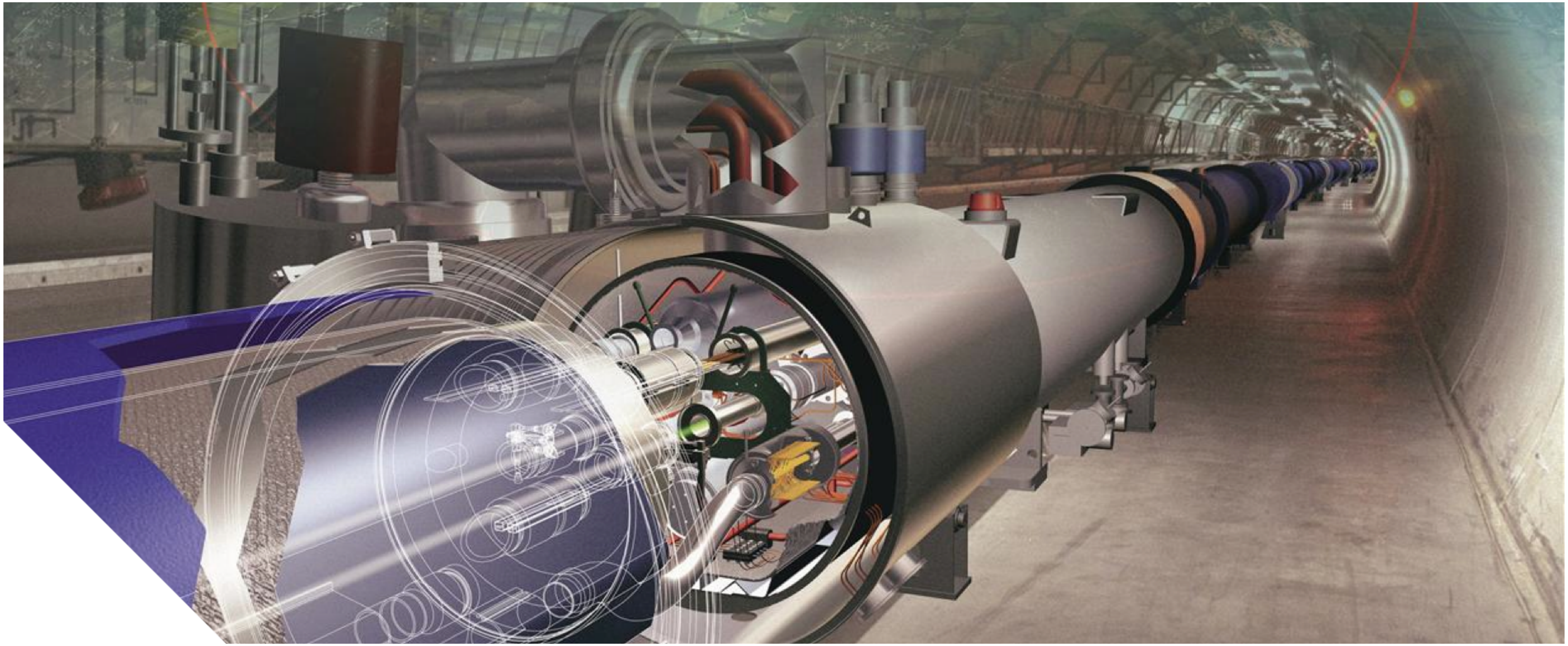
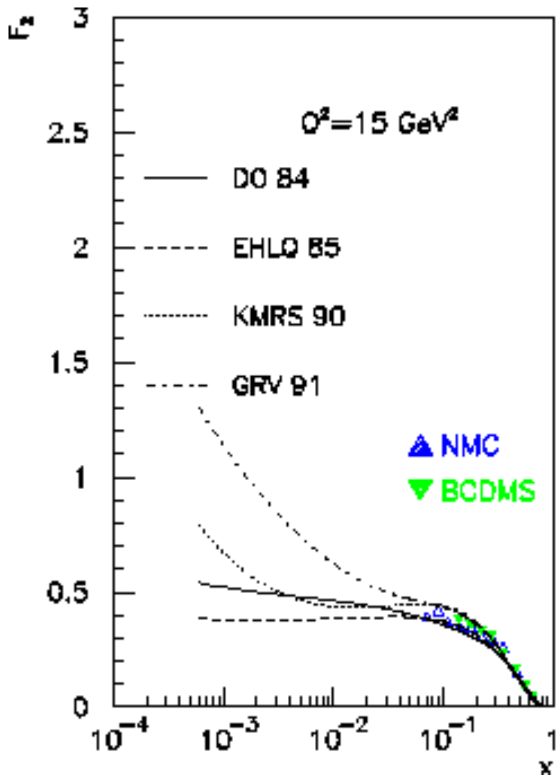


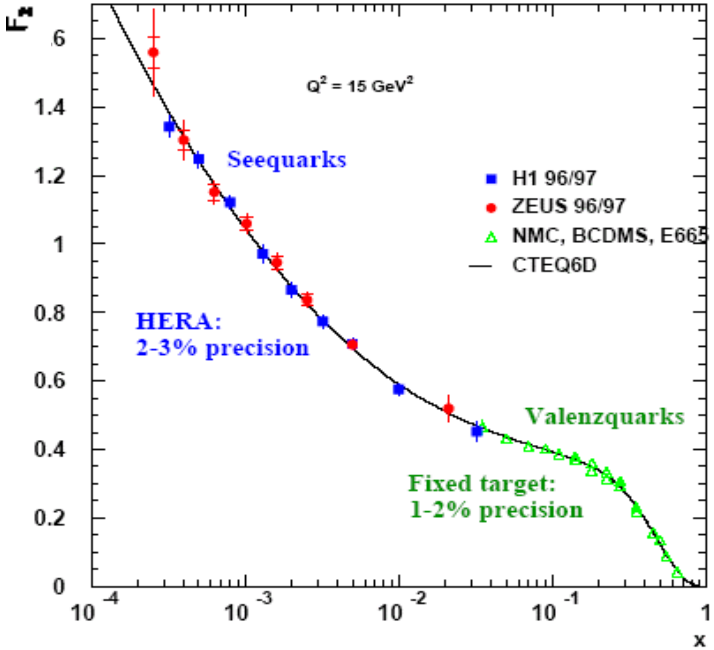
QCD – Lecture 6

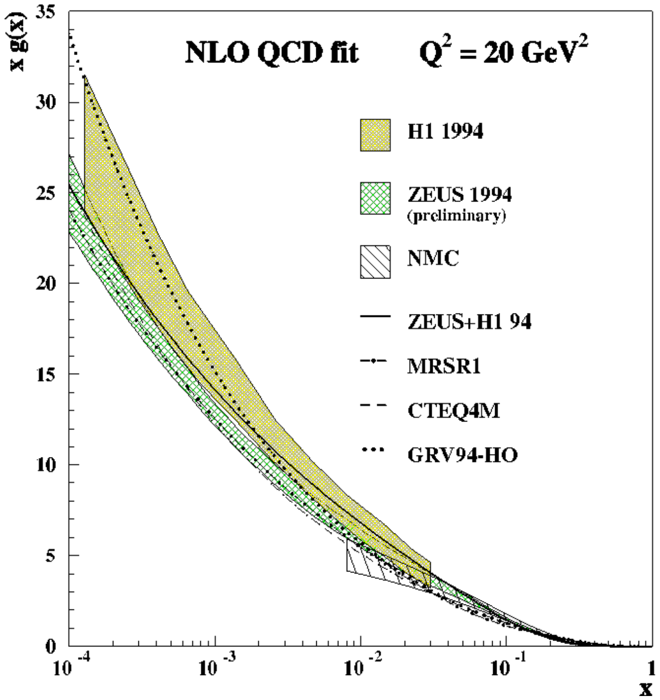
QCD at low x and low Q^2

Claire Gwenlan, Oxford, HT

the rise of the gluon at low x







Before the HERA measurements, most of the predictions for low x behavior of the structure functions and the gluon PDF were wrong

NOW it seems that the conventional NLO DGLAP formalism works TOO WELL!

(there **should be** ln(1/x) corrections and/or non-linear high density corrections for $x < 5\times10^{-3}$)

the rise of the gluon at low x from DGLAP

$$\frac{d\,g(x,Q^2)}{d\ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dy}{y}\left[P_{gq}\left(\frac{x}{y}\right)q(y,Q^2) + P_{gg}\left(\frac{x}{y}\right)g(y,Q^2)\right]$$

- at low x: x/y = z → 0 $P_{gq} \to \frac{2C_F}{z} = \frac{8}{3z}$, $P_{gg} \to \frac{2C_A}{z} = \frac{6}{z}$ (gluon splitting functions are singular)
- Pgg dominates so the equation becomes: $\frac{dg(x,Q^2)}{d\ln Q^2} \simeq \frac{\alpha_s(Q^2)}{2\pi}\int_x^1 \frac{dy}{y}\frac{6}{z}g(y,Q^2)$

  changing variables using: $t = \ln(Q^2/\Lambda^2)$ and with $\alpha_s(Q^2) = 1/(b_0 t) = \frac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$

  gives: $xg(x,Q^2) \simeq \exp\left\{\sqrt{\frac{12}{\pi b_0}\ln(\frac{t}{t_0})\ln(\frac{1}{x})}\right\}$ p234 – Devenish & Cooper-Sarkar

  over x,Q^2 range of HERA data, this solution mimics a power law behavior:

  $xg(x,Q^2) \sim x^{-\lambda_g}$ with $\lambda_g = \left(\frac{12}{\pi b_0}\frac{\ln(t/t_0)}{\ln(1/x)}\right)^{\frac{1}{2}}$

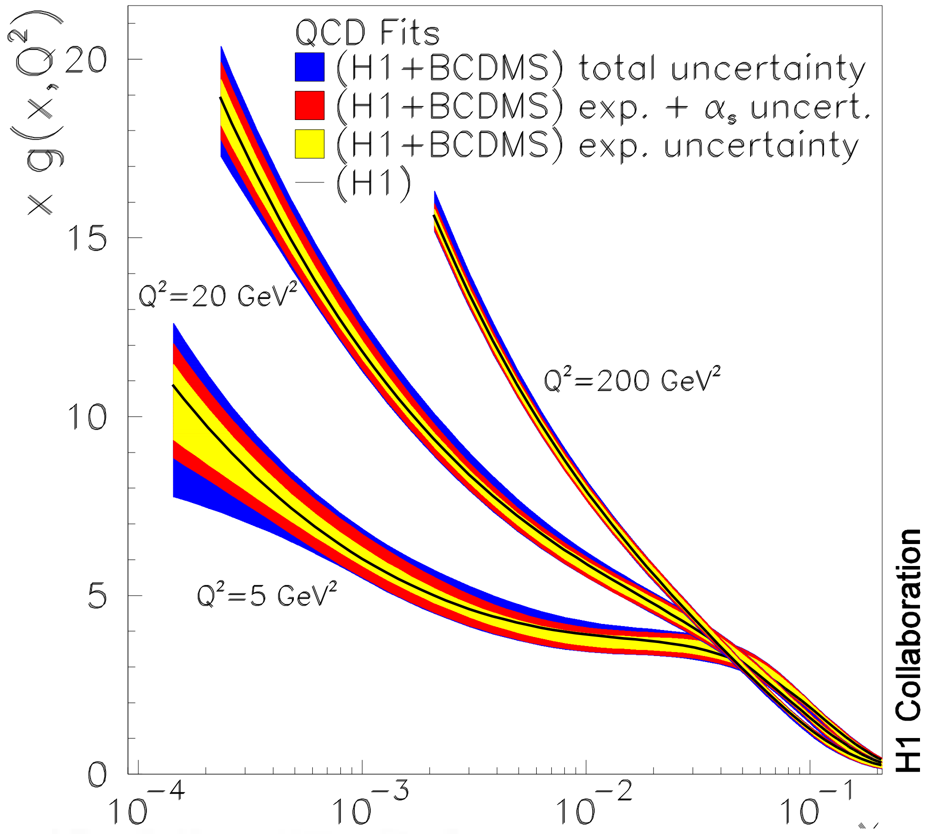
  slope of low x gluon gets steeper as Q^2 increases

