Muon (g-2) Past and Future

Beam Dynamics in the
Muon (g-2) Storage Ring

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Outline of the Talk

• **Brief review of magnetic moments (including the theory of $a_\mu$)**
• **Spin motion in a magnetic field**
• **Overview of the experimental technique**
  – *The precision storage ring magnet*
  – *The fast muon kicker*
  – *The electrostatic quadrupoles*
• **Beam dynamics in the storage ring**
• **The new experiment E969**
• **Outstanding challenges for the future**
• **Summary and conclusions**
Muon: (2nd generation lepton)

Source: weak decay \[ \pi^- \rightarrow \mu^- \bar{\nu}_\mu \]

\[ \tau_\mu = 2.19703(4) \ \mu s \]

Parity Violating Decay \(\Rightarrow\) Polarized Muons

The Pion Rest Frame

\[ m_\mu c^2 = 105.658389(34) \ \text{MeV} \]

"Who ordered that?"
Magnetic Moments: $g$-factors, etc

\[ \vec{\mu}_s = g_s \left( \frac{e}{2m} \right) \vec{s} \]

- $\mu$ – magnetic moment
- $g$ – gyromagnetic ratio
- $s$ – spin

- **Dirac Equation predicts** $g \equiv 2$
- **In nature, radiative corrections make** $g \neq 2$

\[ g = 2 + \frac{\alpha}{\pi} \left( \mu + c_2 \left( \frac{\alpha}{\pi} \right)^2 \mu^* \right) + \ldots \]
Magnetic Moments – ctd.

\[ \mu = (1 + a) \left( \frac{e\hbar}{2m} \right) \text{ where } a = \left( \frac{g - 2}{2} \right) \]

\[ \mu_e = 1.001\,159\,652\,193 \frac{e\hbar}{2m_e} \]

\[ \mu_\mu = 1.001\,165\,923 \frac{e\hbar}{2m_\mu} \]

\[ \mu_p = 2.792\,847\,39 \frac{e\hbar}{2m_p}; \quad g_p \approx 5.586 \]
Unlike the EDM, there is a large SM value for the MDM

The Electron: to the level of the experimental error (4ppb),

\[ a_e(\text{Standard Model}) = a_e(\text{QED with } \gamma, e) \]

Contribution of \( e \), (or anything heavier than the electron) is \( \leq 4 \text{ ppb} \).

For the muon, the relative contribution of heavier particles

\[ \sim \left( \frac{m_\mu}{m_e} \right)^2 \sim 40,000 \quad \Rightarrow \]

\[ a_\mu(\text{SM}) = a_\mu(\text{QED}) + a_\mu(\text{hadronic}) + a_\mu(\text{weak}) \]
Standard Model Value for $(g-2)$

**QED**
- $11,658,470.57(29) \times 10^{-10}$

**Hadron**
- $692.4(6.2) \times 10^{-10}$
- $-10.1(6) \times 10^{-10}$

**Weak**
- $+38.9$
- $-19.4$
- $< 0.1$

$1st + 2nd \text{ Order Weak} = 15.1(4) \times 10^{-10}$

+ higher order terms
Lowest Order Hadronic contribution from $e^+e^-$ annihilation

\[ a_\mu(\text{had}) = \left( \frac{\alpha m_\mu}{3\pi} \right)^2 \int_{4m_\pi^2/s^2}^\infty ds K(s) \left( \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \right) \]
• Assume: CVC, no 2nd-class currents, isospin breaking corrections.

• n.b. \( \tau \) decay has no isoscalar piece, while \( e^+e^- \) does

• Many inconsistencies in comparison of \( e^+e^- \) and \( \tau \) decay:

  - Using \( e^+e^- \) data and CVC to predict \( \tau \) branching ratio gives \( 0.7 \) to \( 3.6 \) \( \sigma \) discrepancies with reality.

  - \( f_\pi \) from \( \tau \) decay has different shape from \( e^+e^- \).

