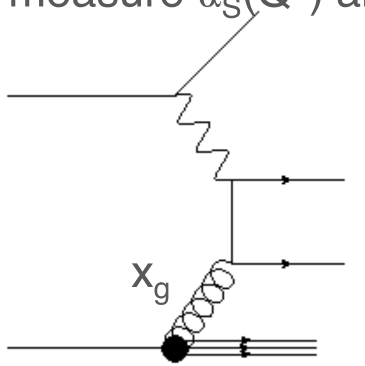
- many PDFs use a fixed value of **αs(MZ)** by default, EG. CT(EQ), NNPDF, HERAPDF, …
- and supply PDFs for various different fixed **αs(MZ)** values

- look what happens when you free **αs(MZ)** and **ONLY** use inclusive (NC/CC) DIS data

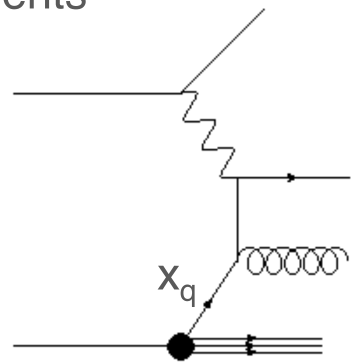
jet measurements and α_S

Jet studies in the Hadron Final state gives us more information

- You can measure $\alpha_S(Q^2)$ and $xg(x,Q^2)$ from 2+1 jet events



BGF



QCDC

+1 means proton remnant

$$\sigma_{2+1} \sim \alpha_s\left[A\, x_g\, g\left(x_g, Q^2\right) + B\, x_q\, q\left(x_q, Q^2\right)\right]$$

(glue) + (quark)

This helps to break the $\alpha_S(Q^2)$ / gluon PDF correlation
Use more information that depends directly on the gluon -- jet cross-sections

To get $x\, g(x,Q^2)$

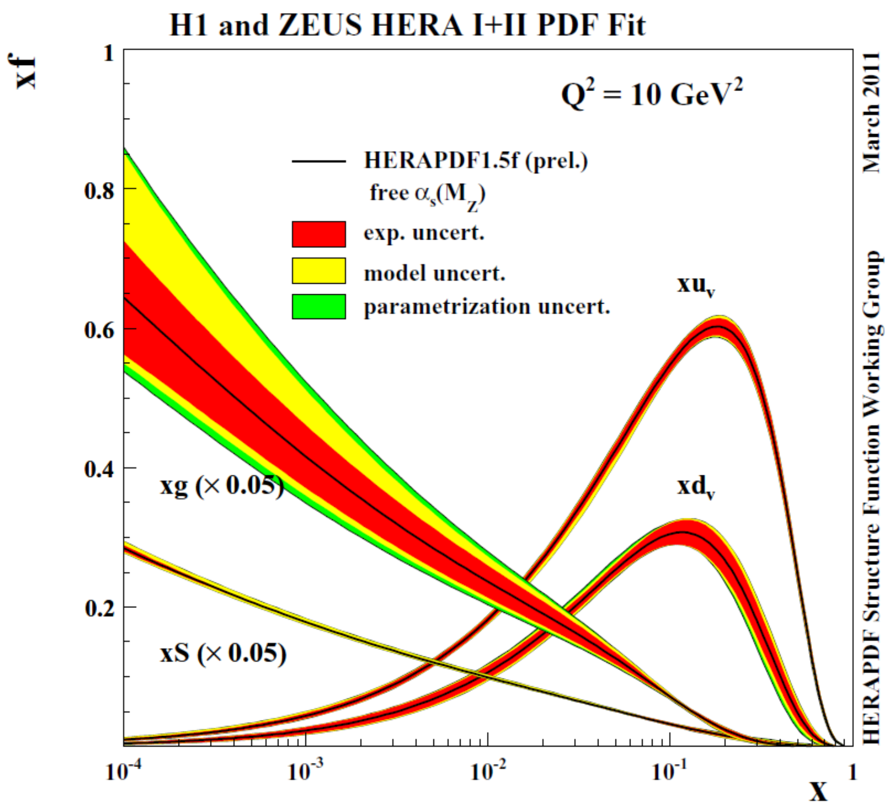
- Assume α_S is known
- Choose kinematic region BGF > QCDC (i.e. low x, Q^2)