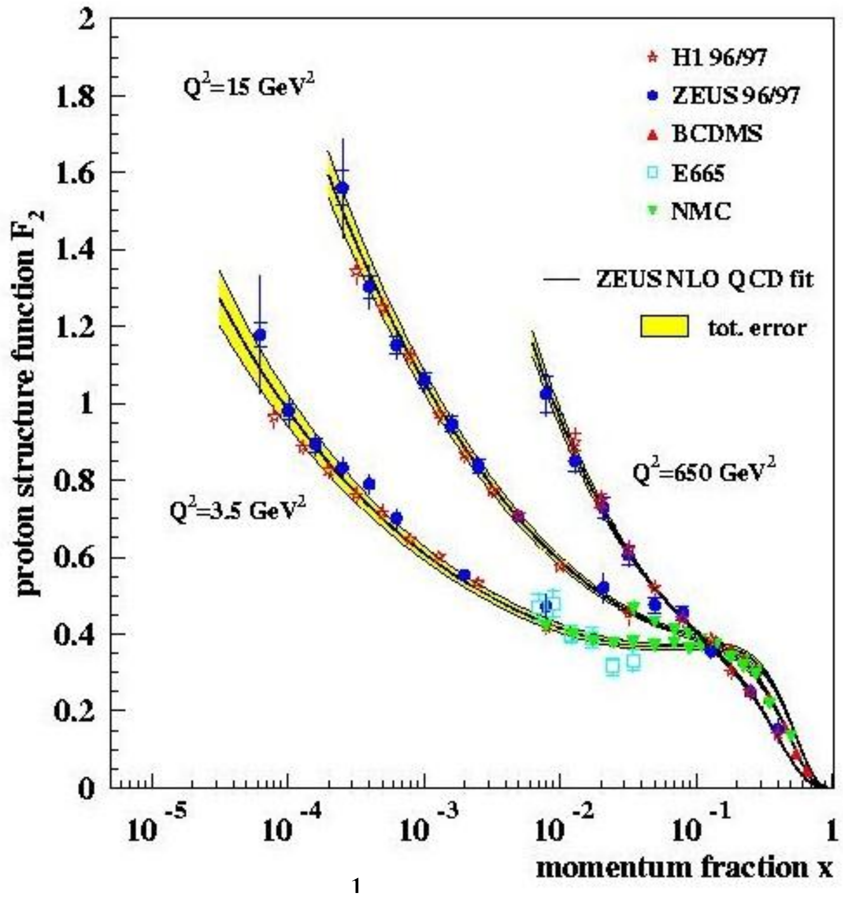
- also, at low x, evolution of F2 becomes gluon dominated, and generates a similarly steep behaviour

  $F_2 \sim x^{-\lambda}$, where $\lambda = \lambda_g - \epsilon$

EG. arXiv:0305165

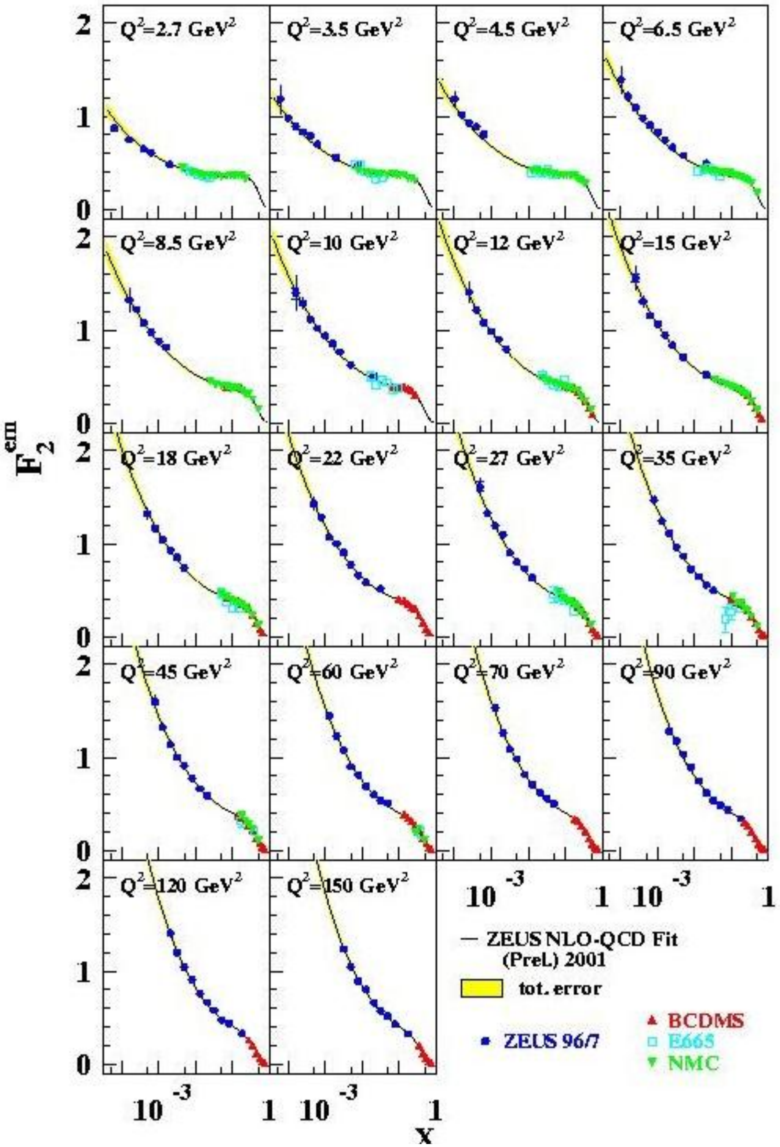
gluon at low x

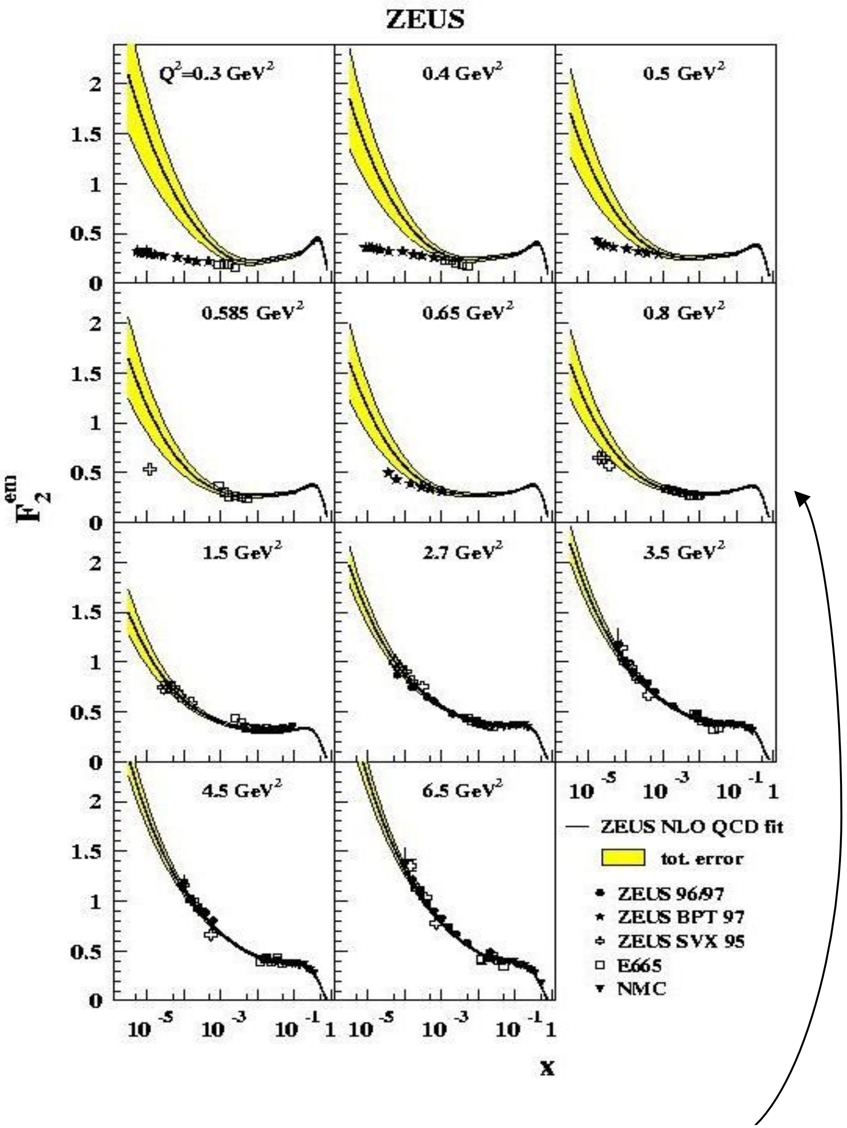




- a flat gluon at low Q^2 becomes very steep **AFTER** Q^2 evolution **AND** F2 becomes **gluon dominated**

$$\boxed{xg(x,Q^2) \sim x^{-\lambda_g}} \quad \text{with} \quad \lambda_g = \left(\frac{12}{\pi b_0}\frac{\ln(t/t_0)}{\ln(1/x)}\right)^{\frac{1}{2}}, \quad t = \ln(Q^2/\Lambda^2)$$





so it was a surprise to see **F_2 steep at small x** for low Q^2, down to Q^2 ~ 1 GeV^2

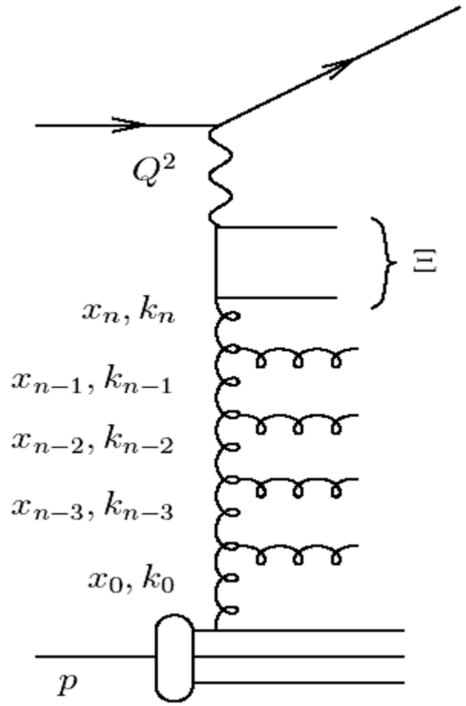
SHOULD perturbative QCD work? α_s is becoming large – α_s at Q^2~1 GeV^2 is ~ 0.4

beyond DGLAP: low x partons and BFKL

- **there is another reason why the application of conventional DGLAP at low x is questionable**

- can be shown that DGLAP equations effectively sum terms in $(\alpha_s \log Q^2)^n$
- diagrammatically, such terms arise from an **n-rung ladder** diagram, and assumes parton emissions are **strongly-ordered in transverse momenta**

$$Q^2 \gg k_{nT}^2 \gg ... \gg k_{1T}^2 \gg Q_0^2$$



- **HO corrections to splitting (and coefficient) functions also contain terms in log(1/x)**
- gives rise to contributions to PDFs of form

$$\alpha_s^P(Q^2)\left(\ln Q^2\right)^q\left(\ln\frac{1}{x}\right)^r$$

conventionally, in DGLAP:

LO: p = q ≥ r ≥ 0 LL(Q2)

NLO: p = q+1 ≥ r ≥ 0 NLL(Q2)

