New Ideas for 6 D Ionization Cooling

Muon Collaboration

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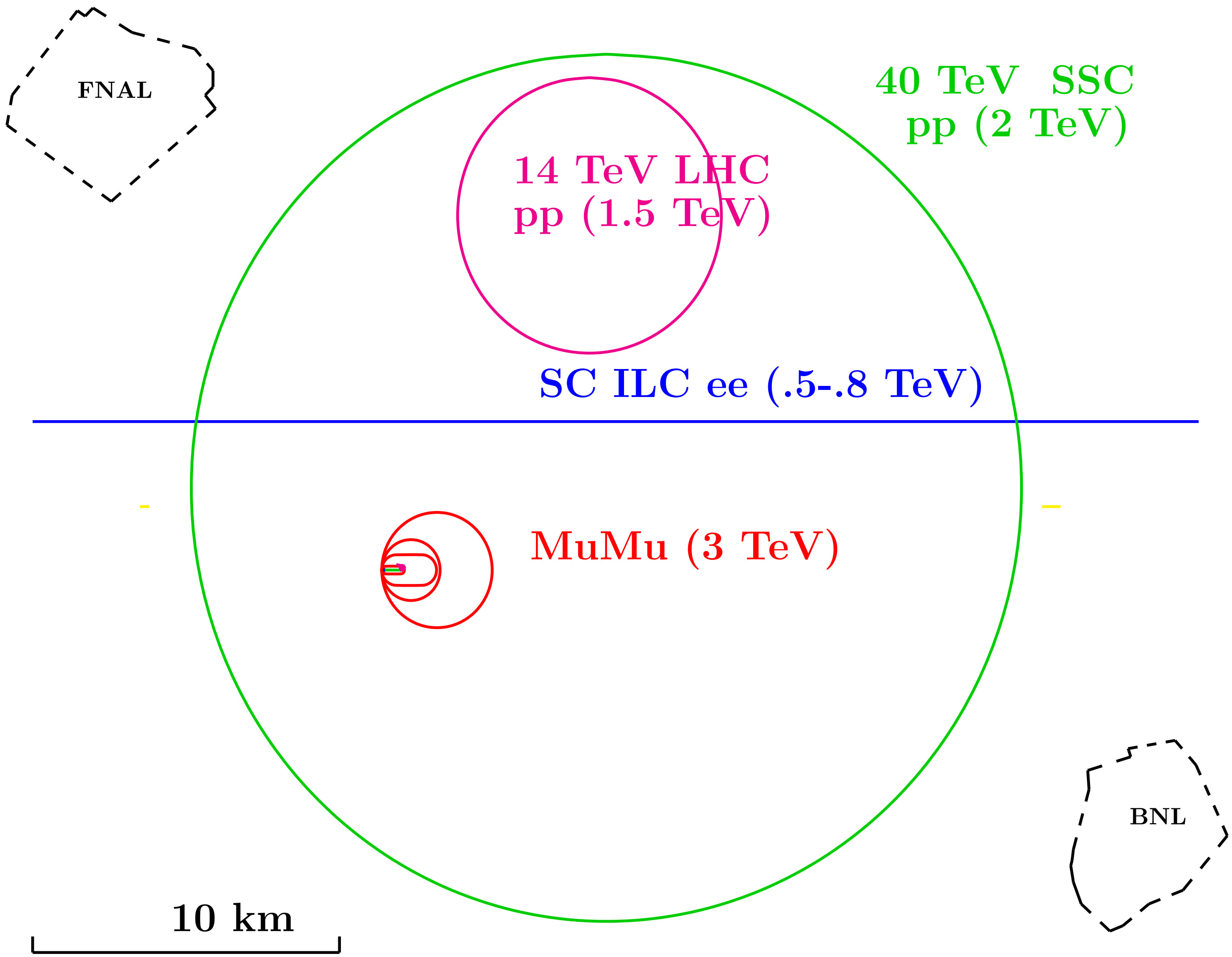
- Muon Collider requirements
- Transverse Ionization Cooling Theory
- Longitudinal Emittance Cooling Theory
- Final Reverse Emittance Exchange
- New Ideas on How to do it
- Conclusion
- More New ideas if we have time

Why a Muon Collider



3 TeV Collider requirements from Snowmass 98

Assume

Average bending field	**T**	**5.2**
Luminosity	$10^{33} cm^{-2}$	**70**

$\mathbf{E_{cm}}$	$\mathbf{N_\mu}$	$\mathbf{N_b \times f}$	$\mathbf{P_\mu}$	$\beta_\perp = \sigma_z$	**dp/p**	$\mathbf{emit_\perp}$	$\Delta\nu$	ϵ_6
TeV	10^{12}	**Hz**	**MW**	**mm**	**%**	**mm**		10^{-12}m
3	**2**	**2×15**	**28**	**3**	**0.16**	**.05**	**.044**	**170**

$$\epsilon_{||} = \beta_v \gamma \, \sigma_z \frac{dp}{p} = 1.5\ 10^4\ 0.003\ \frac{0.16}{100} = 7.2\ 10^{-2} \quad \mathrm{m}$$

$$\epsilon_6 = \epsilon_{||} (\epsilon_\perp)^2 = 7.2\ 10^{-2} \times (50\ 10^{-6})^2 = 180\ 10^{-12} \quad \mathrm{m}^3$$

Initial $\epsilon_6 \approx 2\ (.02)^2 \approx 10^{-3}$ **m**

- **Cooling Required ≈ 1/5000,000**
- **Final Longitudinal Emmittance = 72,000 (pi mm mrad)**
- **Final Transvers Emittance = 50 (pi mm mrad)**

What is Emittance ?

$$\text{normalized emittance} = \frac{\text{Phase Space Area}}{\pi \text{ m c}}$$

If x and p_x are both Gaussian and uncorrelated, then the area is that of an upright ellipse, and:

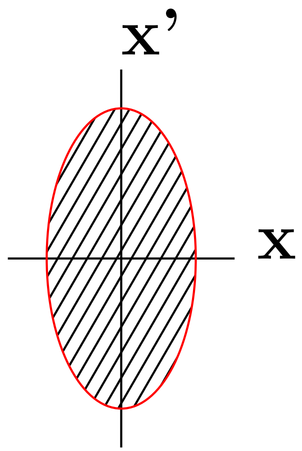
$$\epsilon_\perp = \frac{\pi \ \sigma_{p_\perp} \sigma_x}{\pi \ mc} = (\gamma \beta_v) \sigma_\theta \sigma_x \qquad (\pi \ m \ rad)$$

$$\epsilon_\parallel = \frac{\pi \ \sigma_{p_\parallel} \sigma_z}{\pi \ mc} = (\gamma \beta_v) \frac{\sigma_p}{p} \ \sigma_z \qquad (\pi \ m \ rad)$$

$$\epsilon_6 = \epsilon_\perp^2 \ \epsilon_\parallel \qquad (\pi \ m)^3$$

Note that the π, added to the dimension, is a reminder that the emittance is phase space/π

What is Beta$_\perp(Twiss)$ of Beam



Upright phase ellipse in x' vs x,

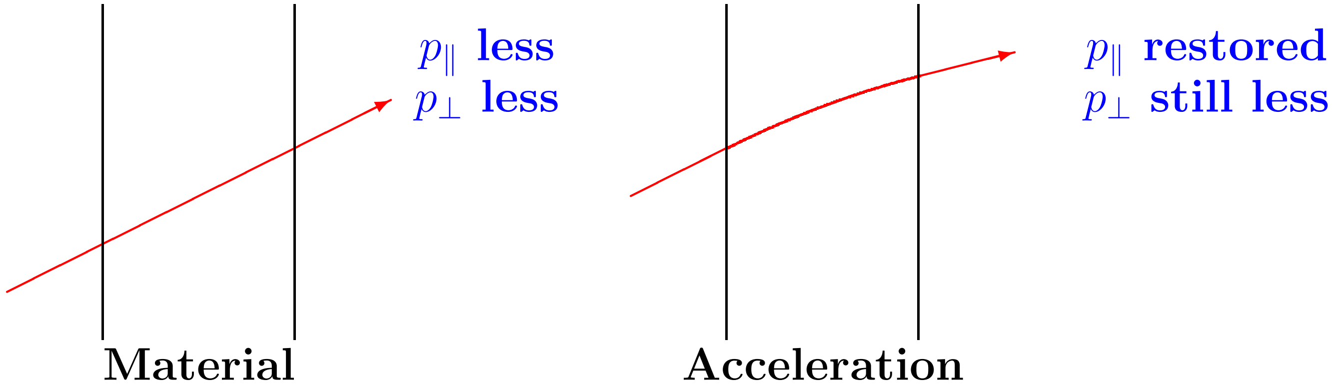
$$\beta_\perp = \left(\frac{\text{width}}{\text{height}}\right) = \frac{\sigma_x}{\sigma_\theta}$$

Strong focus $\rightarrow$ low σ_x and large σ_θ $\rightarrow$ low β

$$\sigma_x = \sqrt{\epsilon_\perp \; \beta_\perp \; \frac{1}{\beta_v \gamma}}$$

$$\sigma_\theta = \sqrt{\frac{\epsilon_\perp}{\beta_\perp} \; \frac{1}{\beta_v \gamma}}$$

Transverse Cooling



Rate of Cooling without scattering

$$\frac{d\epsilon}{\epsilon_{x,y}} = \frac{dp}{p} \; J_{x,y}$$

For the moment the "partition functions"

$$J_{x,y} = 1$$

Explanation later

Minimum (Equilibrium) Emittance

$$\epsilon_{x,y}(min) \quad = \quad \frac{\beta_\perp}{\beta_v \; J_{x,y}} \quad C(mat, E)$$

$$J_{x,y} \;=\; 1 \qquad\qquad C(mat, E) \;\propto\; \frac{1}{L_R \; d\gamma/ds}$$

At minimum of dE/dx (≈ 300 MeV/c)

material	density kg/m^3	dE/dx MeV/m	L_R m	C_o %	A_o %
Liquid H_2	71	28.7	8.65	0.38	1.36
Li	530	87.5	1.55	0.69	1.31
Be	1850	295	0.353	0.89	1.28
C	2260	394	0.47	1.58	1.25
Al	2700	436	0.089	2.48	1.23

