Wave Optics

Problems labeled (*) are essentially standard derivations. These come up regularly on Prelims papers and are well worth learning.

A Standard Questions

1. *In a Young's slits experiment a screen with two narrow slits of separation d is illuminated normally by coherent light of wavelength λ. The resulting pattern is viewed on a screen at a distance $D \gg d$. Derive an expression for the intensity distribution on the screen.

2. In a Young's slit experiment as described above, green light ($\lambda = 550\,\text{nm}$) is used. With $D = 0.7\,\text{m}$ the distance between the zeroth (central) maximum and the fifth maximum is found to be 7 mm. Calculate the separation of the slits.

3. A thin sheet of glass (0.2 mm thick, $n = 1.66$) is placed over one of the slits, on the directly illuminated side. By how many fringes does the pattern shift?

4. The sheet of glass is now removed and the green light source is replaced by a source producing three sharp lines at 450 nm (blue), 550 nm (green) and 650 nm (red) with equal intensities. Sketch the three different fringe patterns and hence describe the overall appearance of the fringes.

5. *A parallel beam of monochromatic light of wavelength λ is directed normally on to a reflection grating with line spacing d. Show that maxima in the intensity of the diffracted light occur at angles θ to the normal such that $n\lambda = d \sin\theta$ where n is an integer. How does the resulting pattern differ from that in a Young's slits experiment?

6. *Describe a practical arrangement for using a diffraction grating as a spectrometer.

7. A grating is used in normal incidence and third order to observe the red line ($\lambda = 656.3\,\text{nm}$) from a hydrogen discharge at a diffraction angle of 36.2°. What is the ruling spacing (in lines per mm) of this grating? What angle would the first order line occur at? The same grating is used to observe the sodium D lines ($\lambda = 589.0\,\text{nm}$ and $\lambda = 589.6\,\text{nm}$). Calculate the angular separation of the two lines at first order and at the largest order which can be observed.

8. *Derive the angular intensity distribution in the diffraction pattern when a slit of width w is illuminated normally with light of wavelength λ, and hence find the Rayleigh (diffraction) limit for a single slit.

9. *Why does diffraction limit the image formed by a lens? State the Rayleigh resolution limit for a circular lens.

B Slightly more interesting questions

1. A convex lens has a radius of 20 mm and a focal length of 100 mm. A point source of light is placed 500 mm behind the lens and a screen is placed 120 mm in front of it. Calculate the size of the illuminated spot on the screen. (Neglect effects due to diffraction at this stage). Now calculate the distance at which the screen should be placed to get the best possible image, and estimate the spot size in this case.

2. The pupil of the human eye is normally around 5 mm in diameter. A person with 20/20 vision should be able to read letters 9 mm high at a distance of 20 m, while in Chapter two of *The Two Towers* Legolas can resolve objects of around 10 cm size at a distance of five leagues (about 24 km). Comment.

3. Satellite TV is transmitted at a frequency of around 15 GHz and received using dish antennae with a diameter of around 0.7 m. Estimate the accuracy with which it is necessary to align a satellite dish.

4. The Hubble space telescope (a reflecting telescope with a main mirror diameter of 2.4 m) was recently used to view the Apollo landing sites on the moon using near UV light ($\lambda \approx 300$ nm). The pictures did not show any evidence of the lunar modules. Should we conclude that the landings were a hoax?

5. We can calculate the diffraction pattern from two finite-width slits by brute force, using the same technique we used for a single slit, except that the integral is now run over *both* slits. Suppose that the two slits are of width b and positioned at distances $\pm d/2$ (with $b < d$ of course). Find the light amplitude and intensity as a function of diffraction angle (do the integral in two pieces) and comment on the form of your result.

(Problems reproduced courtesy of Jonathan A. Jones)