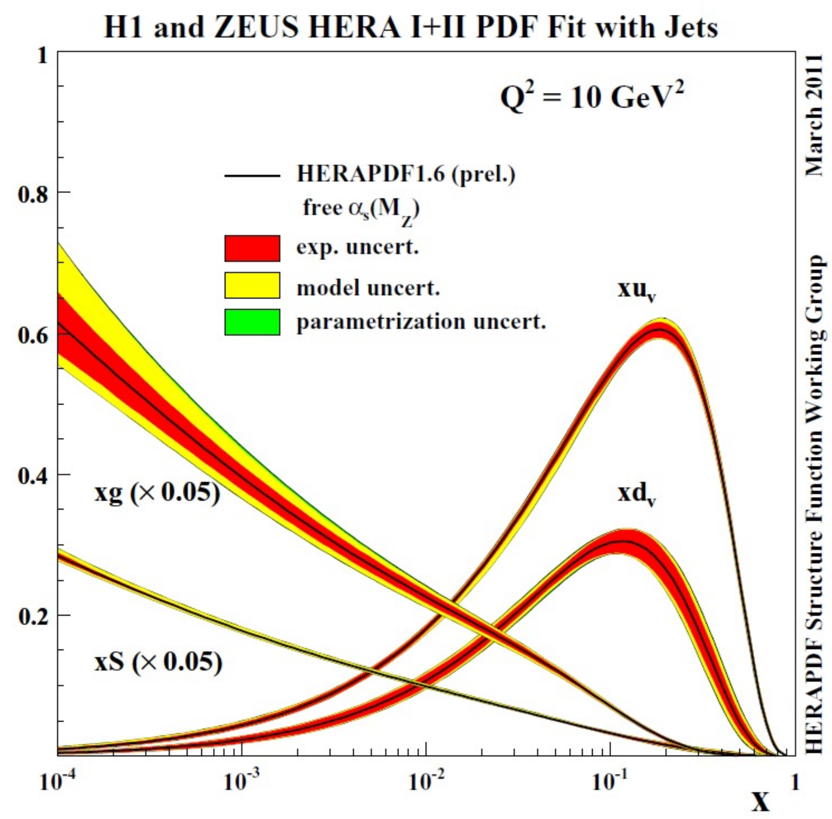
To get $\alpha_S(Q^2)$

- Choose kinematic region where PDFs $xq(x)$, $x\, g(x)$ are well known. (i.e. $x_g > 10^{-2}$, $x_q > 10^{-3} - 10^{-2}$ and $\sigma_{BGF} \sim \sigma_{QCDC}$

In practice, we fit jets in all kinematic regions and hope to determine **xg(x,Q2)** and **αs(Q2)** simultaneously

jet measurements and αs



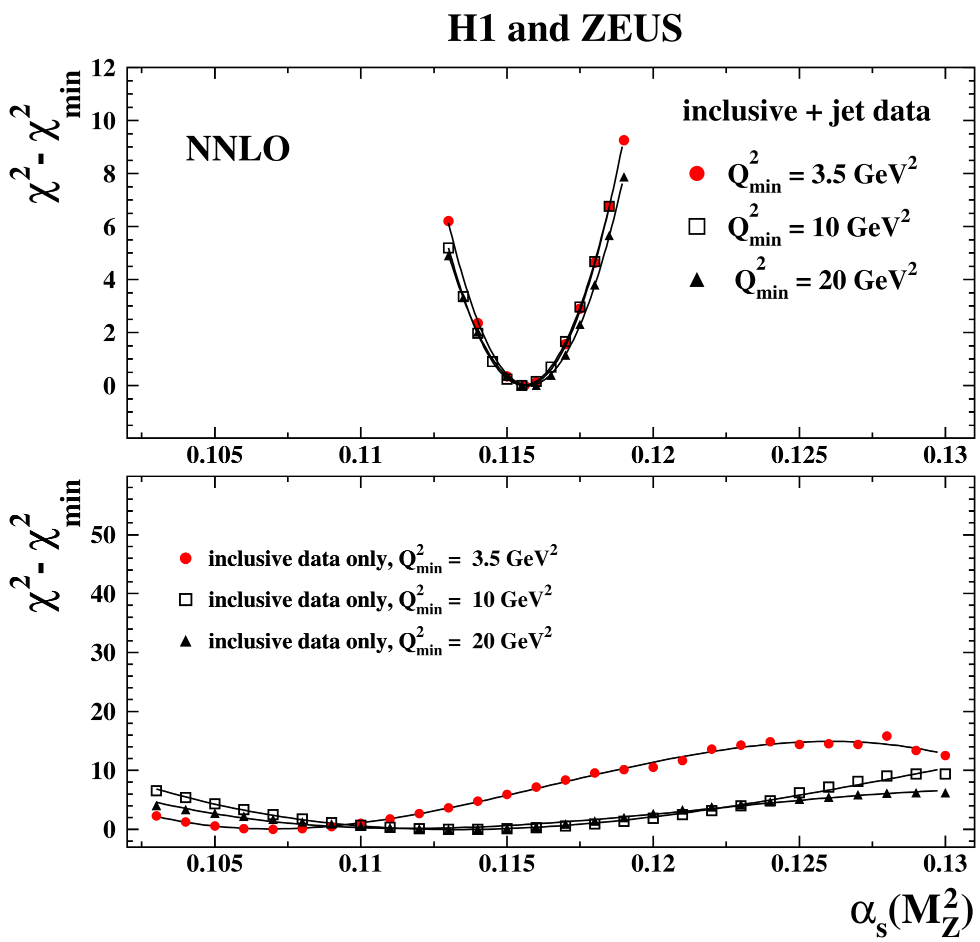


- this is what happens when **αs(MZ)** is kept as a free parameter in the fit, but **DIS jet measurements** are added