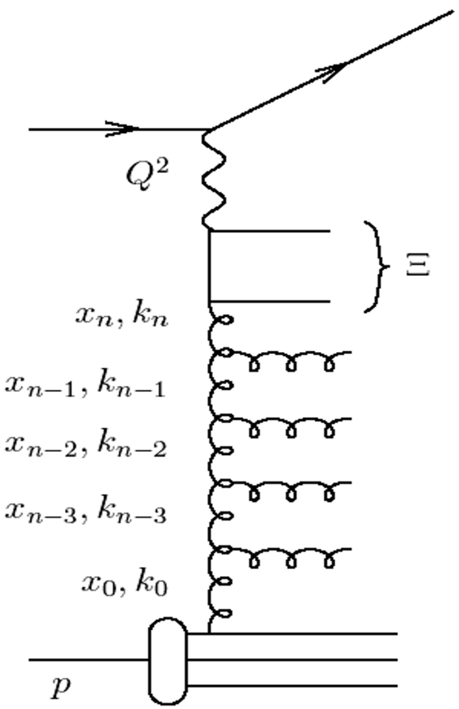
BUT if log(1/x) large, should also consider:

p = r ≥ q ≥ 1 LL(1/x)

p = r+1 ≥ q ≥ 1 NLL(1/x)

BFKL summation

Diagrammatically,



- **DGLAP:**
- Leading Log Approximation (LLA) in log(Q^2) → strong ordering in transverse momentum

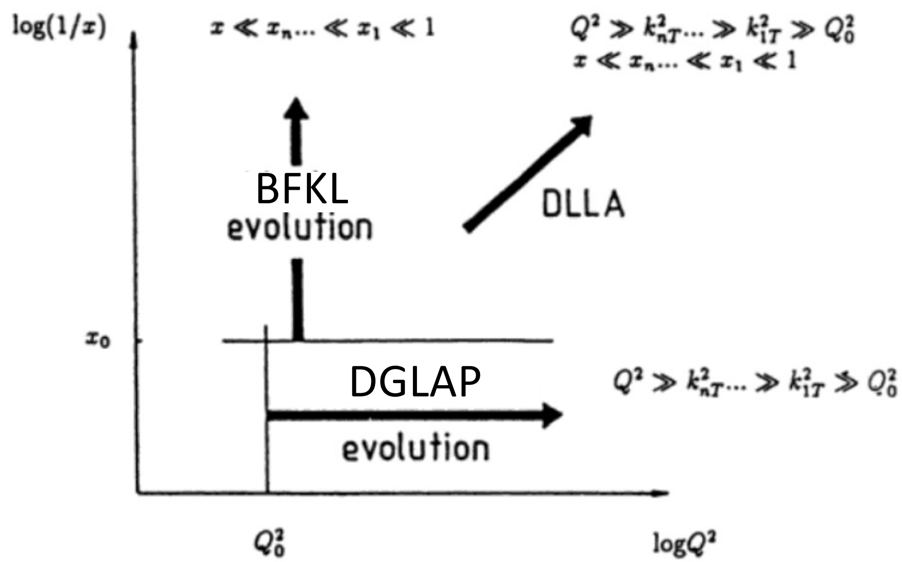
$$Q^2 \gg k_{nT}^2 \gg ... \gg k_{1T}^2 \gg Q_0^2$$

- and at small x, also have strong ordering in x

$$x \ll x_n \ll ... \ll x_1 \ll 1$$

Double Leading Log Approximation (DLLA) sums leading terms in log(1/x) provided they are coupled with leading log(Q^2) terms

- **BFKL (Balitskii, Fadin, Kuraev, Lipatov)**
- sums terms in $(\alpha_s \log 1/x)^n$ independent of log(Q^2)
- ordering in x but NOT in kT
- **predicts x but not Q^2 dependence**



BFKL

- **BFKL equation has structure:** $$\frac{d\mathcal{G}(x,k_T^2)}{d\log(1/x)} = \int dk_T'^2 K(k_T^2, k_T'^2)\mathcal{G}(x,k_T'^2) = \lambda\mathcal{G}$$
- where G is the gluon density unintegrated over kT $$xg(x,Q^2) = \int^{Q^2} \frac{dk_T^2}{k_T^2}\mathcal{G}(x,k_T^2)$$
- at small x, BFKL equation has the solution:

$$xg(x,Q^2) \sim e^{\lambda \log(1/x)} \sim x^{-\lambda} \sim \left(\frac{s}{s_0}\right)^{\lambda}$$

where $\lambda = \frac{3\alpha_s}{\pi} 4\log 2 \sim 0.5$ at $\alpha_s \approx 0.25$

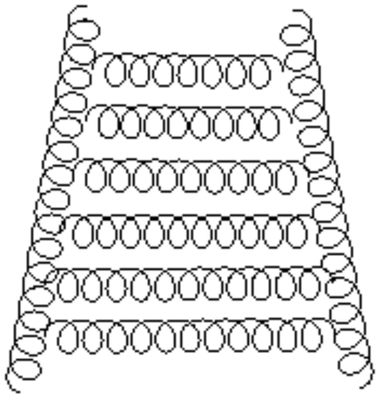
valid around $Q^2 \sim 4$ GeV^2

is the leading eigenvalue of the kernel K

- steeply rising gluon behaviour even at moderate Q^2
- **is this the reason for the steep behavior of F2 at low x?**
- NOTE that this has an analogous form to the Regge-pole exchange behavior of the amplitude (see Lecture 3)
- IS there a BFKL pomeron?

an aside …

- **BFKL were calculating gluon ladder diagrams to try to understand the flavourless Pomeron which dominates hadron-hadron cross sections**



i.e. they were trying to understand the ordinary Regge Pomeron now called the **SOFT** Pomeron

$s^{\alpha-1}$, $x^{1-\alpha}$, α=1.08

BUT their calculation yielded too large a value for α (α=1.5); this is now called the **HARD** Pomeron or **BFKL** Pomeron

- these calculations were rather naïve and NLO corrections suggest a smaller α
- however, DIS data at low x gave the first sign that maybe a **HARD Pomeron** does exist

non-linear effects and saturation

Furthermore, if the **gluon density becomes large**, there may be **NON-LINEAR** effects

gluon recombination gg → g

$$\sigma \sim \alpha_s^2 \, \rho^2 / Q^2$$

may compete with **gluon evolution g → gg**

$$\sigma \sim \alpha_s \, \rho$$

where ρ is the gluon density

EG. **GLR** (Gribov-Levin-Ryskin)

$$\frac{\partial^2 xg(x,Q^2)}{\partial \ln Q^2 \partial \ln(1/x)} = \frac{3\alpha_s}{\pi} xg(x,Q^2) - \frac{81\alpha_s^2}{16Q^2R^2}(xg(x,Q^2))^2$$

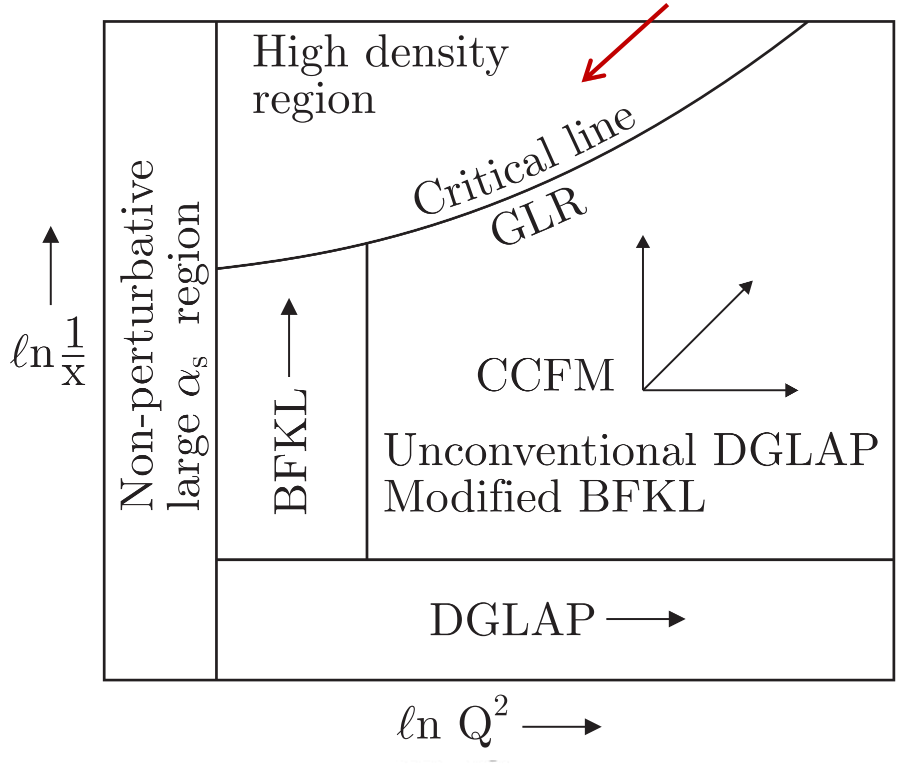
$\alpha_s \, \rho$ $\qquad$ $\alpha_s^2 \, \rho^2 / Q^2$

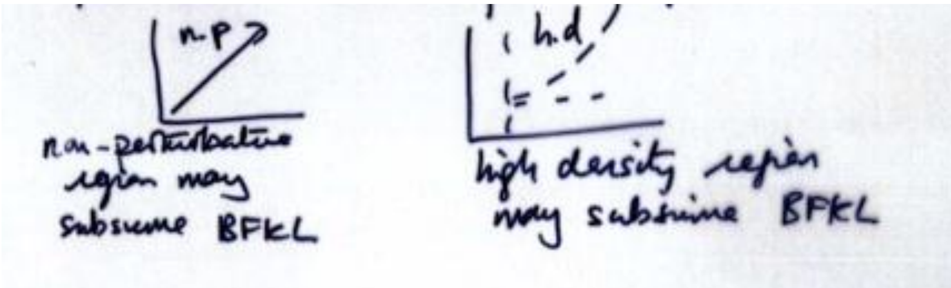
the **non-linear** term slows down the evolution of $xg(x,Q^2)$, which **tames the rise at small x**

the gluon density may even **SATURATE**

(respecting the Froissart bound, $\sigma_{tot} < const.(\ln s)^2$)

colour glass condensate, JIMWLK, BK





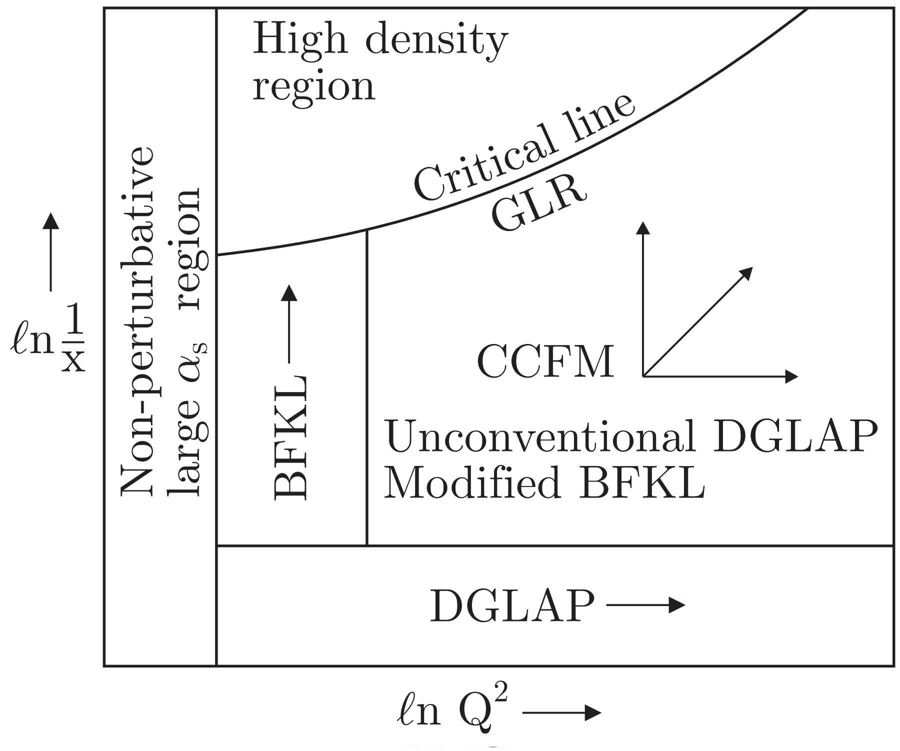
there is plenty of debate about positions of these lines!