- Hydrogen much the best material
- Coefficient A_o is for longitudinal cooling - explanation to come

An Aside: Beam Divergence Angles

$$\epsilon_{x,y}(min) \;=\; \frac{\beta_\perp}{\beta_v \; J_{x,y}} \; C(mat, E)$$

$$\sigma_\theta \;=\; \sqrt{\frac{\epsilon_\perp}{\beta \; \beta_v \gamma}}$$

so for a beam in equilibrium

$$\sigma_\theta \;=\; \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}}$$ independent of emittance

for 75 % of maximum cooling rate, an aperture at 3 σ, and $\beta_v^2\gamma = 2$ the required angular acceptance A of the system must be

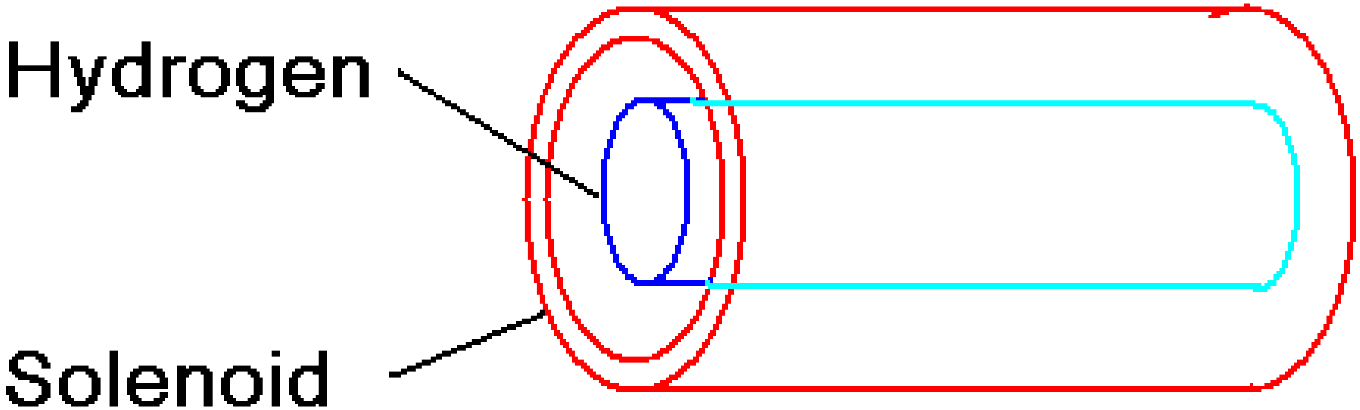
$$A \;=\; 3\sqrt{4} \; \sqrt{\frac{C(mat, E)}{\beta_v^2 \gamma}}$$

Material	H2	Li	Be	C	Al
Ang Acceptance (RaD)	0.25	.35	.4	.54	.66

These are very large angular acceptances !

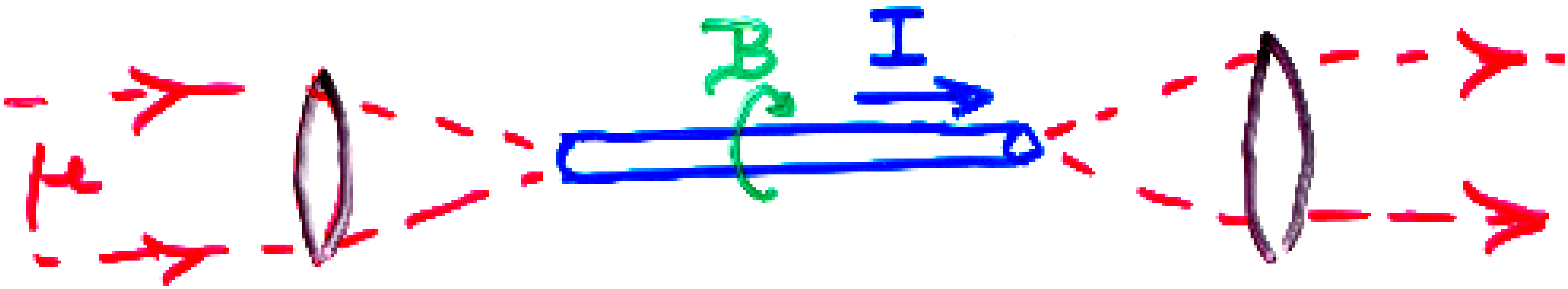
How to get low beta (strong focus) ?

- Strong Solenoid



  - Practical limit is 10 T
  - Expensive

- **Lithium Lens**



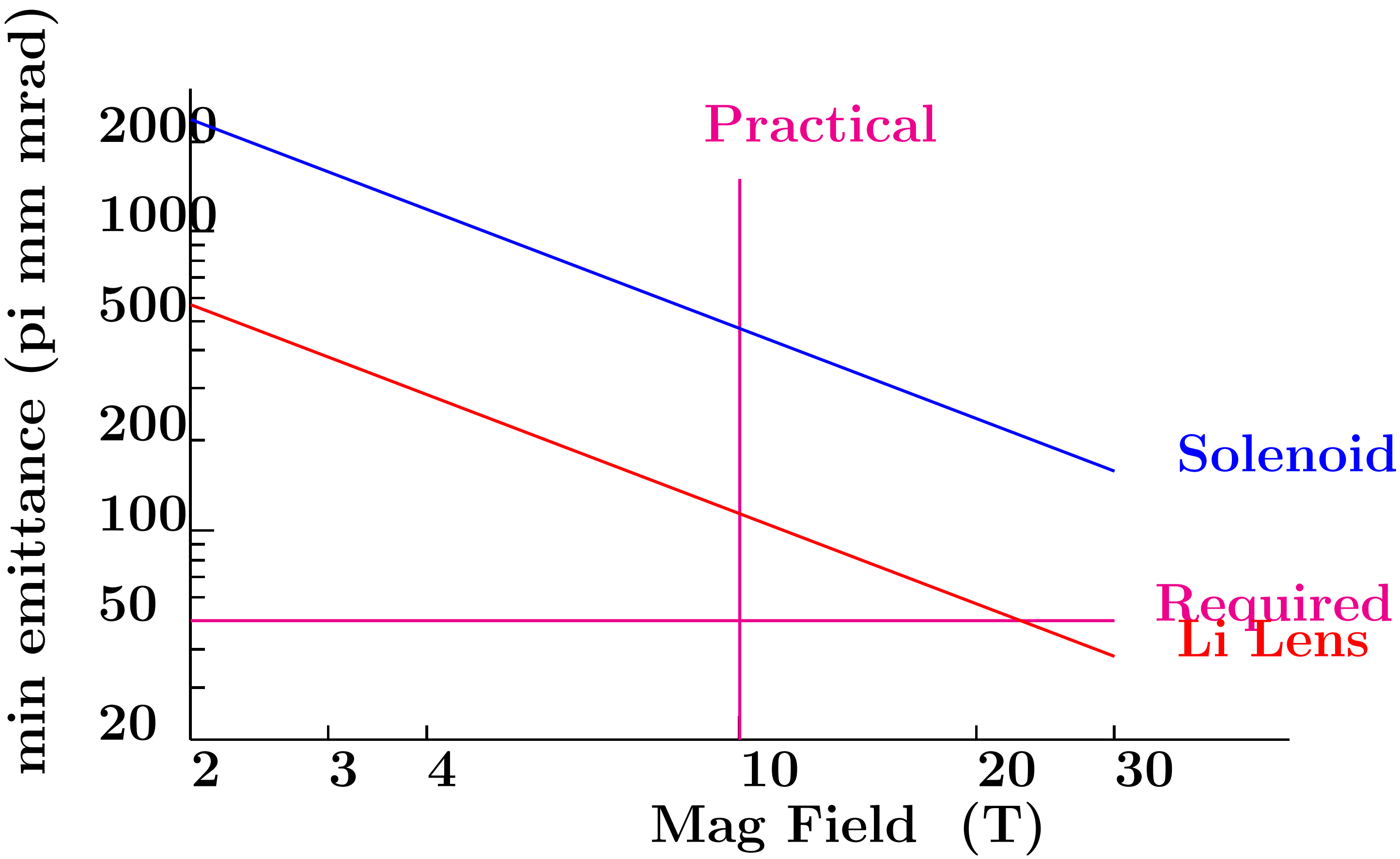
- **For uniform i then perfect lens**

$$I \propto A \propto r^2$$

$$\text{Bending} \propto B \propto I/r \propto r$$

- **Maximum current limited by breaking containment tube**
- **Pressure $\propto$ Surface Field**
- **Current lenses get up to near 10 T**