this is also true when adding in jet measurements from ppbar/pp collisions (Tevatron, LHC)

jet measurements and αs

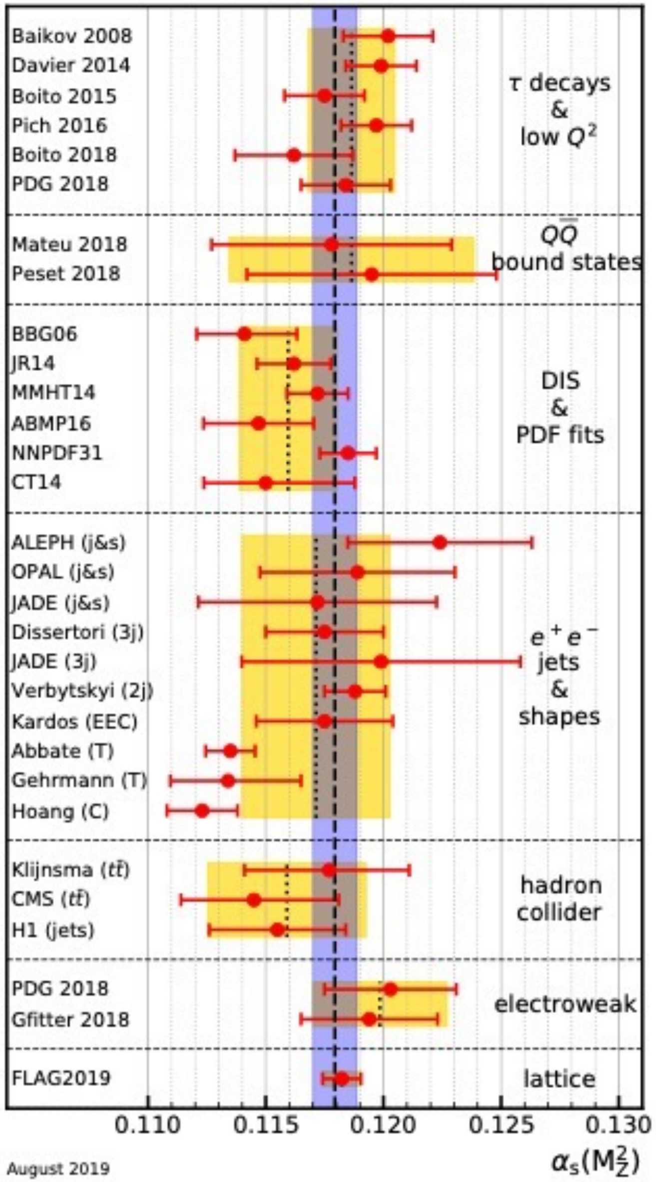
arXiv:2112.01102



- and look at what happens to your ability to determine **αs(MZ)**

$$\alpha_s(M_Z^2) = 0.1156 \pm 0.0011 \text{ (exp) } {}^{+0.0001}_{-0.0002} \text{ (model + parameterisation) } \pm 0.0029 \text{ (scale)}$$

other αs determinations



← αs from QCD fits

ways to measure the gluon distribution

- Scaling violations in DIS $\frac{\partial F}{\partial \ln Q^2} \sim x\, g(x, Q^2)$ particularly useful at low x
- Prompt γ data
  - Older fixed target data did not agree with predictions well- but at higher p_t- for example at ATLAS there is hope.
  - $pN \rightarrow \gamma\, x$
  - $g\, q \rightarrow \gamma\, q$
- Inclusive jet production
  - Tevatron and LHC jet data have been used
  - $p\, p \rightarrow \text{jet} + x$
  - $g\, g \rightarrow g\, q,\ g\, q$
- 2 jets in DIS
  - HERA jet data have been used
  - $\gamma^*\, g \rightarrow q\, g$ (BGF)