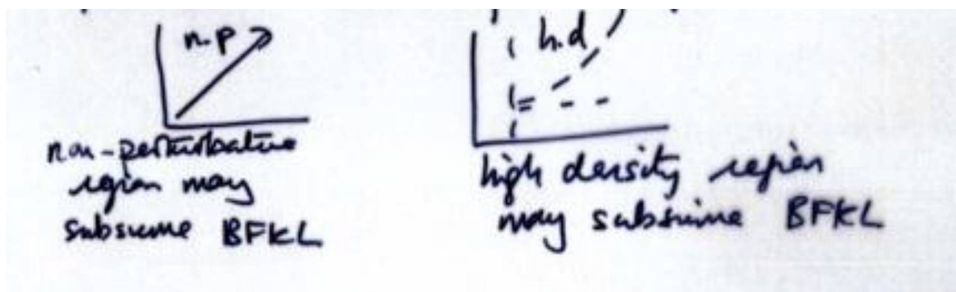
to summarise:

various reasons to worry that conventional LO and NLO log(Q^2) summations, as embodied in the **DGLAP** equations, **may be inadequate**

it was a surprise to see F2 steep at small x even for very, very low Q^2, Q^2 ~ 1 GeV^2

1. should pQCD work? αs is becoming large, EG. αs at Q^2~1 GeV^2 is ~ 0.4
2. there has not been enough lever arm in Q^2 for evolution, but even the starting distribution is steep – **the HUGE rise at low x makes us think:**
3. there **should** be log(1/x) resummation (BFKL) as well as traditional DGLAP resummation – BFKL predicts F2 ~ $x^{-\lambda s}$ with λs=0.5 even at low Q^2
4. and/or there should be non-linear/ high density corrections for x < 5×10^{-3}





what does the data say?

Does the data *need* unconventional explanations?

- *ln($^1/_x$)* terms in the splitting factors
- CCFM
- modified BFKL

Afficionados claim χ^2 improvements over conventional NLLA DGLAP..
But, one seems to be able to use DGLAP by absorbing unconventional behaviour in the boundary conditions i.e. the unknown shapes of the non-perturbative parton distributions at Q_0^2

We measure, $F_2 \sim x\,q$

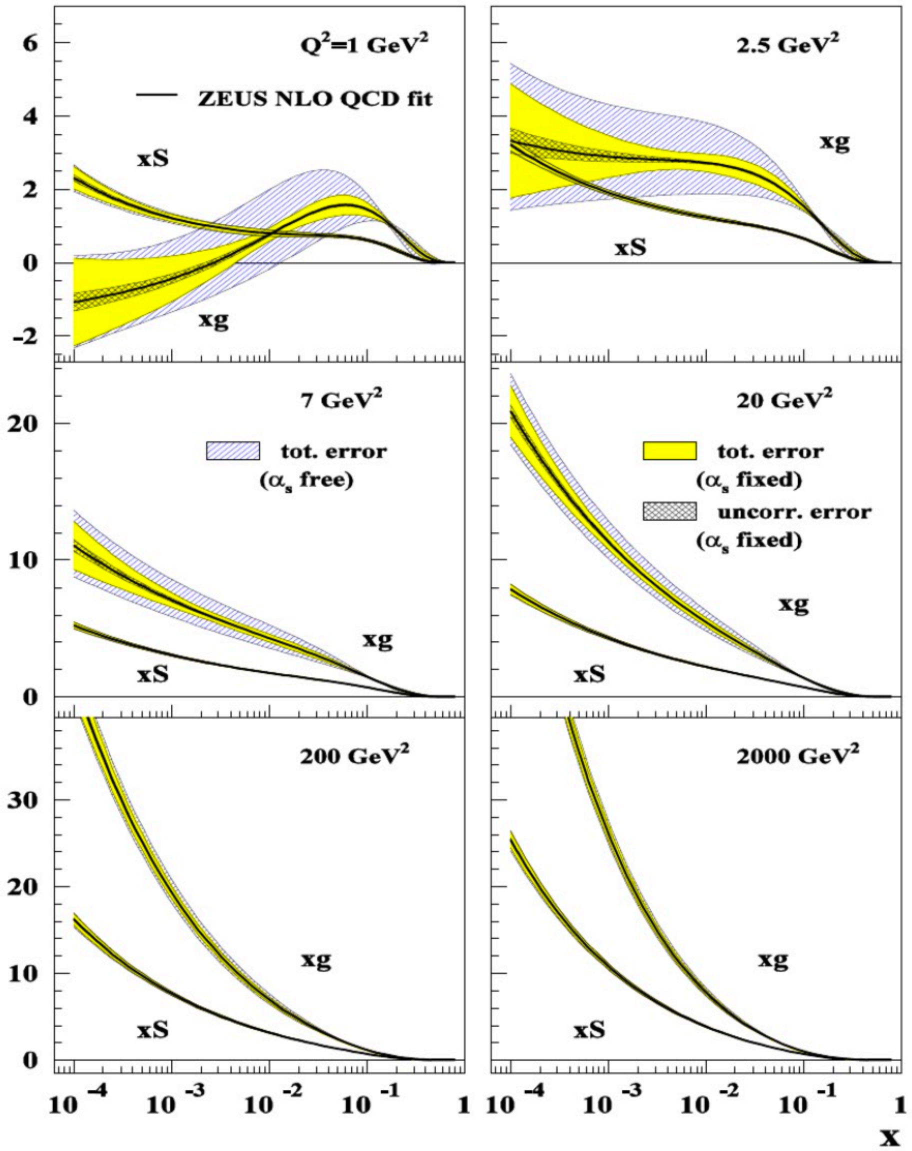
$$\frac{d\,F_2}{d\ln Q^2} \sim P_{qg} \cdot x\,g$$

we can explain unusually steep $\frac{d\,F_2}{d\ln Q^2}$ by:

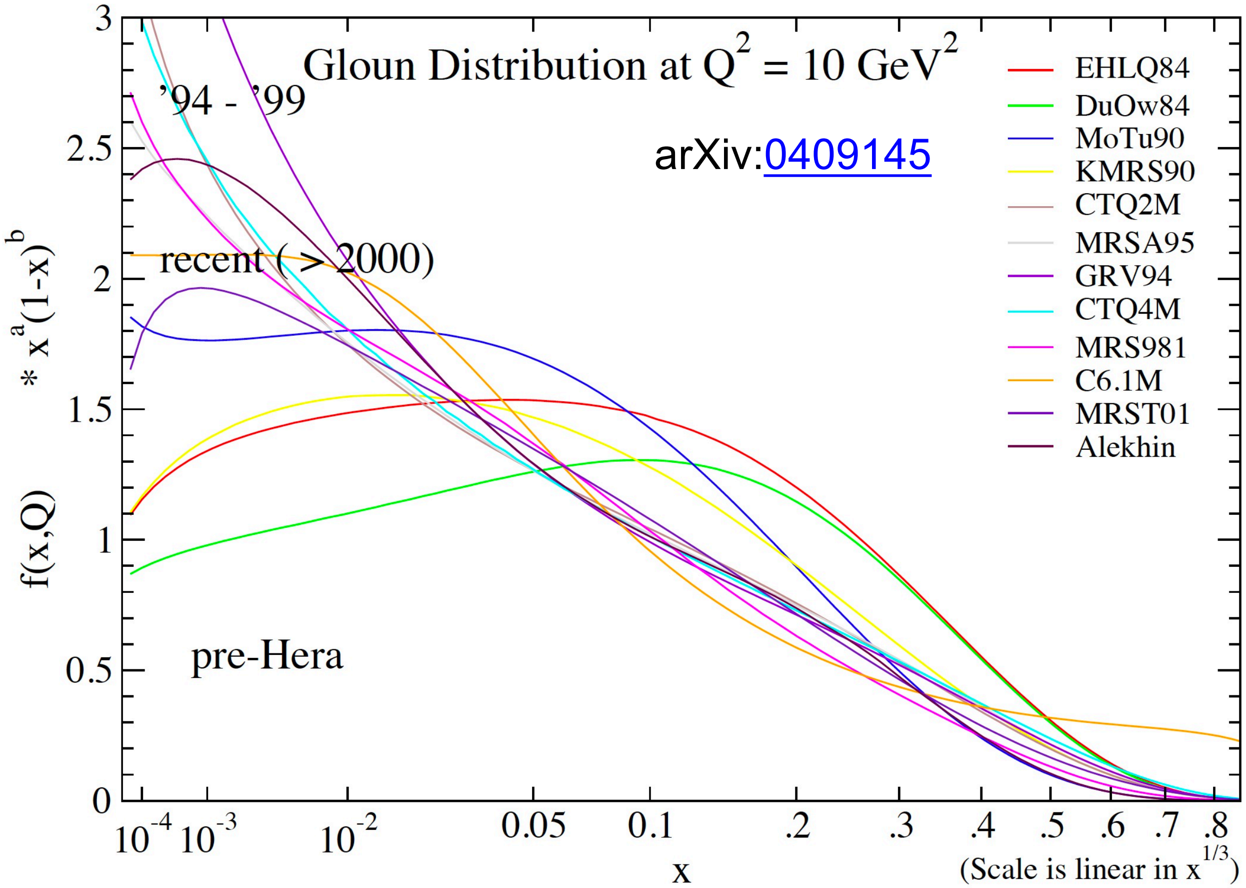
unusual P_{qg} → eg *ln($^1/_x$)*, BFKL
OR unusual $x\,g(x,Q_0^2)$ → "valence-like" gluon etc.

→ measure other gluon sensitive quantities at low x: F_L , $F^2_{c\bar{c}}$

Global Fit (ZEUS + fixed target)



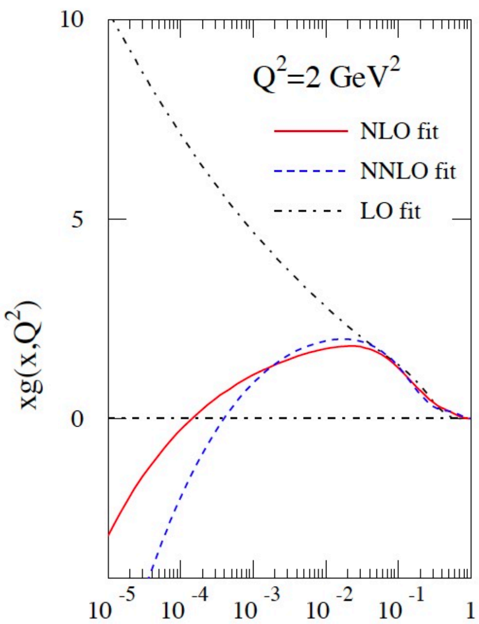
change of the gluon over time

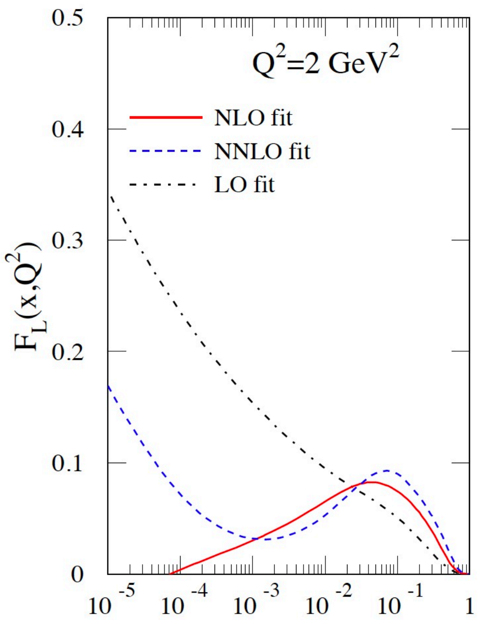


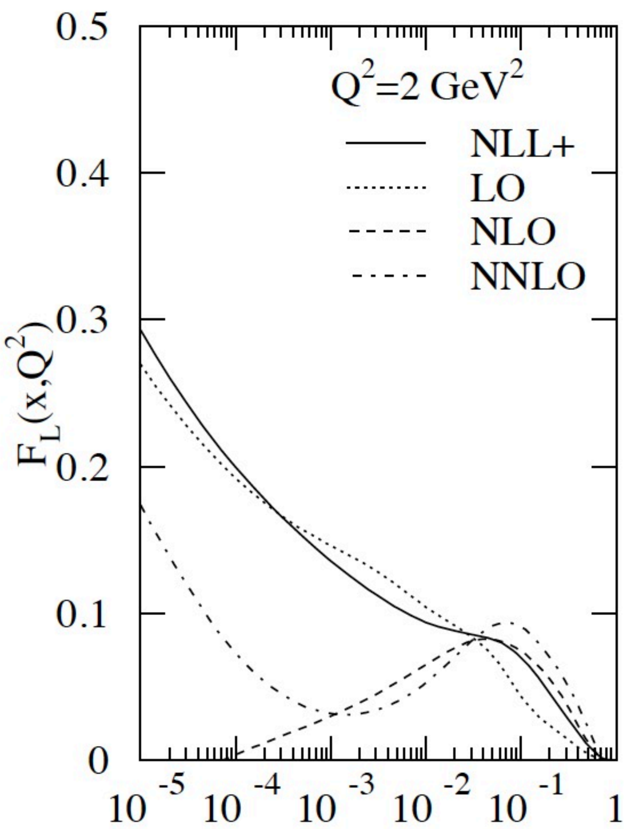
- in fact, when HERA low x data first published, gluon went from being flat to steep at low x
- BUT then when the HERA data proved to **still be steep even at very low Q^2**, DGLAP fits started to produce gluons which turn over again at low x

gluon evolves FAST – in order to evolve so fast upwards it must also evolve fast downwards