Compare Solenoids and Li Lenses

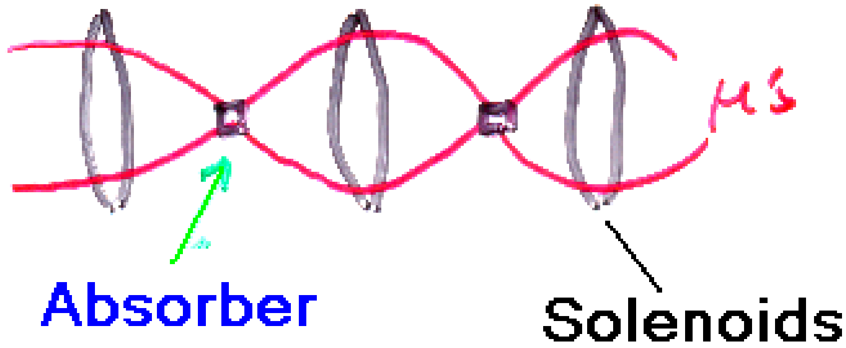


– Even a 20 T Solenoid will not get required emittance

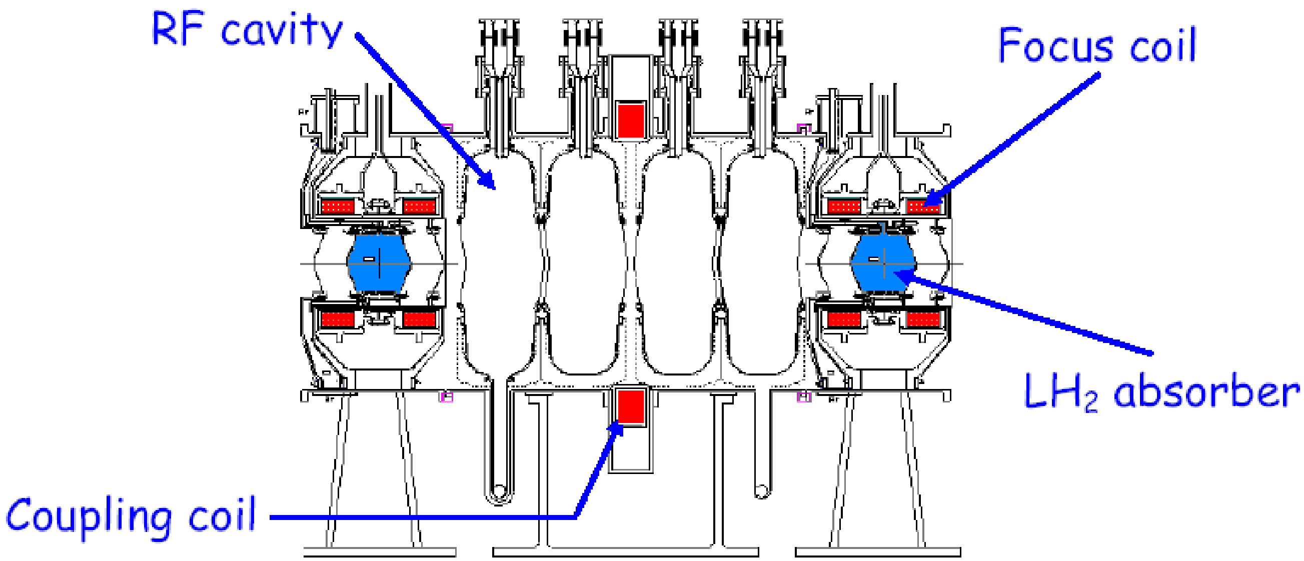
– Existing Li Lenses (10T) will not reach it

– 30 T Li Lens ok, but not developed
and probably impossible from cavitation

- At Multiple foci



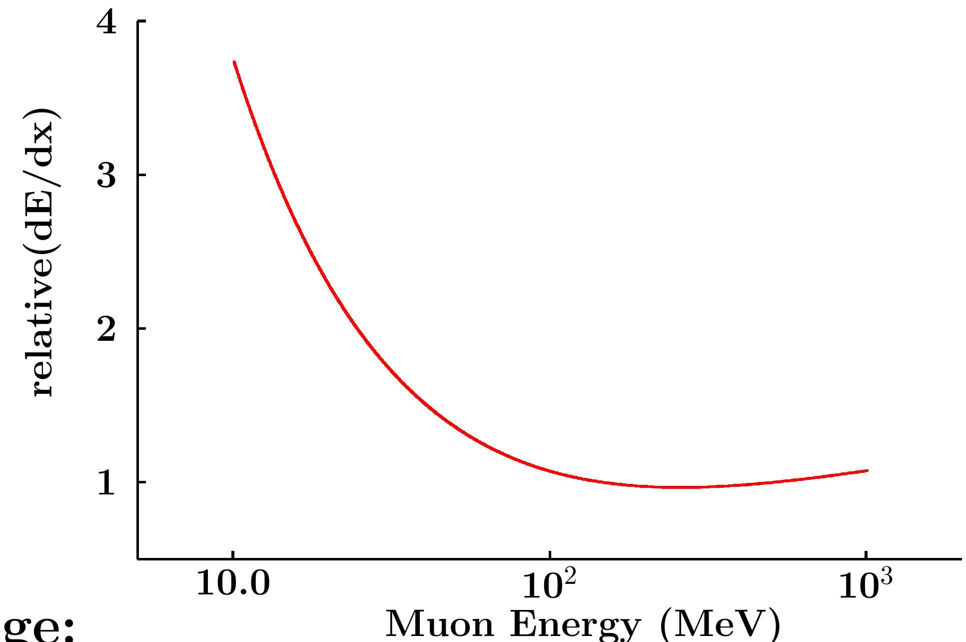
e.g. Mice cells



- Beta of order 1/3 average beta for moderate B (3-6 T)
- Harder as B rises because of coil thickness
- Hard to get emittances < 400 pi mm mrad

Longitudinal Cooling ?

- **At mom $\ll$ 200 MeV/c dp/p is increased (heating)**
- **At mom $\gg$ 200 MeV/c dp/p is weakly reduced (cooling)**
- **We Use $\approx$ 200 MeV/c negligible heating or cooling**



Partition function J_z :

$$\frac{d\epsilon_z}{\epsilon_z} = \frac{dp}{p} \quad J_z$$

$$J_z \approx 0$$

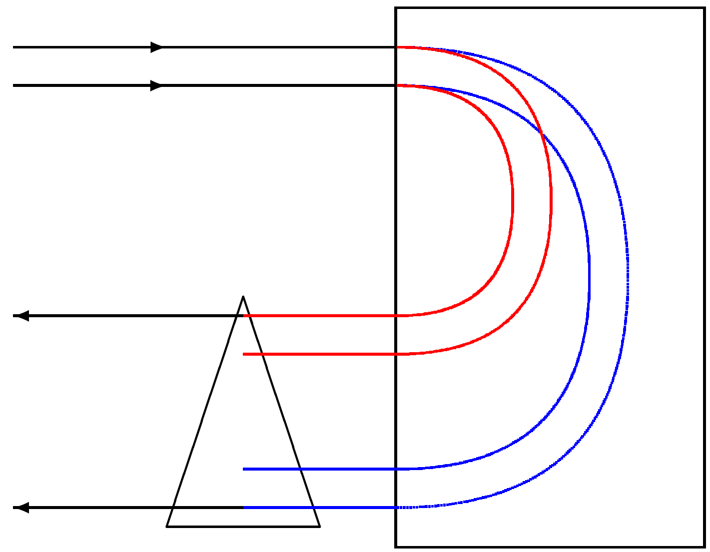
6 dimensional emittance change:

$$\frac{d\epsilon_6}{\epsilon_6} = \frac{dp}{p} \quad J_6$$

where

$$J_6 = J_x + J_y + J_z \approx 2$$

Emittance Exchange



- **dp/p (and Longitudinal emittance) reduced**
- **But σ_y (and transverse emittance) increased**
- **Transverse cooling from mean loss in absorber**
- **"Emittance Exchange" + Transverse Cooling = 6 D cooling**

$$J_x = (J_x)_o + \Delta J_x \qquad J_y = (J_y)_o + \Delta J_y \qquad J_z = (J_z)_o + \Delta J_z$$

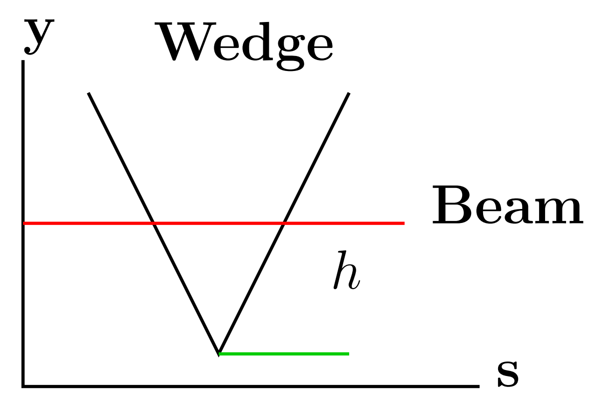
$$\Delta J_x + \Delta J_y + \Delta J_z + = 0$$

e.g. If cooling only by wedges

Rate of Cooling without straggling

$$\Delta J_z(\text{wedge}) \;=\; = \frac{D}{h}$$

where $D = dy/(dp/p)$ **is the Dispersion**



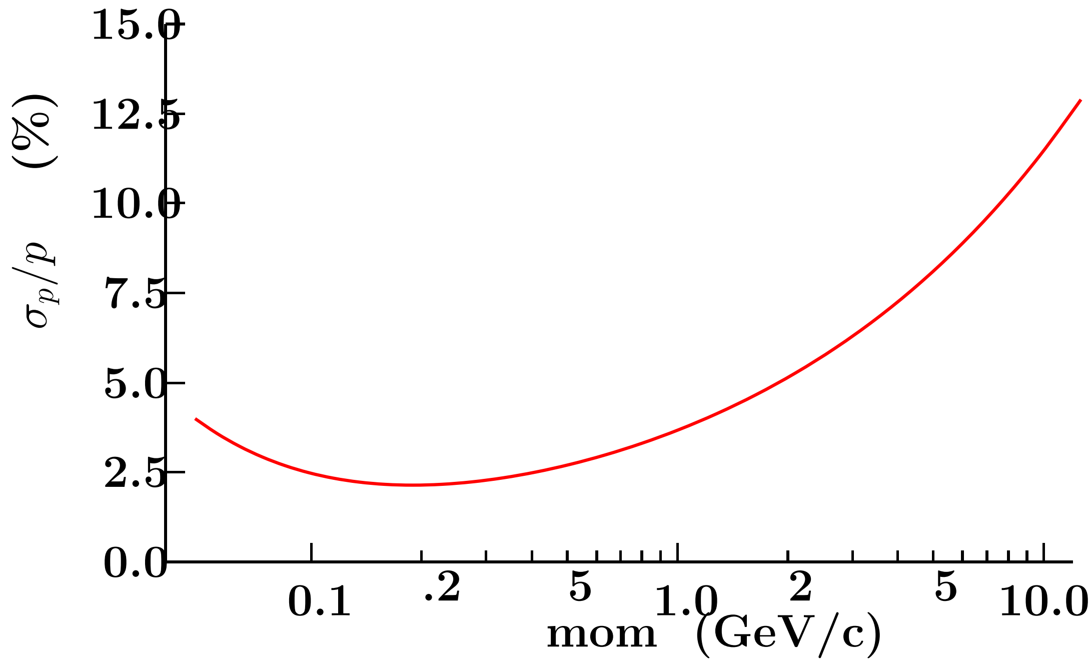
given a finite J_z we get a minimum (equilibrium) dp/p:

$$\frac{\sigma_p}{p} \;=\; A_o \sqrt{\frac{\gamma}{\beta_v^2}\left(1-\frac{\beta_v^2}{2}\right)\quad\frac{1}{J_z}}$$