\( a_\mu \) (Had) is very much a work in progress!
Present precision: $\pm 0.5 \text{ ppm}$

$$a_\mu = 11659208(6) \times 10^{-10} \ (0.5 \text{ ppm})$$

$$a_\mu = \frac{\omega a}{e/mc B}$$

All E821 results were obtained with a “blind” analysis.

$$\Delta a_\mu (E821 - \text{SM}) = (25.2 \text{ to } 26.0 \pm 9.4) \times 10^{-10}$$

An interesting, but not definitive discrepancy with theory.
Why people are interested: SUSY (large $\tan \beta$)

$$a_\mu (\text{SUSY}) \simeq \frac{\alpha (M_Z)}{8 \pi \sin^2 \theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan \beta \left(1 - \frac{4 \alpha}{\pi} \ln \frac{\tilde{m}}{m_\mu}\right)$$

$$\simeq (\text{sgn} \mu) \times 10^{-10} \tan \beta \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2$$
Traditionally,

- For many years, muon \((g-2)\) has provided strong and serious constraints on models of physics beyond the standard model.
Spin Precession Frequencies: $\mu$ in $B$ field

$$\omega_a = \omega_S - \omega_C = \left( \frac{g - 2}{2} \right) \frac{eB}{mc}$$

$$\omega_C = \frac{eB}{mc\gamma}$$

$$\omega_S = \frac{geB}{2mc} + (1 - \gamma) \frac{eB}{\gamma mc}$$

spin difference frequency = $\omega_s - \omega_c$

The highest energy decay $e^\pm$ are along the muon spin direction
Spin Precession Frequencies: $E$ and $B$ field

$$\vec{\omega}_a = -\frac{e}{m} \left[ a_\mu \vec{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right]$$

$\gamma_{\text{magic}} = 29.3$

$B \Rightarrow \langle B \rangle_{\mu-\text{dist}}$

Need to know $\langle B \rangle_{\mu-\text{dist}}$ to $< 0.1$ ppm
Experimental Technique

Protons \rightarrow \text{Pions} \rightarrow \text{Inflector}

\text{(from AGS)}

p=3.1\text{GeV/c}

- Muon polarization
- Muon storage ring
- injection & kicking
- focus by Electric Quadrupoles
- 24 electron calorimeters

\vec{\omega}_a = -\frac{e}{m} a_\mu \vec{B}

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The Beamline

AGS

U-V line

VD3
VD4

V line

Pion Production Target

U line

Pion Decay Channel

Beam Stop

K1-K2

D5

K3-K4

D6

Inflector

g-2 Ring
The Production Target
Decay Channel
Plan View of the Injection Line
mismatch between entrance channel and storage volume, plus an imperfect kick causes coherent beam oscillations
The Inflector

Length = 1.7 m
Central field = 1.45 T
Magnetic Circuits

\[ \Phi \int \frac{dl}{\mu A} = NI \]

Ohm’s law
Schematic of the Magnet

- Inner coil
- Thermal insulation
- Dipole correction coil
- Pole piece
- Wedge
- Pole bump
- Beam region
- Programmable current sheet
- Fixed NMR probes
- Outer coils
- Yoke

p = 7112 mm

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Winding the Coils
The Finished Coils
Coil Interconnect
Inserting a Pole Piece

Kapton insulation to prevent eddy currents from running around the ring, especially during an energy extraction or quench.
muon (g-2) storage ring
Mapping the Field

NMR B-field mapping trolley

Fixed probes monitor dipole and quadrupole field components

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\[ \langle B \rangle_\phi \text{ for 2001} \text{ averaged over azimuth} \]

0.5 ppm contours

\[ \sigma_{syst} \text{ on } \langle B \rangle_\mu \text{-distribution} = \pm 0.03 \text{ ppm} \]
The Kick

Horizontal (radial) Phase Space


x' ± 77mm

ring acceptance

inflector

stored beam x_c

Ideal kick

Beam exiting the inflector

x_c ≈ 77 mm

β ≈ 10 mrad

B·dl ≈ 0.1 Tm

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Injection Simulation

ring acceptance
ring acceptance
ring acceptance
ring acceptance
The Kicker Modulator

Fluorinert (FC40) oil
Kicker Plate Geometry

electrodes
The Kicker Current Pulse

Eddy currents less than 0.1 ppm on Bd 1 after 20 µs

Measured with the Faraday effect.
The Electrostatic Quadrupoles: $\mu^+$ polarity

~ ± 24 kV at full power, 17 kV for beam scraping after injection
The Ring Layout

- Inflector
- Trolley drive
- Traceback chambers
- Trolley garage
- Cryo pump
- Q1
- Q2
- Q3
- Q4

Quads cover 43% of the ring
Scraping the Beam

- $V_0 = \pm 24 \text{ kV}$
- $V_s = 17 \text{ kV}$
- Beam is lifted and moved sideways
- Scraped on collimators to minimize losses
Ring $\beta$-Function

\[ x(s) = A \sqrt{\beta_x} \cos(\psi + \delta) \]
\[ \psi(s) = \sqrt{K_s} \]

\[ \sqrt{\frac{\beta_{\text{max}}}{\beta_{\text{min}}}} = 1.03 \]

for 4-fold symmetry
Weak Focusing Betatron

Field index: \( n = \frac{\kappa R_0}{\beta B_0} \approx 0.135 \)

\[
f_y = f_C \sqrt{n} \approx 0.37 f_C ;
\]

\[
f_x = f_C \sqrt{1 - n} \approx 0.929 f_C
\]