FL and the gluon



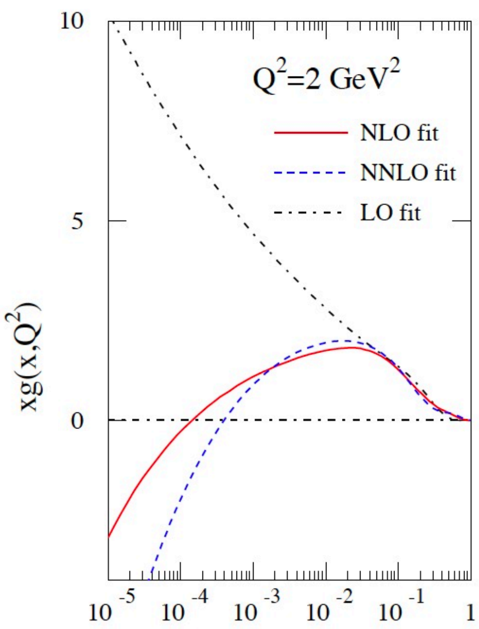


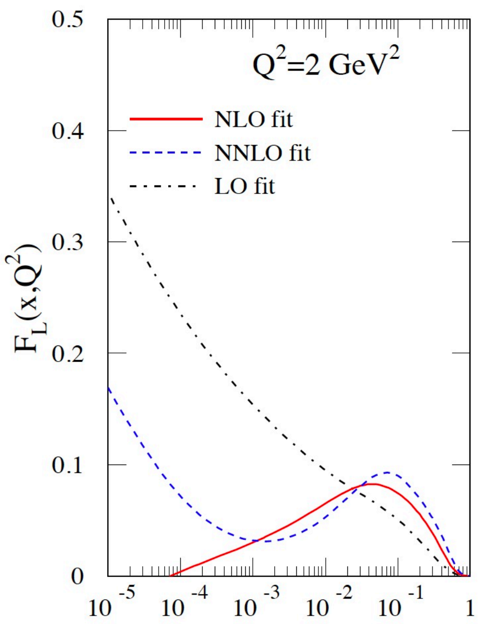


- negative gluon predicted at low x, low Q^2 from NLO DGLAP **remains at NNLO (worse)**
- corresponding FL not negative (at NNLO!) **but has peculiar shape**
- including **log(1/x) resummation in calculation** of splitting functions (BFKL inspired) improves shape, plus X^2 of global fit improves

no one found this VERY convincing until recently….
when log(1/x) BFKL resummation worked out in detail and applied to NNPDF fits, giving NNPDF3.1xs
arXiv:1710.05935

FL and the gluon





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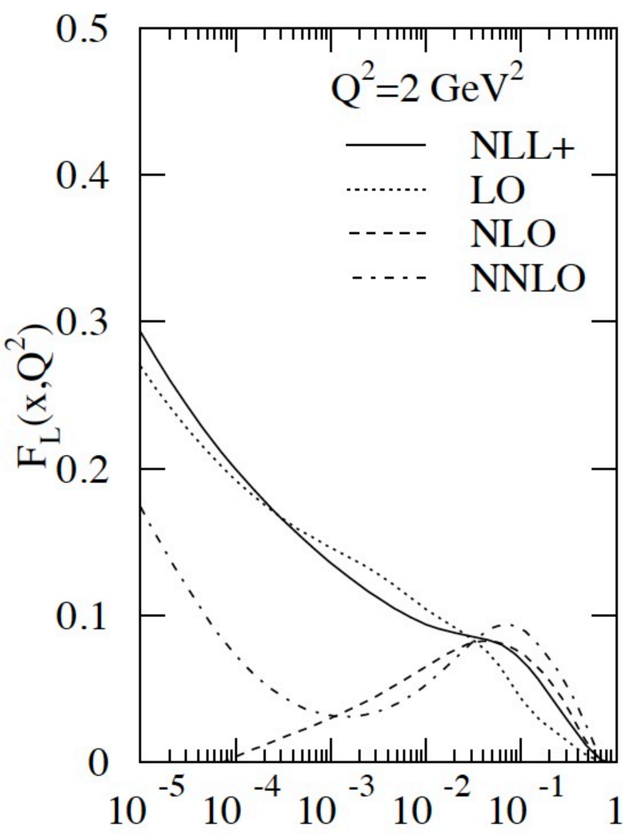


- including **log(1/x) resummation in calculation** of splitting functions (BFKL inspired) improves shape, plus X^2 of global fit improves

arXiv:1710.05935 – why not sooner? a) it is a difficult calculation – program is called HELL (High Energy Leading Log resummation); and b) measurements not precise enough until final HERA combination, arXiv:1506.06042

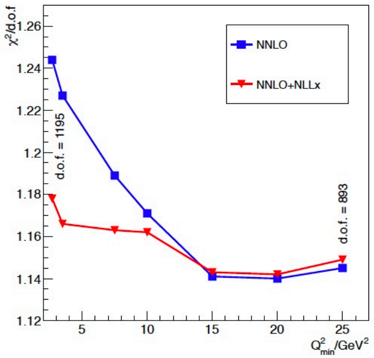
impact of log(1/x) resummation

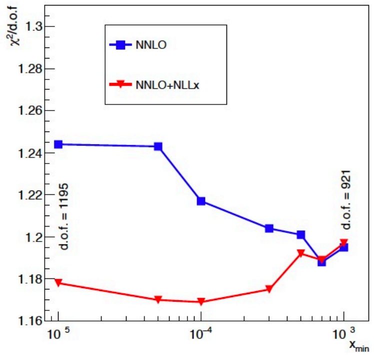
consequences of this on HERAPDF fit:

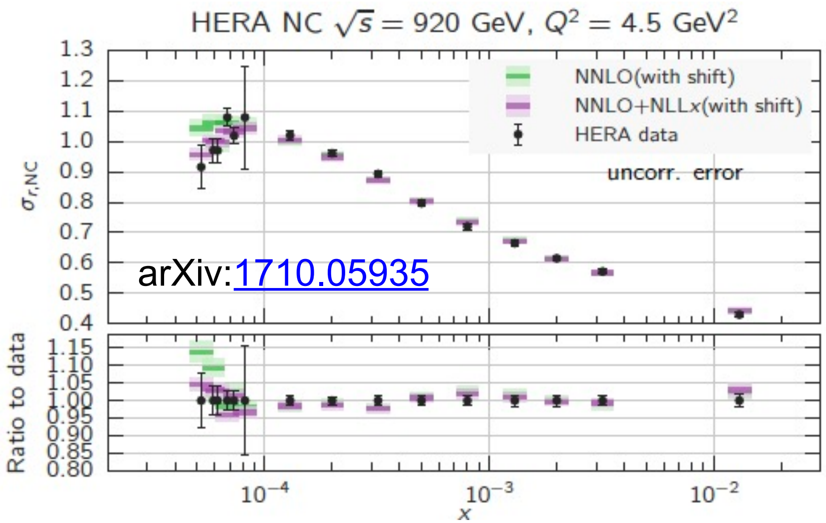
arXiv:1802.00064

1. X^2 VASTLY improved – not just a bit →
2. improvement comes at low x and low Q^2

	NNLO fit with new settings	NNLO +NLLx fit with new settings
Total χ^2/d.o.f	1446/1178	1373/1178
subset NC 920 χ^2/n.d.p	446/377	413/377
subset NC 820 χ^2/n.d.p	70/70	65/70
subset charm χ^2/n.d.p	48/47	49/47
correlated shifts inclusive	102	77
correlated shifts charm	15	11
log term inclusive	20	-3
log term charm	-2	-1







3. … and affects the high-y/low x turnover of the cross section, y=Q2/(s.x), which fits much better because FL predicted to be larger

$$\sigma_{\rm red} = F_2 - \frac{y^2}{Y_+} F_L$$

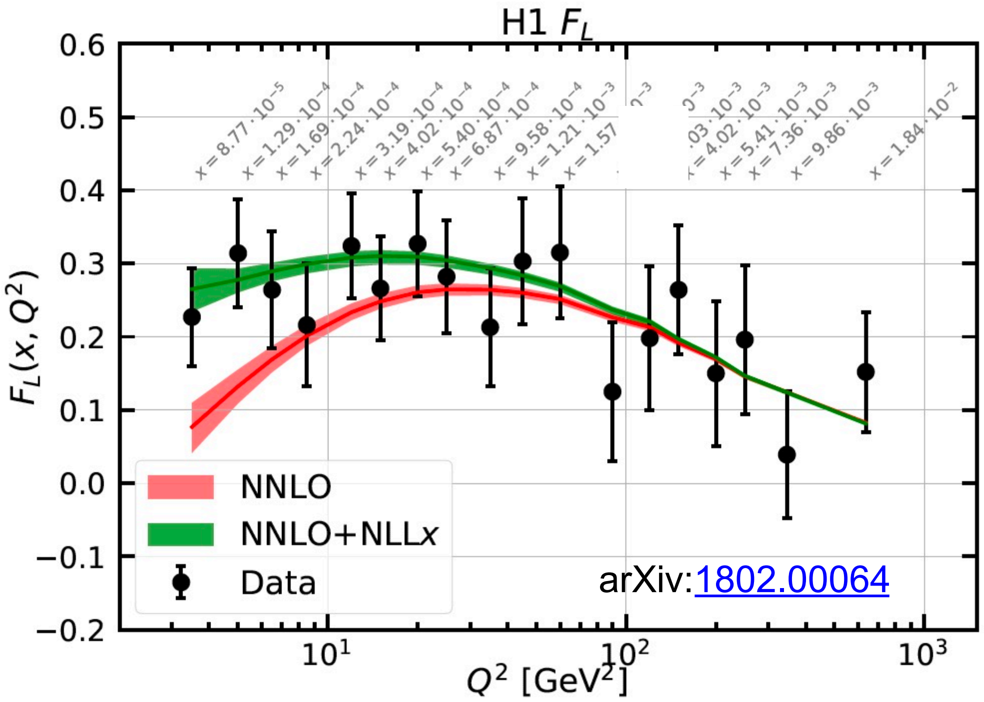
4. FL gluon dominated and gluon now has more reasonable shape…