The values of A_o were given in the above table

For Hydrogen, $A_o \approx$1.36 %, but it is almost the same for other materials

For $J_z = 2/3$ minimum (equilibrium) dp/p



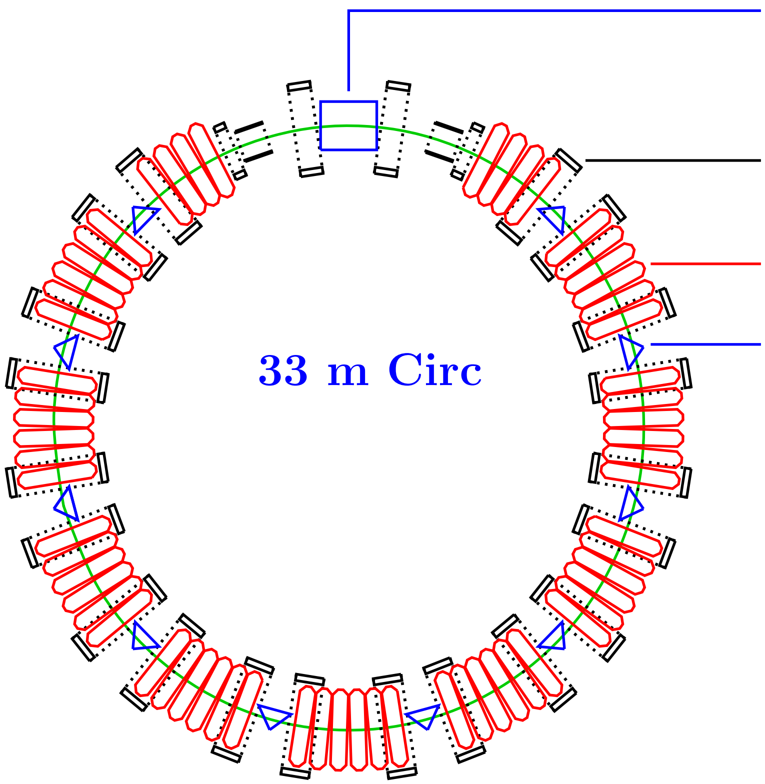
Minimum at 2.5% around 200 MeV/c

$$\epsilon_z = \frac{\sigma_p}{p} \times \beta_v \gamma \ \sigma_z$$

σ_z, the bunch length depends on the RF gradient and frequency
less at higher frequency

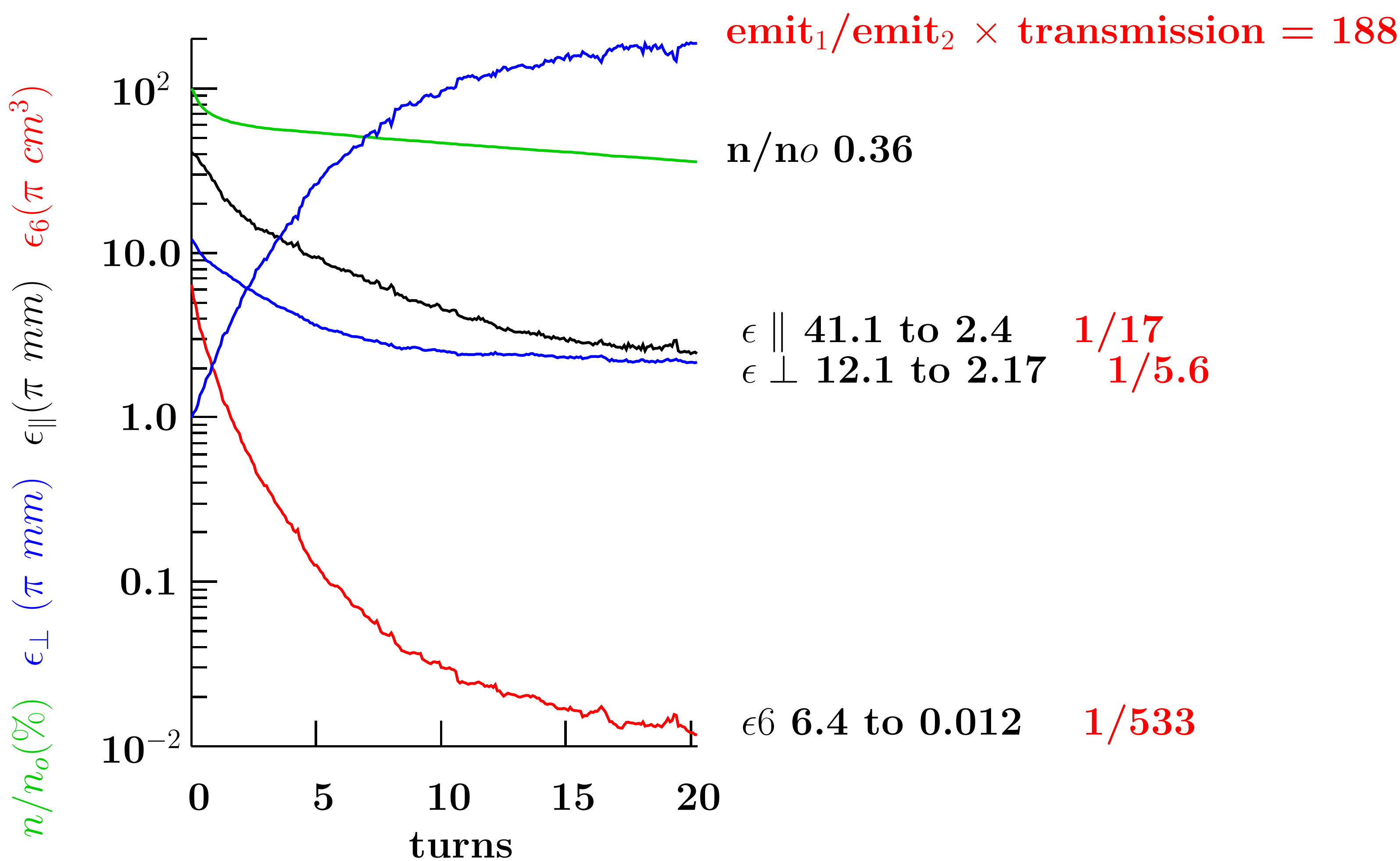
e.g. 6 D cooling in "RFOFO" Ring with Wedges

- Lattice similar to MICE
- Bending gives dispersion
- Wedge absorbers: Cooling also in longitudinal
- Many turns in Ring gives more cooling at lower cost



- Could be converted to Helicoil
- No Injection/extraction
- Better performance by tapering
- But more expensive

performance



Final Long Emittance 2400 (pi mm mrad)

Second ring at 400 MHz → ≈ 1400 (pi mm mrad)

c.f. 7200 (pi mm mrad) Req for Collider

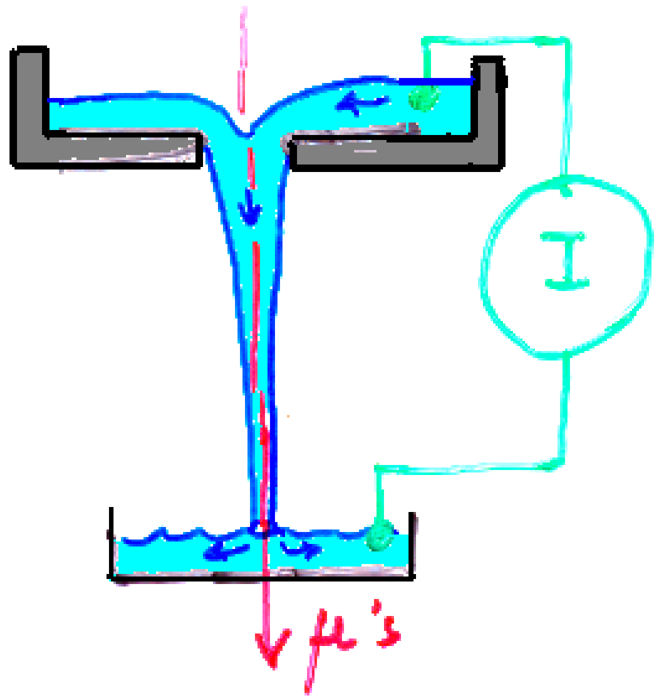
Conclusion as of 2 years ago

- **Longitudinal emittance can be Achieved**
- **Transverse emittance not Achieved**
  - **$\approx$ 800 pi mm mrad Possible with 10 T Solenoid**
  - **$\approx$ 400 pi mm mrad Possible with Lattice**
  - **$\approx$ 200 pi mm mrad Possible with Lithium lend**
  - **Required = 50 pi mm mrad**

- **Final Reverse Emittance Exchange Proposed with wedges**
  But is found to be hard in practice
  See below

New idea Li Jet ?

- No containing tube to break
- Use Magnetic field to stabilize (and form ?) jet
- Jet larger at nozzle to avoid damage
- Ends in indestructible pool



Is this crazy ?

