Introduction to QCD and Deep Inelastic Scattering: Problems
Hilary Term

1. The Callan-Gross equation $2xF_1 = F_2$, or $F_L = 0$, is applicable for virtual photon scattering off spin-1/2 quarks. What would we predict for spin -0 quarks?

2. $$\frac{d^2\sigma(\nu h)}{dxdy} = \frac{G_F^2 s}{\pi}\Sigma_i \left[xq_i(x) + (1-y)^2 x\bar{q}_i(x)\right] , \tag{1}$$

   and

   $$\frac{d^2\sigma(\bar{\nu} h)}{dxdy} = \frac{G_F^2 s}{\pi}\Sigma_i \left[(1-y)^2 xq_i(x) + x\bar{q}_i(x)\right] . \tag{2}$$

   Use equations (1), (2) to calculate the total cross-sections $\sigma(\nu N)$ and $\sigma(\bar{\nu} N)$ in terms of the integrals

   $$\int_0^1 dx\, xq(x) = Q \quad \text{and} \quad \int_0^1 dx\, x\bar{q}(x) = \bar{Q}.$$

   Hence prove that

   $$\frac{\bar{Q}}{Q} = \frac{3-R}{3R-1}, \quad \text{where} \quad R = \frac{\sigma(\nu N)}{\sigma(\bar{\nu} N}.$$

3. For (anti-)neutrino neutral current (NC) scattering

   $$F_2^{\nu,\bar{\nu}}(x,y) = \Sigma_i x(q_i(x) + \bar{q}_i(x))(v_i^2 + a_i^2) , \tag{3}$$

   $$xF_3^{\nu,\bar{\nu}}(x,y) = \Sigma_i x(q_i(x) - \bar{q}_i(x)) 2v_i a_i , \tag{4}$$

   whereas for (anti-)neutrino charge current (CC) scattering

   $$F_2^{\nu,\bar{\nu}}(x,y) = 2\Sigma_i x(q_i(x) + \bar{q}_i(x)) , \tag{5}$$

   $$xF_3^{\nu,\bar{\nu}}(x,y) = 2\Sigma_i x(q_i(x) - \bar{q}_i(x)) . \tag{6}$$

   Use Eqns. (3)–(6) to establish Eqn. (7)

   $$\frac{\sigma_{NC}(\nu N) - \sigma_{NC}(\bar{\nu} N)}{\sigma_{CC}(\nu N) - \sigma_{CC}(\bar{\nu} N)} = \frac{1}{2} - \sin^2\theta_W . \tag{7}$$

4. $$\begin{aligned} F_2^{\nu p} &= 2x(d(x) + s(x) + \bar{u}(x) + \bar{c}(x)) , \\ xF_3^{\nu p} &= 2x(d(x) + s(x) - \bar{u}(x) - \bar{c}(x)) , \end{aligned} \tag{8}$$

   $$\begin{aligned} F_2^{\bar{\nu} p} &= 2x(u(x) + c(x) + \bar{d}(x) + \bar{s}(x)), \\ xF_3^{\bar{\nu} p} &= 2x(u(x) + c(x) - \bar{d}(x) - \bar{s}(x)) . \end{aligned} \tag{9}$$

   Consider Eqns. (8), (9) for large x. Hence express the ratio $F_2^{\nu p}/F_2^{\bar{\nu} p}$ in terms of a ratio of valence quark distributions.

5. Experimental information on valence and sea distributions may also be obtained from charged lepton-nucleon scattering. Express F_2^{lN} for an isoscalar target as a combination of valence and sea distributions. The difference $F_2^{lp} - F_2^{ln}$ (which may be deduced experimentally from data on proton and deuterium targets) can be expressed purely in terms of valence distributions. Justify this statement and hence derive the result of the Gottfried sum-rule

$$\int_0^1 \frac{dx}{x}(F_2^{lp} - F_2^{ln}) = \frac{1}{3}.$$

By examining the assumptions you made in the derivation, suggest why this sum-rule is violated.