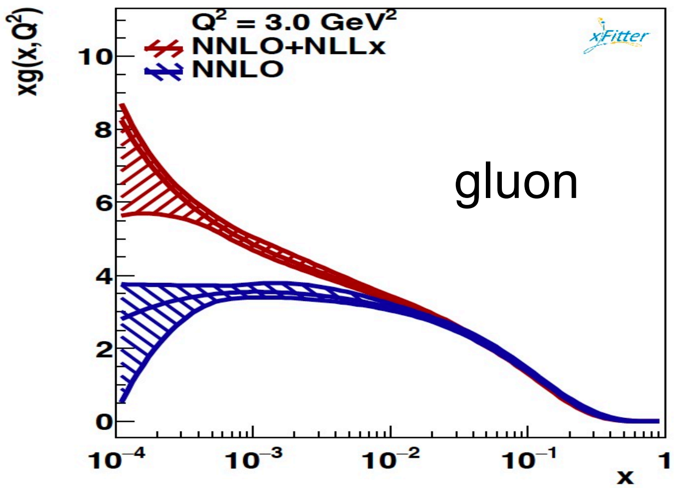
impact of log(1/x) resummation

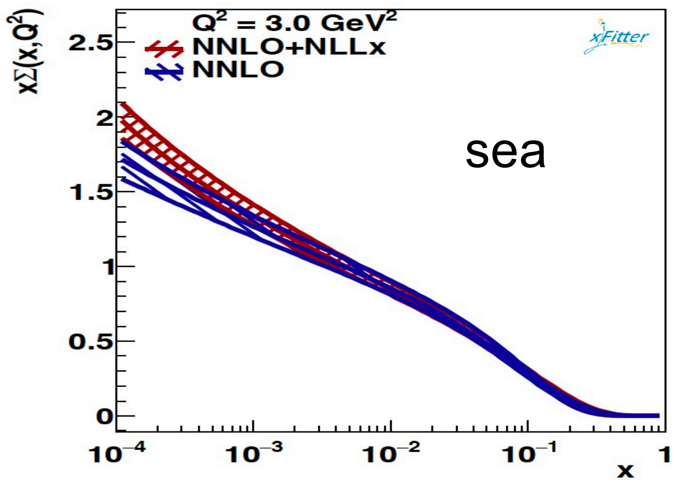
in NLO DGLAP, F_L given by:

$$F_L(x,Q^2) = \frac{\alpha_S}{4\pi} x^2 \int_x^1 \frac{dz}{z^3} \cdot \left[\frac{16}{3} F_2(z,Q^2) + 8 \sum e_q^2 \left(1 - \frac{x}{z}\right) zg(z,Q^2)\right]$$

and at low x this becomes gluon dominated



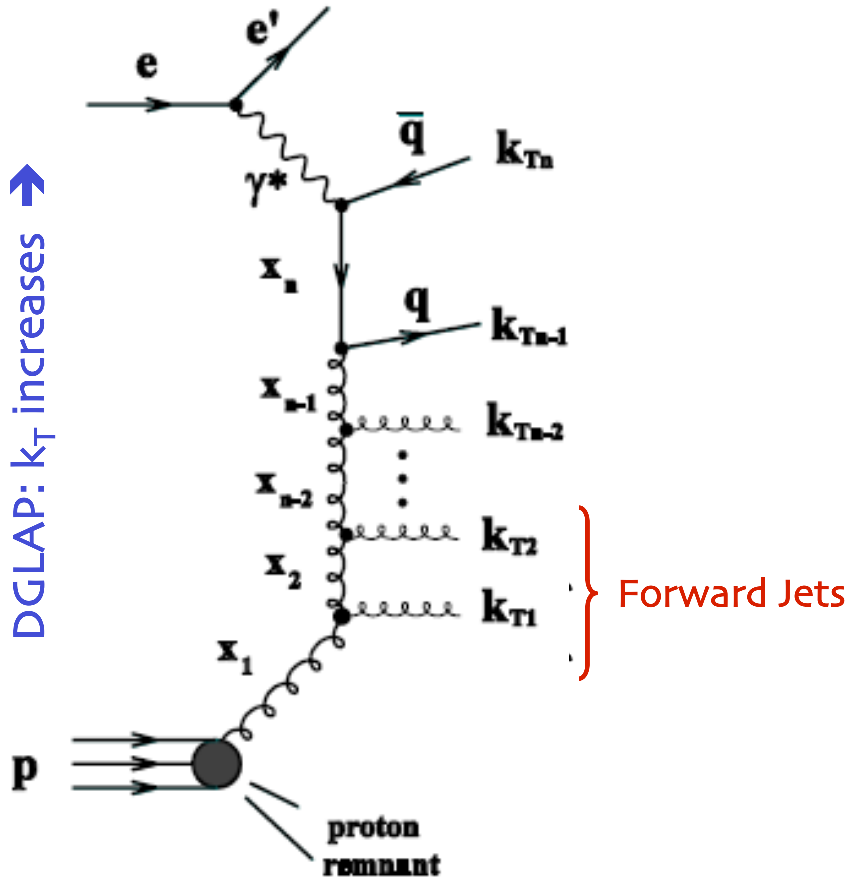




- measured FL much better described when NLLx = next-to-leading-log (1/x) resummation applied
- gluon shape, and its relationship to shape of sea, now much more reasonable

other methods to search beyond DGLAP?

measurements in dedicated final states where DGLAP might be insufficient to describe parton dynamics



EG. Forward jet measurements at HERA

DGLAP evolution strongly ordered in k_T

$k_{T,1}{}^2 \ll k_{T,2}{}^2 \ll \ldots \ll Q^2$

→ **LOW** probability for forward jets with $E_T^{jet} \sim Q^2$

BUT this is not so for k_T unordered BFKL evolution

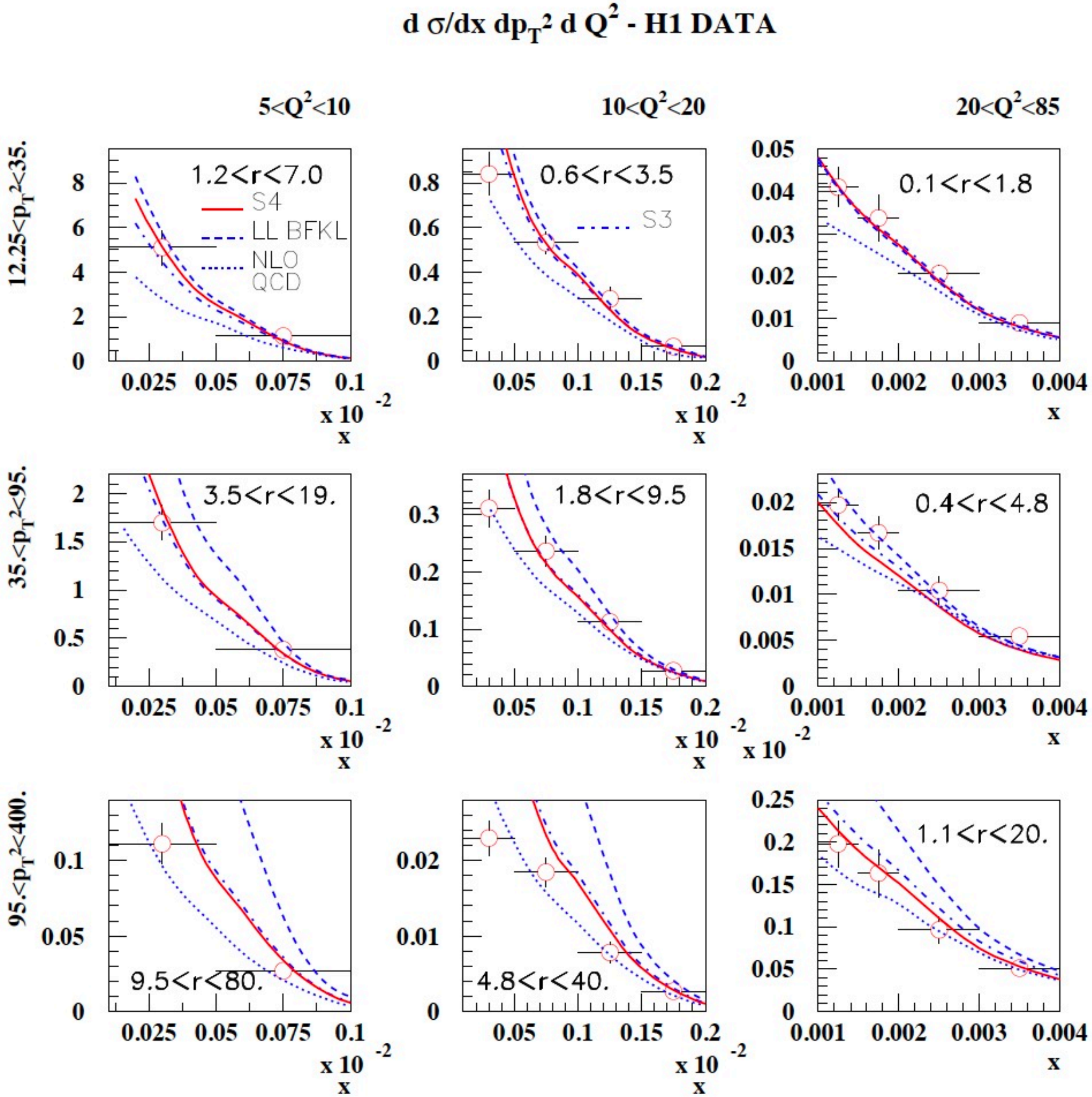
measurements have often served to instead highlight that conventional jet calculations were not very well developed E.G. are discrepancies due to missing HOs in DGLAP or BFKL effects? – **there has been much recent progress in this regard**

forward jets at HERA

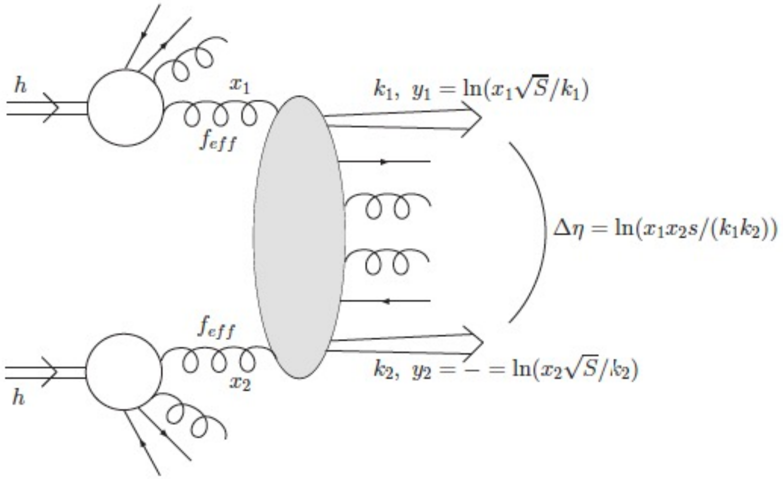
arXiv:0508055

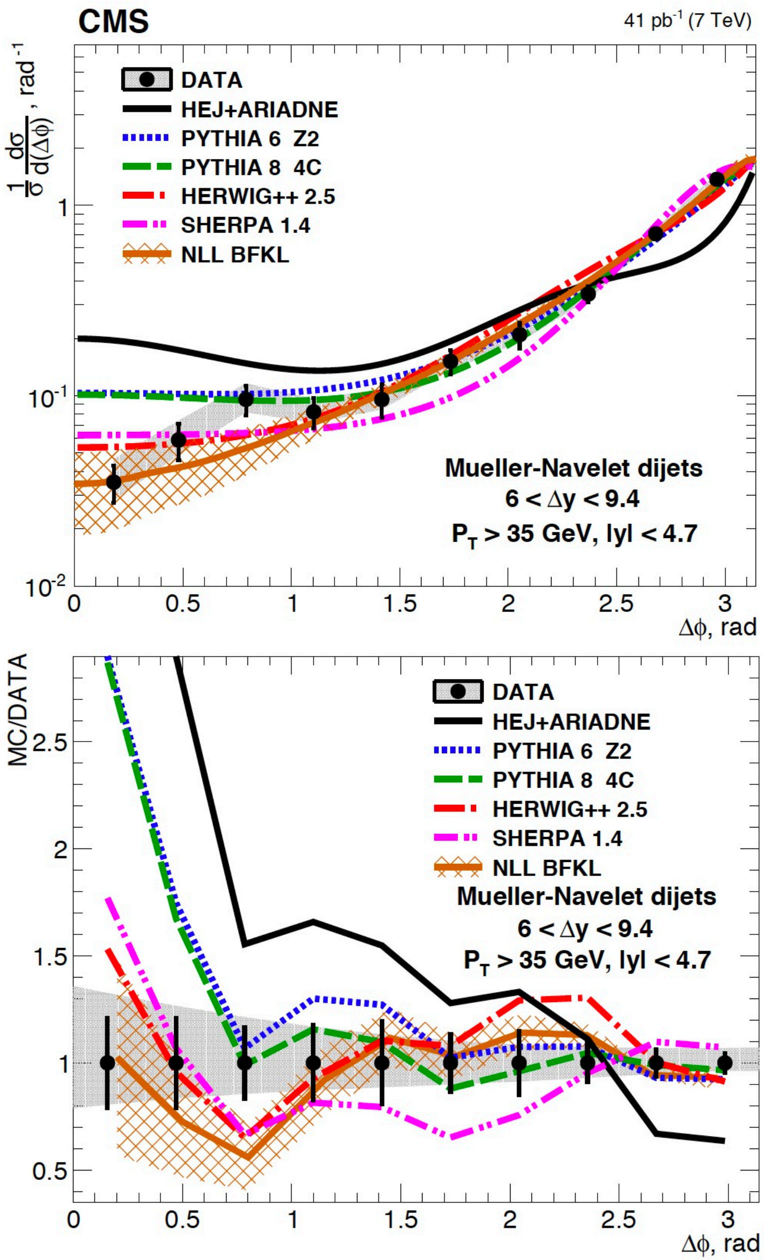
arXiv:0612261

$r = p_T^2/Q^2$



Mueller-Navelet jets at the LHC





same kind of process at the LHC

NLL BFKL = analytical calculation at parton level

HEJ = LL BFKL inspired (ARIADNE for parton shower)