Old solution: Reverse Emittance Exchange at End

- Assume 10 T Li Lens for transverse emittance
- Assume RFOFO Ring for Longitudinal emittance
- This is not quite fair, but reasonable

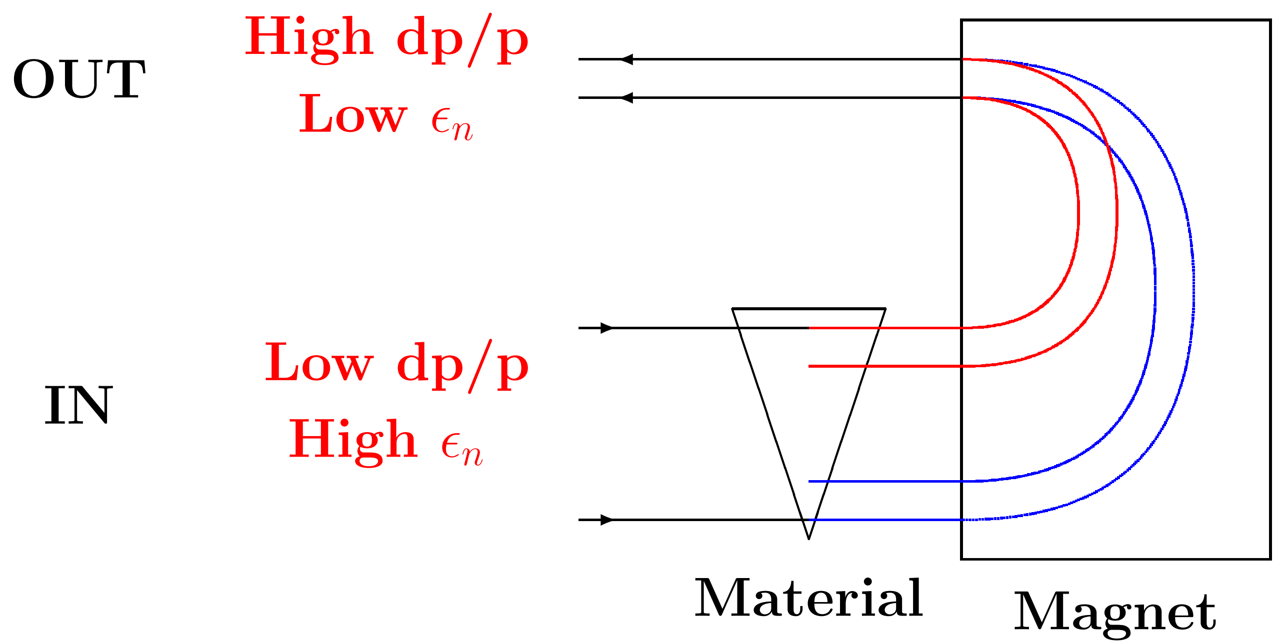
	Required	Achievable	Achievable/Req
Transverse	$50\ 10^{-6}$	$200\ 10^{-6}$	4.0
Longitudinal	$70\ 10^{-3}$	$1.5\ 10^{-3}$	1/50
6 D	$180\ 10^{-12}$	$60\ 10^{-12}$	1/3

- Required 6D emittance seems achievable
- Longitudinal emittance even too small !
- But Transverse emitance too Large

Suggests Final Reverse emittance Exchange

1. Wedges with wrong Dispersion Old Method
2. By use of septa (potato slicer) New idea
3. Very Low energy in Li Lens New idea

1) Wedges with wrong Dispersion (Old Idea)



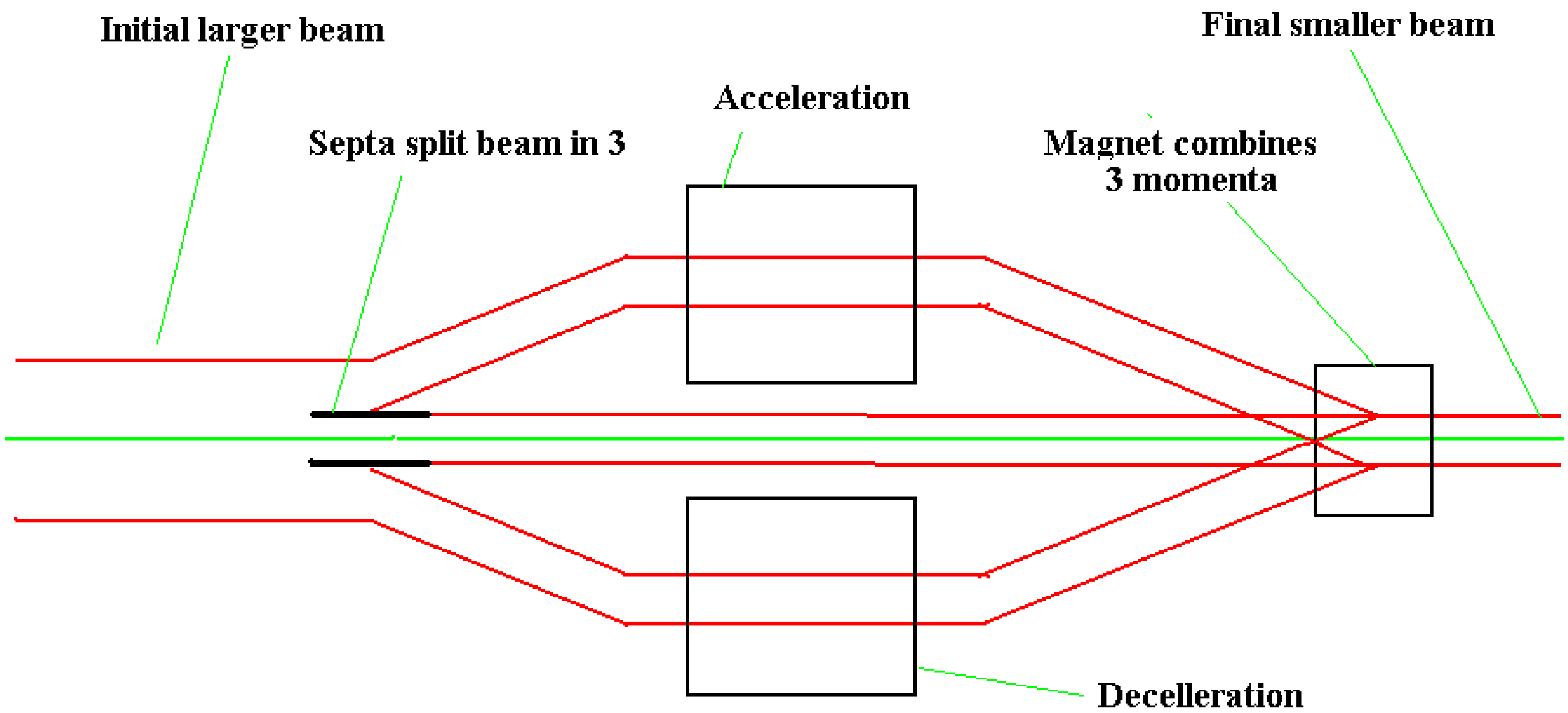
Require 4 times smaller equilib transverse emittance

thus $J_{x,y} = (J_{x,y})_o \times 4 = 1 \times 4 = 4$

and $J_z = 2 - 2 \times J_{x,y} = 2 - 8 = -6$

- **Required transverse emittance achieved, but**
- **Required longitudinal emittance lost**

2) Potato Slicer (New idea)



- This can be done at any momentum
- Gaussian shapes of beams, and septa, lead to dilution
- Realization may be hard

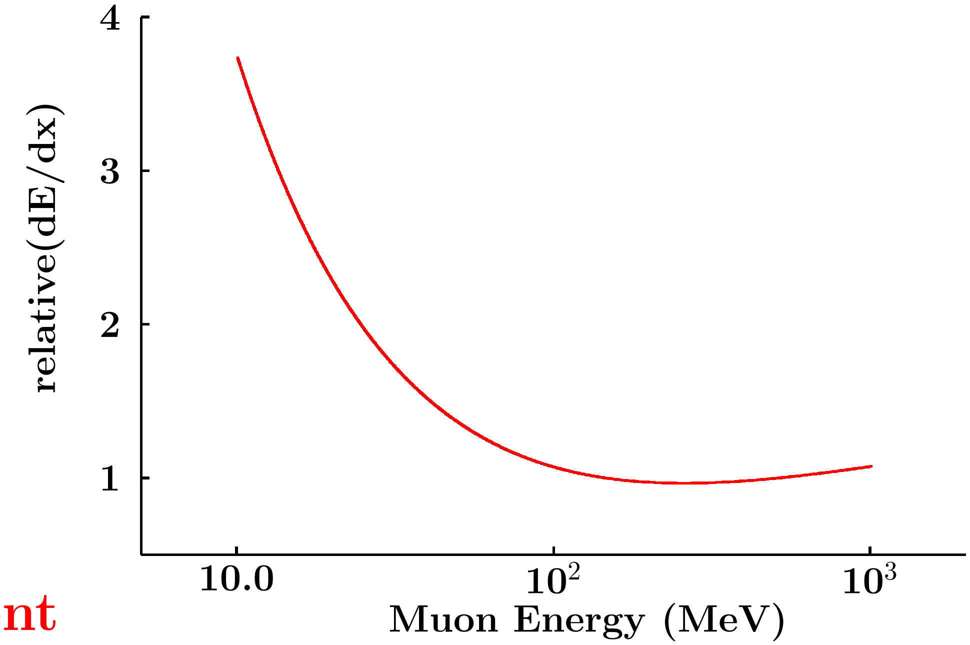
Needs study, but must work at some level

3) Li Lens at very low Energy

Remember:

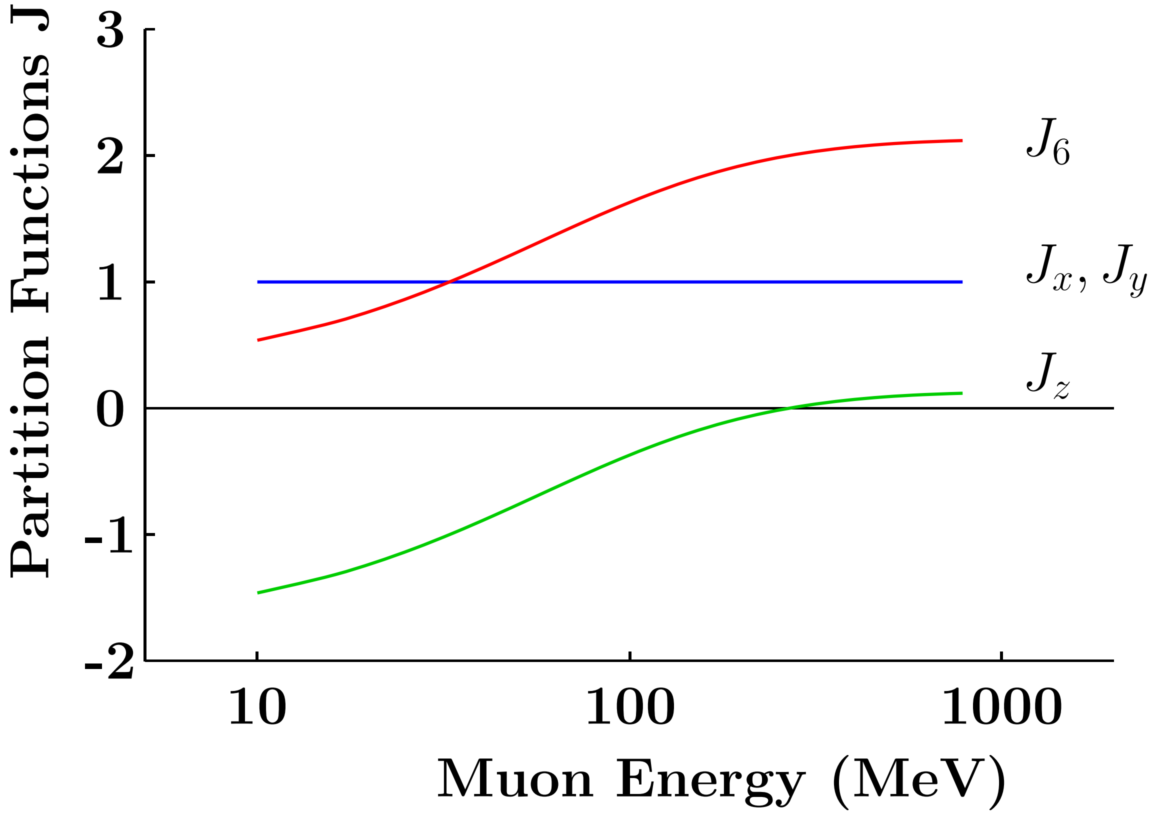
$$\epsilon_{x,y}(min) = \frac{\beta_\perp}{\beta_v \; J_{x,y}} \; C(mat,E)$$

$$J_{x,y} = 1 \qquad\qquad C(mat,E) \propto \frac{1}{L_R \; d\gamma/ds}$$



- **dE/dx × 4 at 10 MeV**
- **C(mat,E) = 1/4 10 MeV**
- **Equilib. emittance × 1/4 = 50 (pi mm mrad)**
- **Now meets trans. requirement**

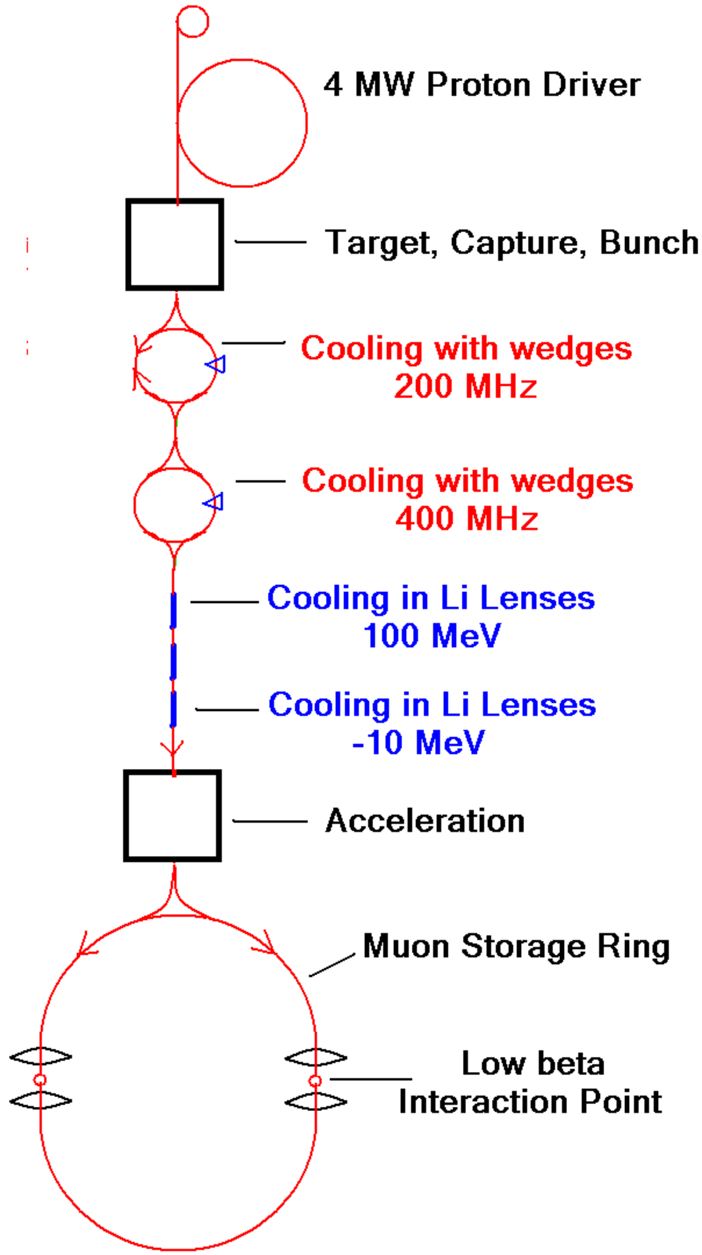
Effect on Longitudinal emittance



- **Long. Emittance will rise from $J_z = -1$**
- **But J_6 remains positive**
- **So 6D emittance should not rise**
- **Effectively: Reverse Emittance Exchange**

Looks good, but needs study

Schematic of Collider



Conclusion

- Solenoid lattices cannot reach required transverse emittance
- But they can lower longitudinal emittance below requirement

- Li lenses cool to lower trans. emittances than solenoid lattices
- But at moderate momenta cannot achieve the trans. req.

- We need "Emittance exchange"
  - A solenoid focused reverse wedge does this in principle
    But seems to fail in practice
  - A Potato slicer should work, but dilutes 6D emittance

  - Li Lens at low energy gives "effective emittance exchange"
    And seems to meet the requirements
    but has not yet been simulated

