Wave conventions: the good, the bad and the ugly

There are many different conventions in use for describing waves, leading to widespread confusion. Here I summarise the main conventions in use and comment on their quality.

1 The good

1.1 The Quantum mechanics convention

This is my favourite convention and I use it throughout my wave optics course. A wave travelling from left to right is described by

$$y_{\rightarrow} = \exp[\mathrm{i}(kx - \omega t)] \tag{1}$$

while a wave travelling from right to left is described by

$$y_{\leftarrow} = \exp[\mathrm{i}(-kx - \omega t)]. \tag{2}$$

Note that a positive term before the x means a wave travelling towards positive x. This convention is used in quantum mechanics where a wave of the form (**??**) corresponds to a particle moving with velocity $\hbar k/m$.

1.2 The electromagnetism convention

In electromagnetism (where we usually use j rather than i for the square root of -1) a wave travelling from left to right is described by

$$y_{\rightarrow} = \exp[\mathrm{j}(\omega t - kx)] \tag{3}$$

while a wave travelling from right to left is described by

$$y_{\leftarrow} = \exp[\mathrm{j}(\omega t + kx)]. \tag{4}$$

This convention concentrates on the frequency which is usually the term of interest in electromagnetism.

1.3 Derived conventions

The quantum mechanics convention can be made equivalent to the electromagnetism convention[1] by choosing $\mathrm{j} = -\mathrm{i}$, so that j is the *other* square root of -1, and these two conventions are fundamentally equivalent. However it is quite common to use the electromagnetism convention and replace j by i leading to

$$y_{\rightarrow} = \exp[\mathrm{i}(\omega t - kx)] \tag{5}$$

[1]Thanks to Geoff Brooker for pointing this out

for a wave travelling from left to right, while a wave travelling from right to left is described by

$$y_{\leftarrow} = \exp[\mathrm{i}(\omega t + kx)]. \tag{6}$$

These conventions are commonly used in some waves course. Note that a positive term before the x now means a wave travelling towards negative x.

2 The bad

There is another convention which is fairly widely used in treatments of waves, in which a wave travelling from left to right is described by

$$y_{\rightarrow} = \exp[\mathrm{i}(kx - \omega t)] \tag{7}$$

while a wave travelling from right to left is described by

$$y_{\leftarrow} = \exp[\mathrm{i}(kx + \omega t)], \tag{8}$$

so that the direction of the wave is encoded in the sign of the *frequency* rather than the wavenumber. Although *in principle* there is nothing wrong with this, in practice it leads to many problems and so is best avoided. In particular it leads to problems when treating reflections of waves from inhomogeneities, as it means that the reflected wave has a different frequency from the transmitted and incident waves.

This convention is frequently used in mathematical treatments of waves, as it is closely linked to d'Alembert's solution for the wave equation

$$y = f(x - ct) + g(x + ct) \tag{9}$$

but other than that it has little or nothing to recommend it.

3 The ugly

So far all the waves have used complex exponential form; it is, of course, possible to use all these conventions with cosine or sine forms instead. In general this is not recommended as it leads to messy equations and lots of horrible algebra. It can also lead to further confusion as some of the conventions become indistinguishable, as sign information is partially lost.

4 Advice

Use complex exponentials where possible. Stick to one convention, preferably one of the "good" conventions. If in doubt the quantum mechanics convention has a lot to recommend it. However, bear in mind that examiners will not always stick to "your" conventions, so be prepared to be flexible!