• Detector acceptance depends on the radial coordinate \( x \). The beam moves coherently radially relative to a detector with the "Coherent Betatron Frequency" (CBO)

\[
f_{\text{CBO}} = f_C - f_x = (1 - \sqrt{1 - n}) f_C
\]
Coherent Betatron Frequency

\[ f_{CBO} = f_C - f_x = (1 - \sqrt{1 - n}) f_C \]

\[ \lambda_{CBO} \approx 14 \text{ turns} \]

CBO amplitude modulates the signal in the detectors.
\[ \nu_x = \sqrt{1 - n} \]
\[ \nu_y = \sqrt{n} \]
Muon Decay

$\mu$-decay: parity violating

$$\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$$

The Muon Rest Frame

Highest energy $e^+$ are along muon spin

The positron carries the muon spin
Electron Detectors

Muon momentum

Muon spin

Sci-Fi Calorimeter module

Measures Energy and time

Spin forward, more high energy e

Spin backward, less high energy e

400 MHz digitizer
$4 \times 10^9 \ e^-, \ E_{e^-} \geq 1.8 \ \text{GeV}$

$f(t) = N_0 e^{-\lambda t} \left[1 + A \cos \omega a t + \phi \right]$
In the 1999 Data Set: A Surprise

Nature gives us 5 parameters:

\[ f(t) = N_0 e^{-\lambda t} \left[ 1 + A \cos(\omega_0 t + \phi) \right] \]

Storage ring plus bunched beam gives us more:

Fourier Transform of the residuals from a 5-parameter fit (from 1 detector).
# Frequencies in the $(g-2)$ Ring

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Expression</th>
<th>Frequency</th>
<th>Period</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_a$</td>
<td>$\frac{e}{2\pi mc} a_\mu B$</td>
<td>0.23 MHz</td>
<td>4.37 µs</td>
</tr>
<tr>
<td>$f_c$</td>
<td>$\frac{v}{2\pi R_0}$</td>
<td>6.7 MHz</td>
<td>149 ns</td>
</tr>
<tr>
<td>$f_x$</td>
<td>$\sqrt{1 - n f_c}$</td>
<td>6.23 MHz</td>
<td>160 ns</td>
</tr>
<tr>
<td>$f_y$</td>
<td>$\sqrt{n f_c}$</td>
<td>2.48 MHz</td>
<td>402 ns</td>
</tr>
<tr>
<td>$f_{CBO}$</td>
<td>$f_c - f_x$</td>
<td>0.477 MHz</td>
<td>2.10 µs</td>
</tr>
<tr>
<td>$f_{VW}$</td>
<td>$f_c - 2f_y$</td>
<td>1.74 MHz</td>
<td>0.574 µs</td>
</tr>
</tbody>
</table>
Fiber Beam Monitors

x monitor
callbrate

y monitor
callbrate

x fiber

Harp 1 (Horizontal), Fiber 4

ns
Measuring the Tune

**x fiber**

Harp 1 (Horizontal), Fiber 4

**y fiber**

Harp 2 (Vertical), Fiber 4
The Tunes During Scraping

- The tune change with scraping is clearly visible from the fiber harps

![Graph showing the change in tune with scraping](image)
CBO in the 2001 Data Set

\[ f(t) = N_0 e^{-\lambda t} \left[ 1 + A \cos(\omega_at + \phi) \right] \]

Residuals from fitting the 5-parameter function
Beam Debunching after Injection

\[ e^+ \text{ Time Spectrum: } t = 6 \, \mu s \]

\[ e^+ \text{ Time Spectrum: } t = 36 \, \mu s \]

\[ \frac{\Delta p}{p} = \pm 0.5\% \]
Fourier Transform vs. Debunching Model

Debunching model

modified FT

Arbitrary units

<\textit{R}>= 711.436 \text{ cm}, \text{ RMS Width} = 0.955

<\textit{R}>= 711.460 \text{ cm}, \text{ RMS Width} = 0.999

Magic Radius 711.2 cm
Peak 711.093 cm

Radius [cm]
Exclusion/Limitations on New Physics

\[ \Delta a_\mu (\text{New Physics}) = a_\mu (\text{SM}) - a_\mu (\text{Exp}) \]
Can we improve the sensitivity of this confrontation between experiment and theory?