PYTHIA6, **PYTHIA8**, **HERWIG++** = LO DGLAP $2 \rightarrow 2$ + LL parton shower

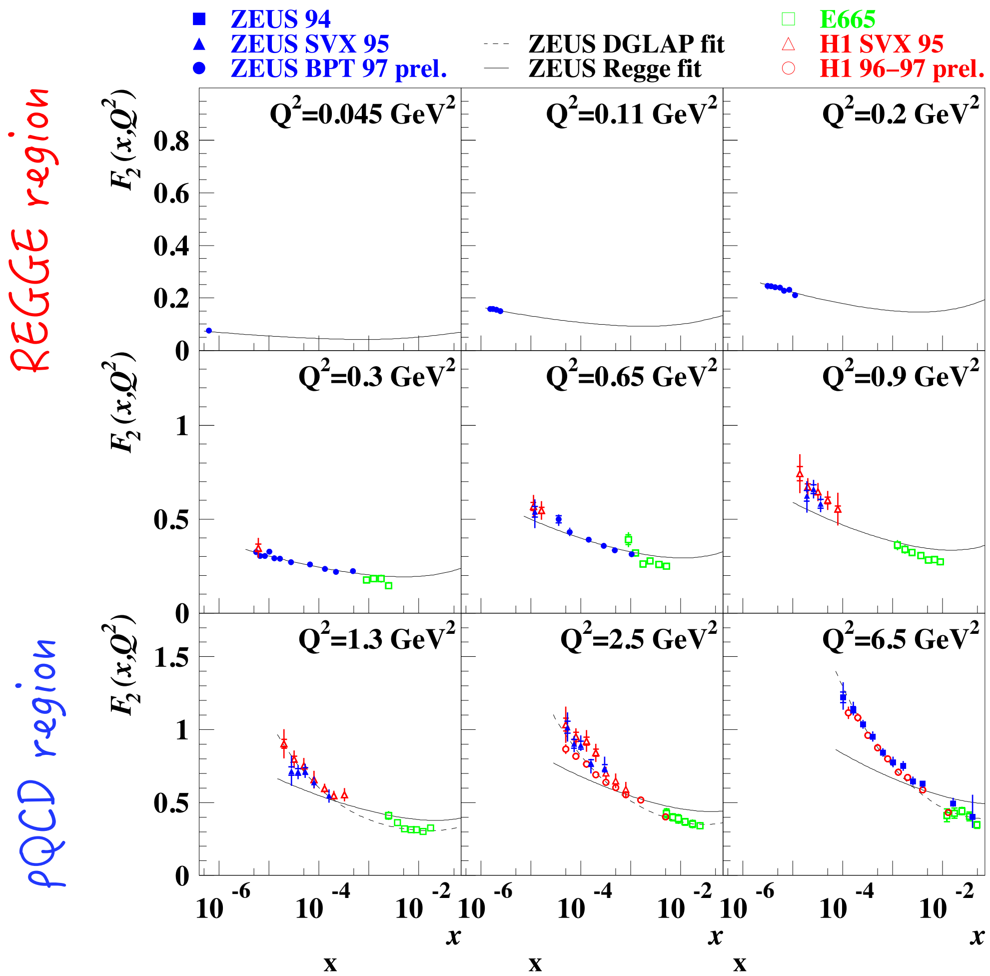
SHERPA= LO DGLAP $2 \rightarrow 2$+Njets + LL parton shower

arXiv:1601.06713

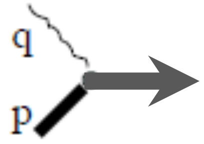
what about the very low Q^2 region ?

LINEAR **DGLAP** evolution doesn't work for $Q^2 < 1$ GeV^2

WHAT does? – REGGE ideas?



small x is high W², $x = Q^2/(2p\cdot q) \sim Q^2/W^2$



$(p+q)^2 = Mx^2 = W^2$

$\sigma(\gamma^* p) \sim (W^2)^{\alpha-1}$ ← Regge prediction for high energy cross sections

α is the intercept of the Regge trajectory

α=1.08 for the SOFT POMERON

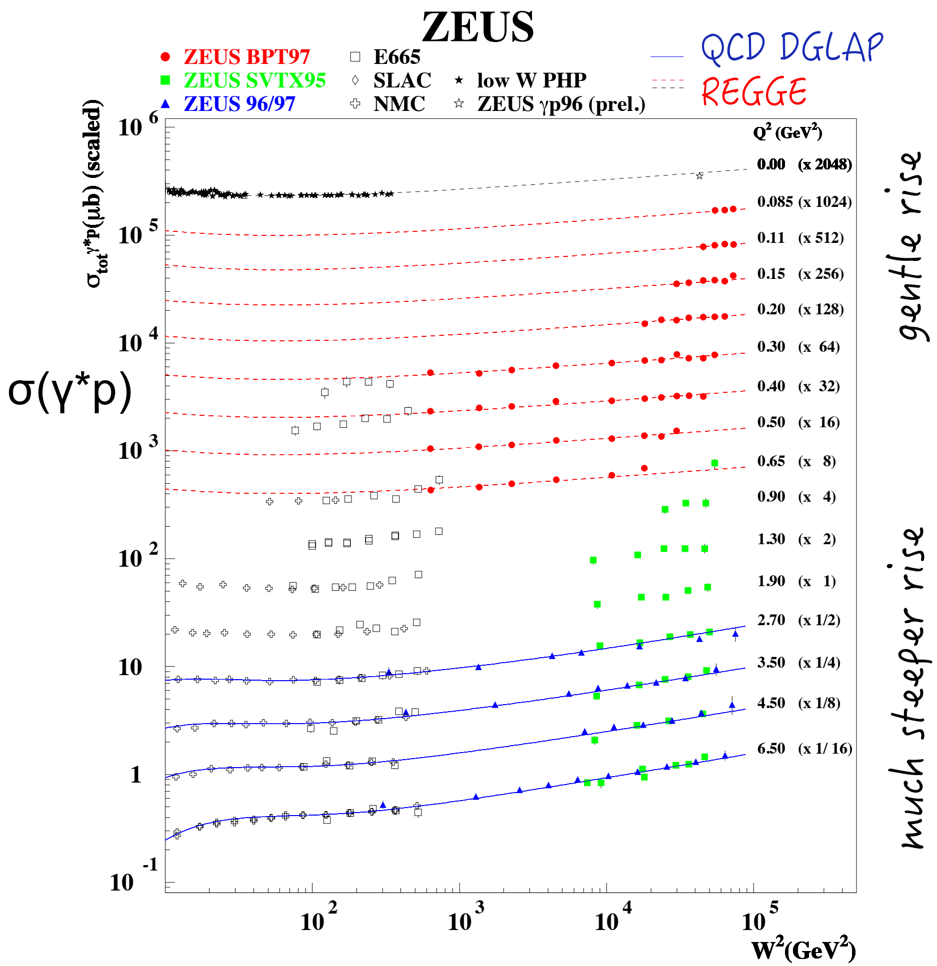
such energy dependence is well established from the SLOW RISE of all hadron-hadron cross sections – including $\sigma(\gamma p) \sim (W^2)^{0.08}$ – for real photon-proton scattering

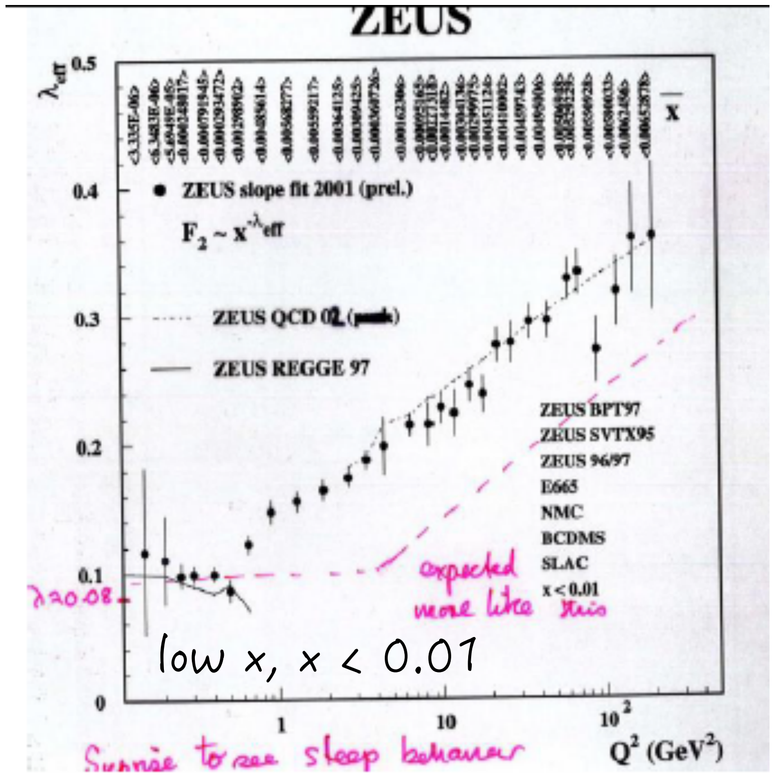
for virtual photons, at small x $\sigma_{tot}^{\gamma^* p} = \frac{4\pi^2\alpha}{Q^2} F_2$

$\sigma \sim (W^2)^{\alpha-1} \rightarrow F_2 \sim x^{1-\alpha} = x^{-\lambda}$

so a SOFT POMERON would imply **λ**=0.08; gives only a very gentle rise of F2 at small x

for $Q^2 > 1$ GeV^2 we have observed a much stronger rise …





the slope of F2 at small x, $F_2 \sim x^{-\lambda}$

is equivalent to a rise of $\sigma(\gamma^* p) \sim (W^2)^{\lambda}$

which is only gentle for $Q^2 < 1$ GeV^2

$$F_2 \sim x^{-\lambda(Q^2)}, \quad \lambda_{\rm eff}(Q^2) = \frac{d \ln F_2}{d \ln(1/x)}$$

so is there a **HARD POMERON** corresponding to this steep rise?