- Much more Study is needed
- There are many other problems

- But there is reason to hope

New Idea: Emittance Exchange using path length differences

S. Derbenev, R. Johnson

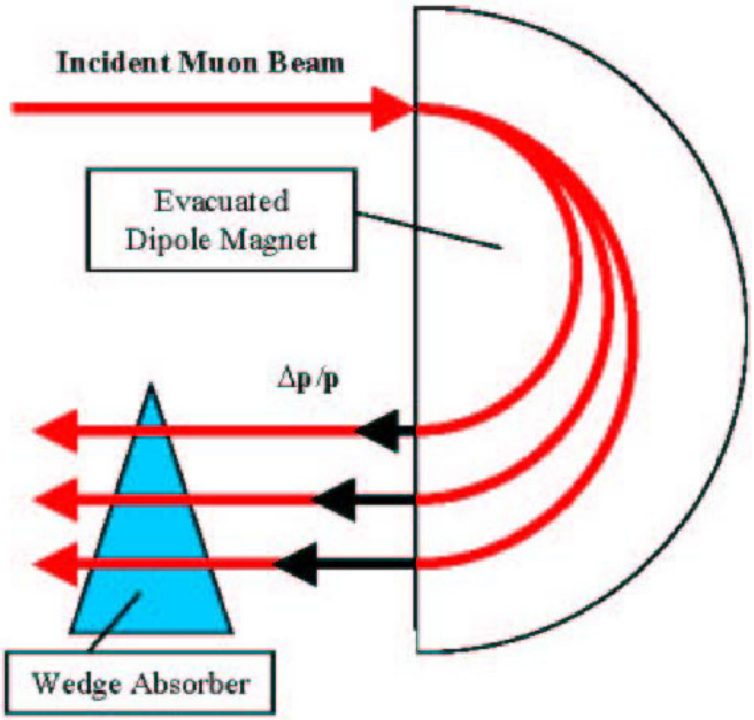


Figure 1. Use of a Wedge Absorber for Emittance Exchange

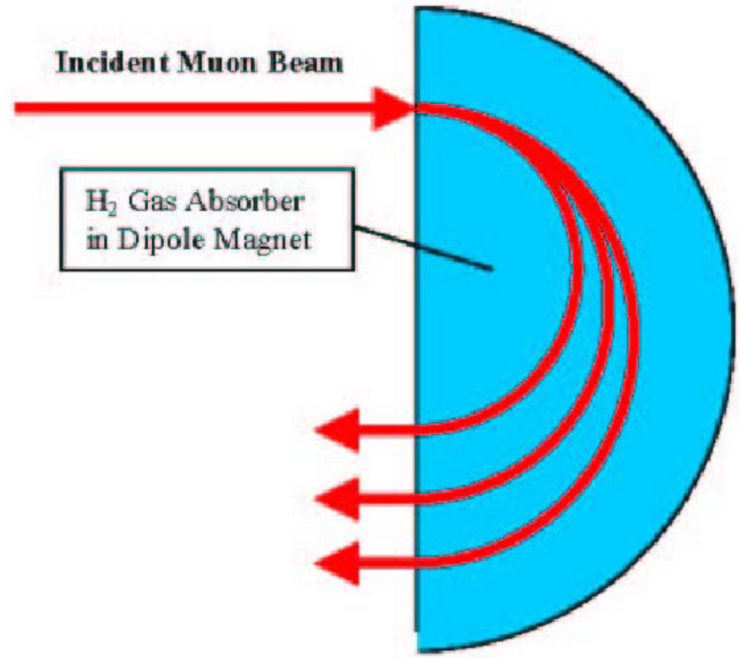
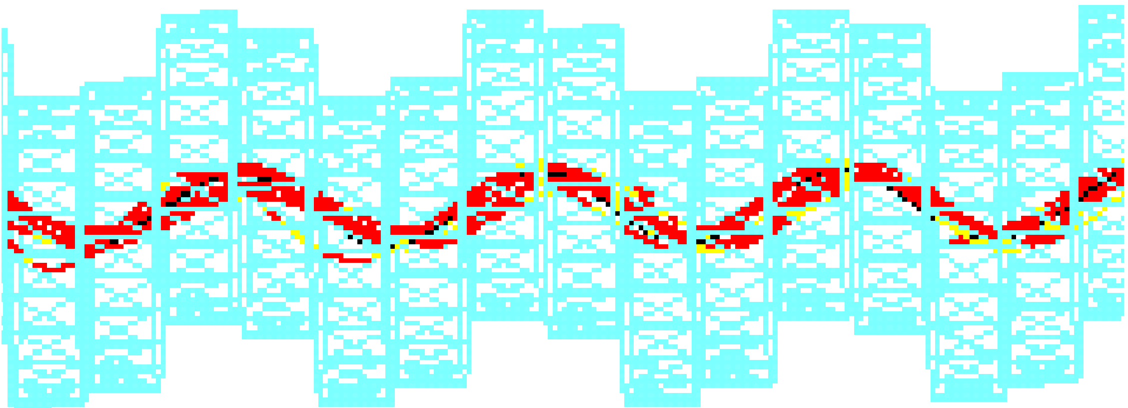
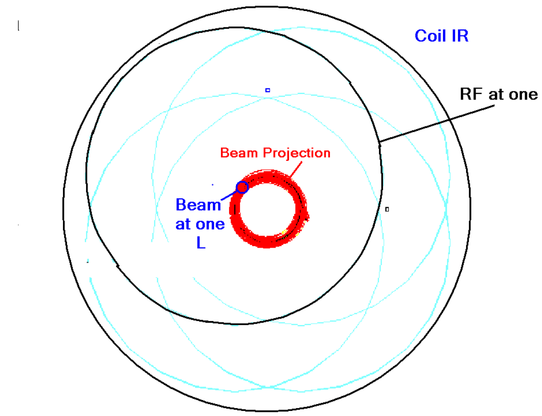


Figure 2. Use of Continuous Gaseous Absorber for Emittance Exchange

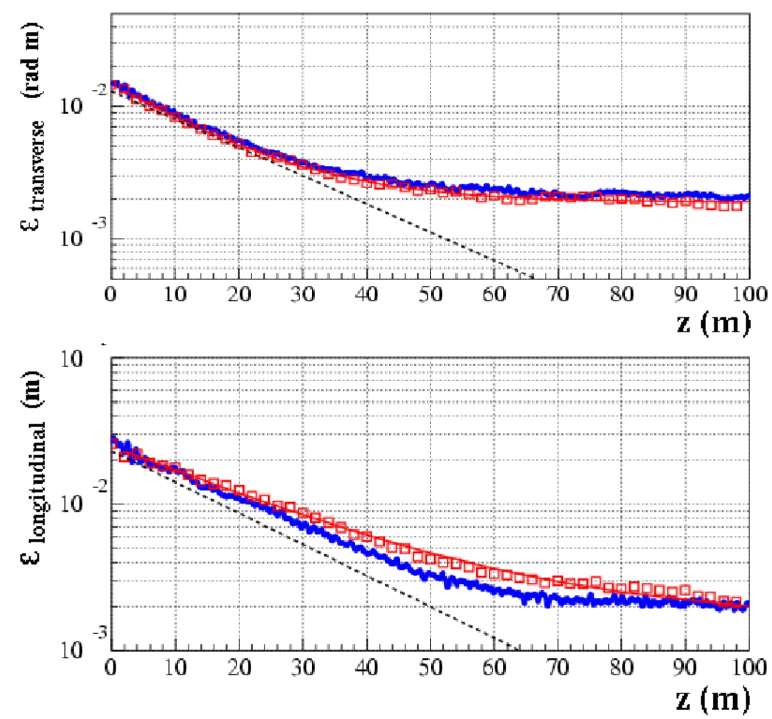
New idea Gas in a Helical Channel

(Derbenev, Rol Johnson, Muons Inc.)



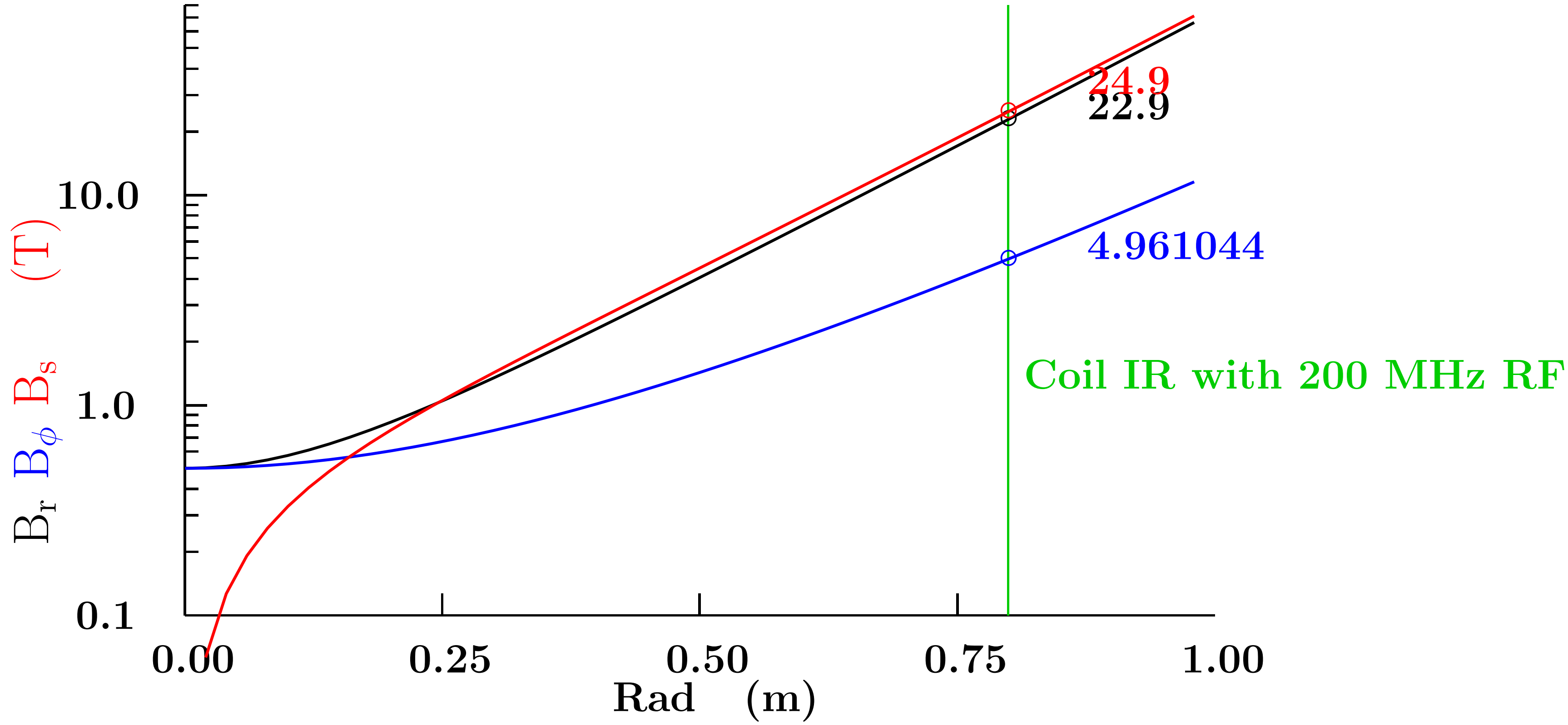


- Partly for higher acc gradients
  Not yet demonstrated
- Cooling in 6 dimensions
  of order 1000
- Moderate fields at beam
  Bz=3.5 T. Br=.5 T
- Better Performance than
  RFOFO Ring



But Helix Fields at Coils > 24 T

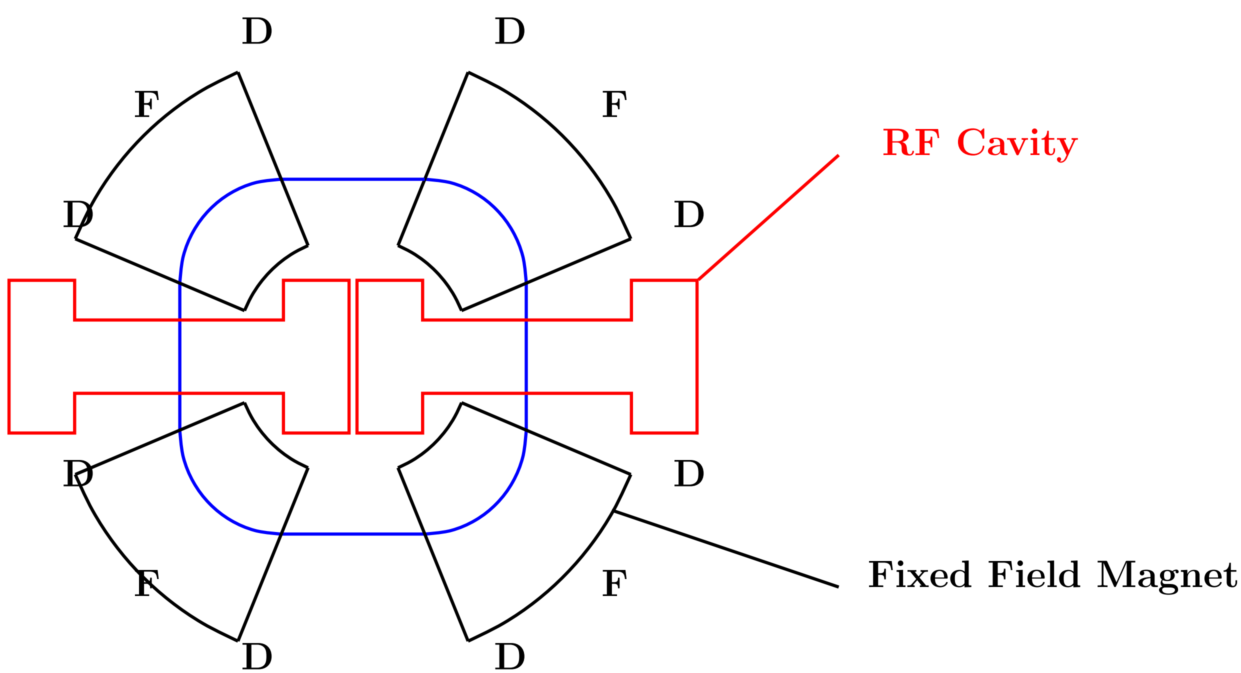
For: $\lambda = 1$ **m**
$B_\perp = 0.5$ **T**