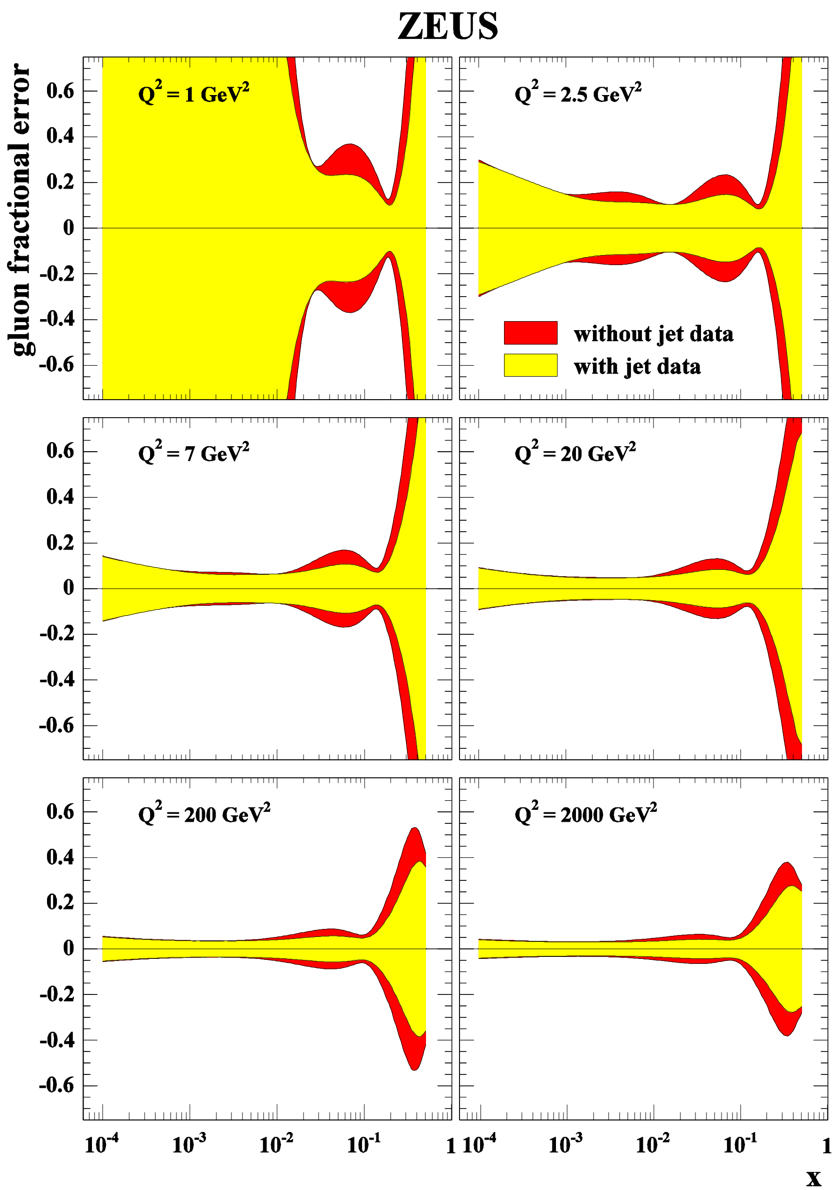
Future:

- Open charm
  - So far tells us more about charm schemes than about the gluon

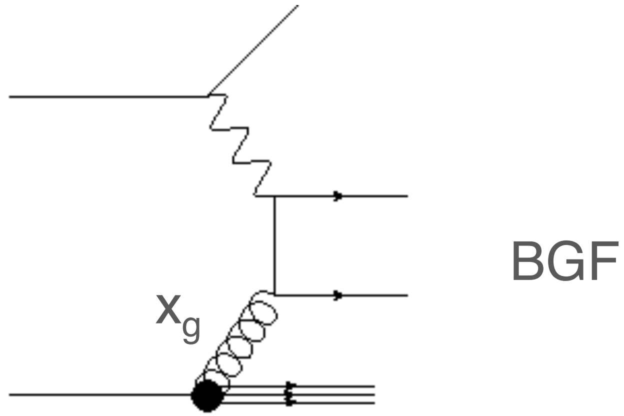
  F_2^{cc}, J/ψ, D_s,D^* prodn from $\gamma^*\, g \rightarrow c\, c$
  - Measurement of F_L (at small x) HERA FL measurements are used
  -

$$x\, g(x, Q^2) = \frac{3}{5} \times 5.8 \left[\frac{3\pi}{49\, s} F_L(0.4\, x, Q^2) - \frac{1}{2} F_2(0.8\, x, Q^2) \right]$$

EG. HERA-II and Tevatron Run-II have improved our knowledge



hep-ph/0503274



- **example:** decrease in **gluon PDF** uncertainty from using ZEUS jet data ("ZEUS-Jets" PDF fit)
- **DIRECT measurement** of the gluon distribution
- ZEUS jet measurements much more precise than Tevatron jets – small energy scale uncertainties (we will see examples of impact of LHC jet data later…)

extras