QCD POMERON, $\alpha(Q^2) - 1 = \lambda(Q^2)$

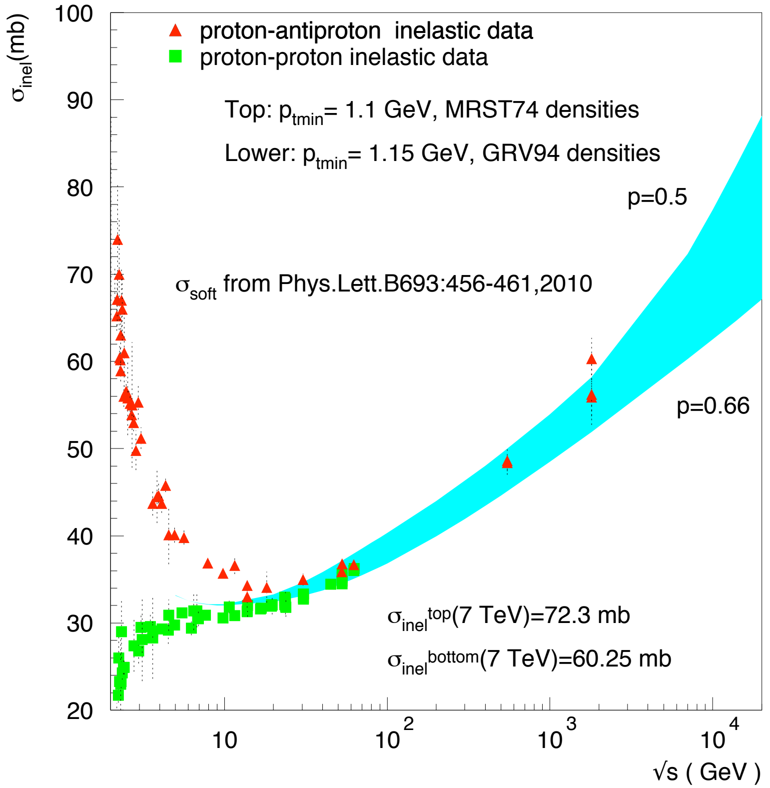
BFKL POMERON, $\alpha - 1 = \lambda = 0.5$

mixture of HARD and SOFT Pomeron to explain the transition $Q^2=0$ to high Q^2 ?

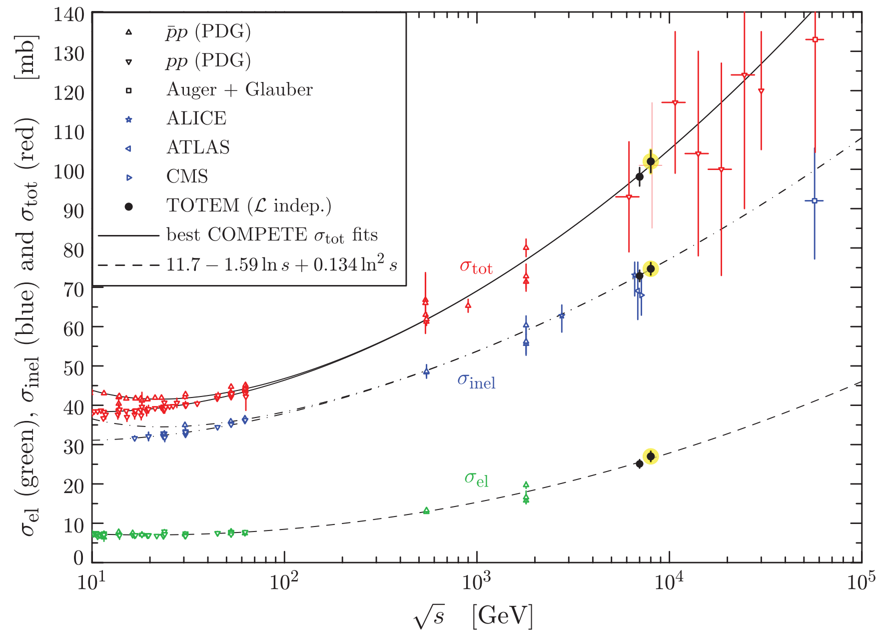
Do we understand the rise of hadron-hadron cross sections at all?

Could there always have been a HARD POMERON – is this why the effective Pomeron intercept is 1.08 rather than 1.00?

Does the HARD Pomeron mix in more strongly at higher energies? What about the LHC?



pre-ATLAS prediction with uncertainty from assumptions on mixing in of HARD Pomeron



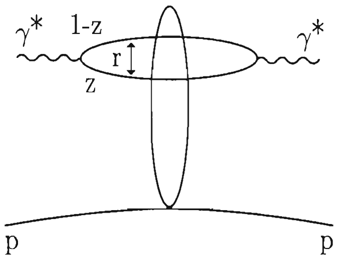
if anything TOTEM results look even steeper

what about the Froissart bound?

colour dipole models

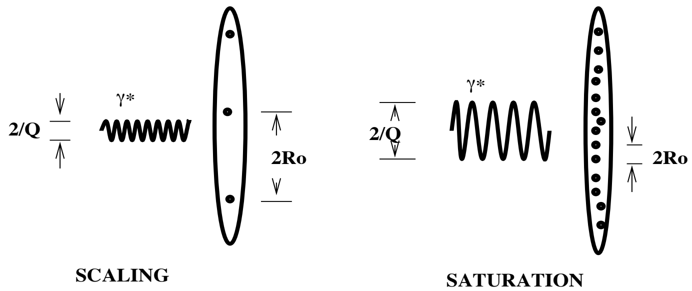
DIPOLE MODELS provide another way to model the transition $Q^2=0$ to high Q^2

at low x, γ* → qqbar; the long lived (qqbar) dipole scatters from the proton



EG. GBW (Golec-Biernat & Wusthoff), arXiv:9807513

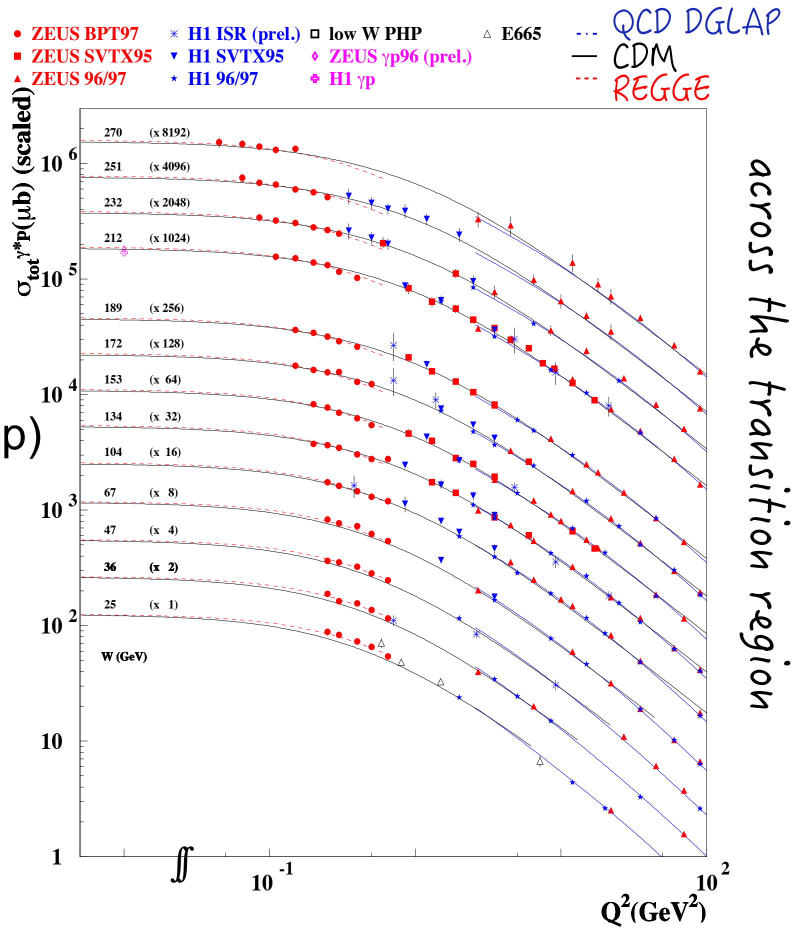
dipole-proton cross section depends on relative size of the dipole **r ~ 1/Q c.f.** separation of gluons in target **R_0**



$$\sigma = \sigma_0 \left(1 - \exp\left(\frac{-r^2}{4R_0^2(x)}\right)\right), \quad R_0^2(x) = \frac{1}{Q_0^2}\left(\frac{x}{x_0}\right)^{\lambda} \sim \frac{1}{xg(x)}$$

r/R_0 SMALL $\rightarrow$ large Q^2, x $\sigma \sim r^2 \sim 1/Q^2$	r/R_0 LARGE $\rightarrow$ small Q^2, x $\sigma \sim \sigma_0$ – SATURATION

σ(γ*p)



NOW there are HERA data right across the transition region

$\sigma_{tot}^{\gamma^* p} = \frac{4\pi^2\alpha}{Q^2} F_2$ is general (for **small x**)

σ(γp) finite for real γ (F2 → 0 as Q^2 → 0)

At high Q^2, σ(γ*p) ~ $1/Q^2$ ⇒ F2 ~ flat

BJORKEN SCALING

extras

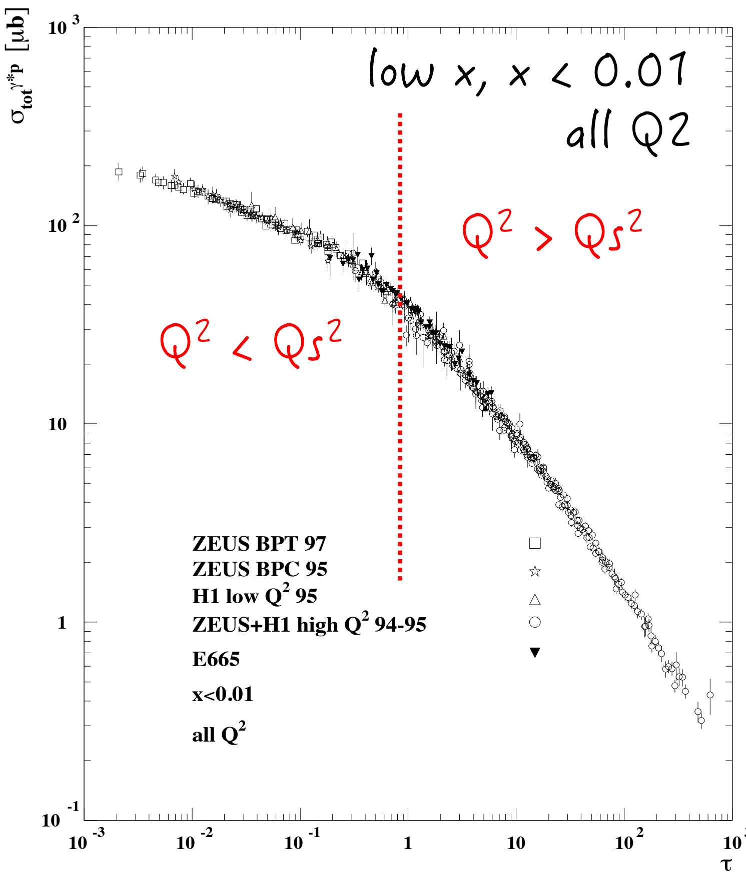
geometric scaling for total σγ*p at low x

GBW – write: $\sigma = \sigma_0 \left(1 - \exp(-1/\tau)\right)$

which involves only

$$\tau \sim Q^2 R_0^2(x) \sim \frac{Q^2}{Q_0^2}\left(\frac{x}{x_0}\right)^{\lambda}$$

INDEED, **for small x**, x < 0.01, σ(γ*p) depends only on τ, not on x,Q^2 separately →
(**NOT true at high x**)



τ is a new scaling variable, **applicable at small x**

can be used to define a **saturation scale**:

$$Q_s^2 \sim 1/R_0^2(x) \sim x^{-\lambda} \sim xg(x)$$

such that saturation extends to higher Q^2 as x decreases

EG. arXiv:0007192 , arXiv:0109010

some understanding of this scaling, of saturation and of dipole models is coming from work on non-linear evolution equations applicable at high density – Colour Glass Condensate; JIMWLK; BK

can be significant consequences for high energy cross sections EG. neutrino cross sections – also predictions for heavy ions-RHIC, diffractive interactions – Tevatron, HERA and the LHC – even some understanding of hadronic physics