- **Increasing pitch: hurts ds/dp**
- **Decreasing helix B: hurts ds/dp**
- **Lowering RF λ $\rightarrow$ lower emit + higher B's**
- **Exploring emittance exchange before bunching and RF**

New idea: With Gas in a Ring

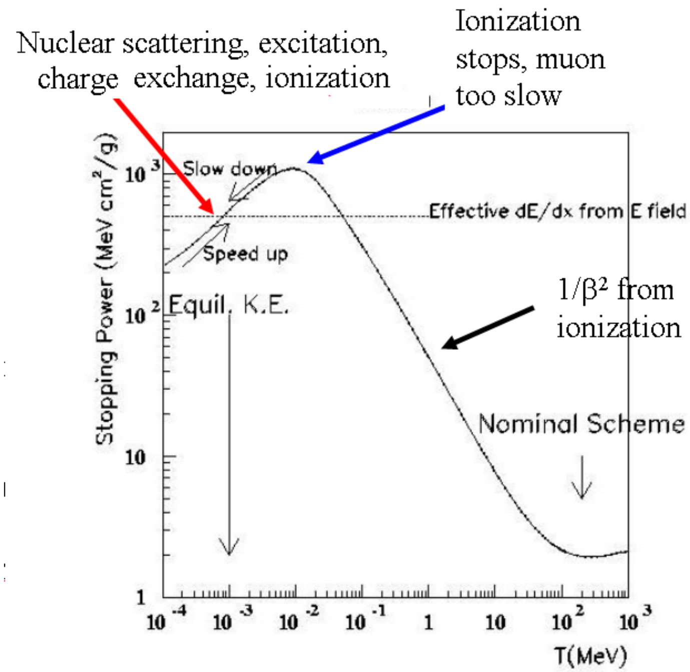
A. Garren, H Kirk

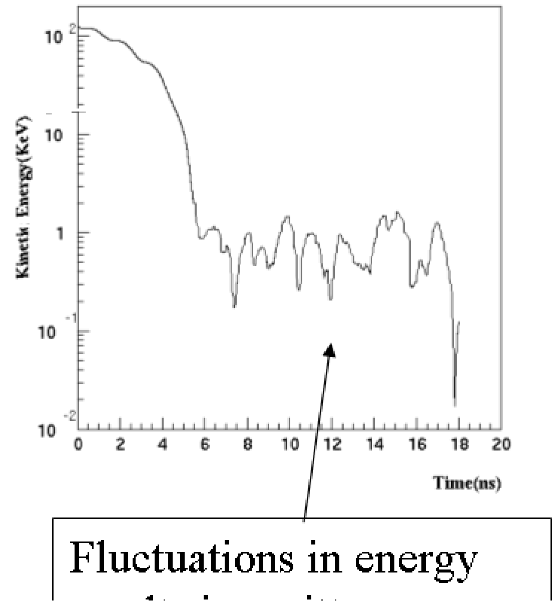


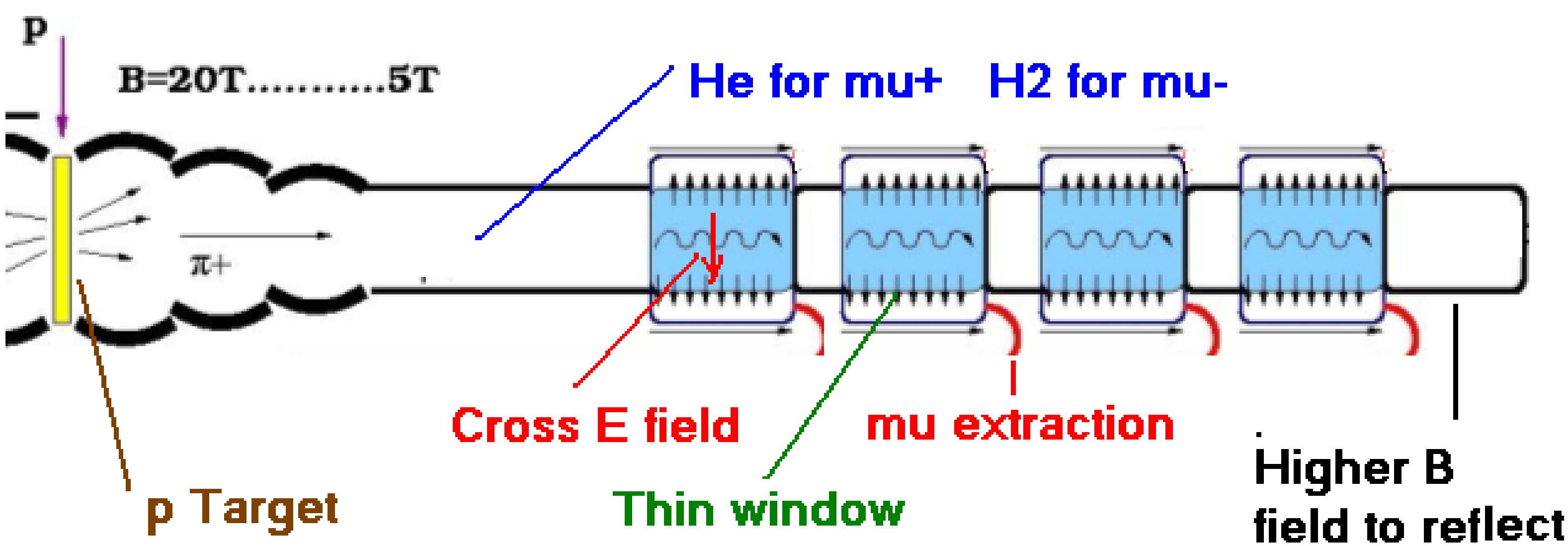
- 2 T fields
- 6 D cooling simulated
- Small: diam= 2 m
- Injection/Extraction hard
- Not as good as RFOFO Ring
- But Demonstration Experiment ?

Old idea: Friction Cooling

Caldwell, Columbia

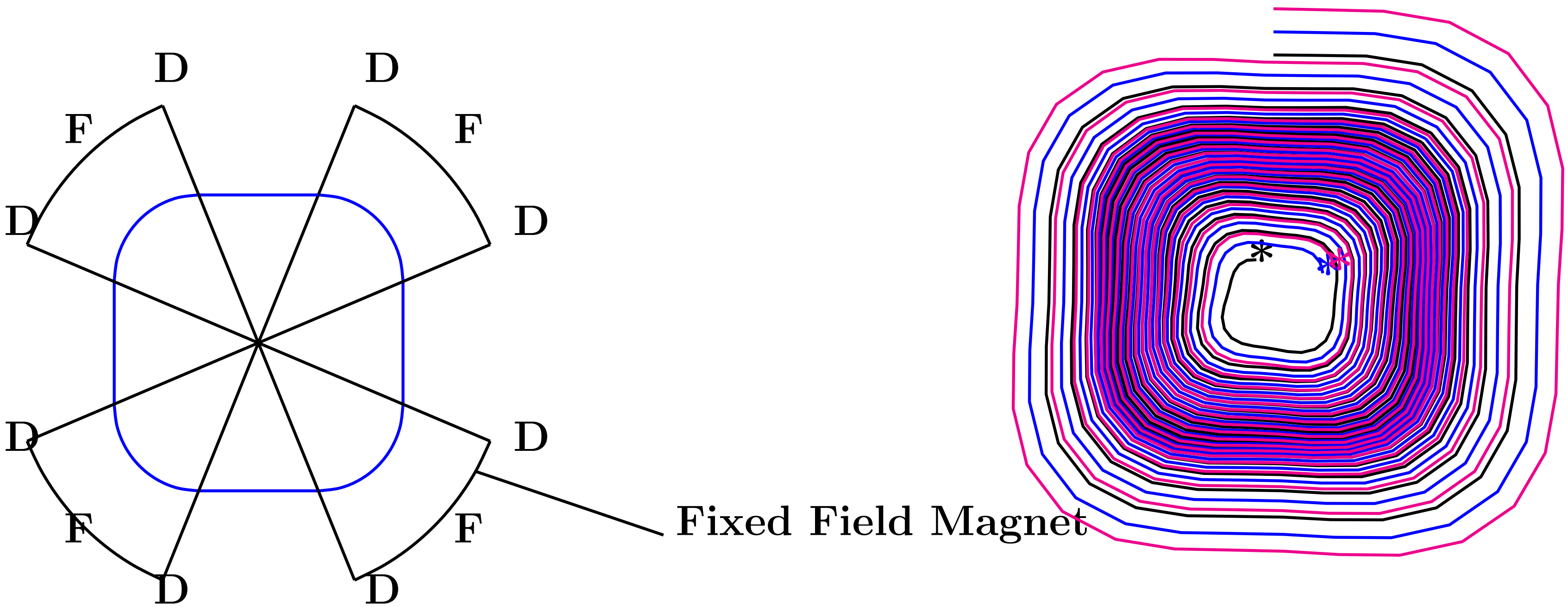






New idea: Inverse Cyclotron for Friction Cooling

D Summers, A Garren, H Kirk



- fine wedges and gas give graded density with radius
- Ionization Injection simulated
- Axial electric field extracts very cold muons (Caldwell)
- Smaller final volume than Caldwell scheme
- $\rightarrow$ Even Less final emittance ($<$ 50 pi mm mrad)
- Work In progress