6. Neutrino and anti-neutrino scattering data on proton and isoscalar targets potentially give a lot of information on the different parton flavour distributions. Suggest how the 4 cross-sections for the charged-current νp, $\bar{\nu} p$, νn, $\bar{\nu} n$ processes, might be combined to extract the distributions u_v, d_v, $\bar{u}$, $\bar{d}$, assuming that $s = \bar{s} = 0$.

7. The momentum sum rule can also be checked from charged lepton hadron data. Define

$$\epsilon_u = \int_0^1 dx\, x(u + \bar{u}), \quad \epsilon_d = \int_0^1 dx\, x(d + \bar{d}),$$

and hence express $\int_0^1 dx F_2^{ep}$ and $\int_0^1 dx F_2^{en}$ in terms of these integrals (again assuming $s = \bar{s} = 0$). Experimental results yield

$$\int_0^1 dx F_2^{ep} \sim 0.18 \quad \text{and} \quad \int_0^1 dx F_2^{en} \sim 0.12.$$

Deduce the fraction of nucleon momentum taken by gluons, ϵ_g.

8. Calculate the colour factor for gluon exchange between a q and a $\bar{q}$ in a colour singlet state $\frac{1}{\sqrt{3}}(r\bar{r} + b\bar{b} + g\bar{g})$.

9. Use the equation

$$\alpha_s(Q^2) = \left[b_0 \ln\left(\frac{Q^2}{\Lambda^2_{\text{QCD}}}\right)\right]^{-1} \quad (10)$$

to plot α_s as a function of Q^2 between 1 and 10^4 GeV^2, where $b_0 = \frac{11}{12\pi}C_A - \frac{2}{3}T_F N_f = \frac{33-2N_f}{12\pi}$. Assume $N_f = 3$ and $\Lambda_{\text{QCD}} = 200$ MeV. What happens to α_{qed} over the same energy range? Estimate the size of the NLO correction to $\alpha_s(Q^2)$ for Q^2 values 1, 10, 1000 GeV^2.

10. The LO DGLAP splitting functions are:

$$\begin{aligned}
P_{qq}(z) &= C_F\left[\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right], \\
P_{qg}(z) &= T_R[z^2+(1-z)^2], \\
P_{gq}(z) &= C_F\left[\frac{1+(1-z)^2}{z}\right], \\
P_{gg}(z) &= 2C_A\left[\frac{1-z}{z}+\frac{z}{(1-z)_+}+z(1-z)\right]+\beta_0\delta(1-z),
\end{aligned}$$

where $\beta_0 = 11/6C_A - 2/3N_fT_R$.

Consider,

$$P(z) = \delta(1-z) + \alpha_s/2\pi P_{qq}(z) + O(\alpha_s^2).$$

Calculate $\int_0^1 P(z)dz$ and $\int_0^1 zP(z)dz$ and comment on their significance.

Discuss the relationship between the different functions, i.e. why is $P_{qq}(z)$ nearly but not quite equal to $P_{gq}(1-z)$.

Calculate and comment upon the sum-rules:

$$\int_0^1 dz\, P_{qq}(z) = 0, \tag{11}$$

$$\int_0^1 dz\, z[P_{qq}(z)+P_{gq}(z)] = 0, \tag{12}$$

$$\int_0^1 dz\, z[2n_f P_{qg}(z)+P_{gg}(z)] = 0. \tag{13}$$

11. DIS processes are not the only processes which gives knowledge on parton distributions. Suggest some other processes which might yield useful knowledge on i) sea quark densities ii) the gluon density, and explain how? What are the major sources of uncertainty today, and how might our knowledge be improved?

12. DIS has also been used for classic determinations of α_s, outline how this is done. Suggest other ways of measuring α_s. Why is it important to make measurements at many different energy scales?

13. Try to explain what is meant by the term Pomeron, and what is its relationship to the gluon distribution and physics at low x ($x < 0.001$). Why is the steep rise of the gluon distribution at low x a challenge for conventional QCD calculations done within the leading log approximation (LLA)? What other "unconventional" QCD calculations have been made? Does current data require unconventional explanations?