- **Yes**
  - E969 at BNL has scientific approval to go from 0.5 ppm → 0.2 ppm
  - funding decision will be made in spring 2006

Will Theory Improve beyond 0.6 ppm?

- **Yes**
  - better R measurements from: KLOE, BaBar, Belle, SND and CMD2 at Novosibirsk
  - More work on the strong interaction

- Theory could eventually improve to ~0.2 ppm
Strategy of the improved experiment

- More muons – E821 was statistics limited $\sigma_{\text{stat}} = 0.46$ ppm, $\sigma_{\text{syst}} = 0.3$ ppm
  - Backward-decay, higher-transmission beamline
  - Double the quadrupoles in the $\pi$ decay line
E821: forward decay beam

This baseline limits how early we can fit data
E969: backward decay beam

Approximately the same muon flux is realized
Then we double the number of quadrupoles in the decay channel

Expect for both sides
No hadron-induced prompt flash

x 2

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Improved transmission into the ring

Inflector aperture

Storage ring aperture

E821 Closed End

P969 Proposed Open End

x 2
### E969: Systematic Error Goal

<table>
<thead>
<tr>
<th>Systematic uncertainty (ppm)</th>
<th>1998</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>E969 Goal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Magnetic field – $\omega_p$</td>
<td>0.5</td>
<td>0.4</td>
<td>0.24</td>
<td>0.17</td>
<td>0.1</td>
</tr>
<tr>
<td>Anomalous precession – $\omega_a$</td>
<td>0.8</td>
<td>0.3</td>
<td>0.3</td>
<td>0.21</td>
<td>0.1</td>
</tr>
</tbody>
</table>
## Systematic errors on $\alpha$ (ppm)

<table>
<thead>
<tr>
<th>systematic</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile-up</td>
<td>0.13</td>
<td>0.13</td>
<td>0.08</td>
</tr>
<tr>
<td>AGS Background</td>
<td>0.10</td>
<td>0.10</td>
<td>*</td>
</tr>
<tr>
<td>Lost Muons</td>
<td>0.10</td>
<td>0.10</td>
<td>0.09</td>
</tr>
<tr>
<td>Timing Shifts</td>
<td>0.10</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>E-Field, Pitch</td>
<td>0.08</td>
<td>0.03</td>
<td>*</td>
</tr>
<tr>
<td>Fitting/Binning</td>
<td>0.07</td>
<td>0.06</td>
<td>*</td>
</tr>
<tr>
<td>CBO</td>
<td>0.05</td>
<td>0.21</td>
<td>0.07</td>
</tr>
<tr>
<td>Beam Debunching</td>
<td>0.04</td>
<td>0.04</td>
<td>*</td>
</tr>
<tr>
<td>Gain Change</td>
<td>0.02</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td><strong>total</strong></td>
<td><strong>0.3</strong></td>
<td><strong>0.31</strong></td>
<td><strong>0.21</strong></td>
</tr>
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* = 0.11
Timescales in the ring

- Muon lifetime $\tau_\mu = 64.4 \mu s$

- Cyclotron period $\tau_C = 149$ ns

- Scraping time (E821) 7 to 15 $\mu$ s

- Total counting time $\sim 700 \mu$ s

- Total number of turns $\sim 4000$
Relative Amplitude of the CBO effect

<table>
<thead>
<tr>
<th>systematic</th>
<th>1999</th>
<th>2000</th>
<th>2001</th>
<th>E969</th>
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<tr>
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<td>0.21</td>
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<td>0.31</td>
<td>0.21</td>
<td>0.11</td>
</tr>
</tbody>
</table>
Lost Muons and CBO are Major Issues

- Two schemes proposed to eliminate CBO and losses
  - Drive CBO with an oscillating dipole to scrape, then slip the phase by $\pi$ and damp it
    - Suggested by Yuri Orlov
  - Pulsed Octupole for 30 turns
    - Suggested by Yuri Shatunov
Oscillating Dipole Solution

• Use Fiber Harps to measure phase of CBO

Sample Parameters

$L = 0.5 \text{ m}$
$N = 20 \text{ turns}$
$E_{x0} = 7.4 \text{ kV/cm}$
$f = 470 \text{ kHz}$
CBO cure: betatron phases mixing by nonlinear fields

\[ \Delta B_x + i \Delta B_y = B_0 \sum_n (b_n + ia_n)(x + iy)^n \]

octupole: \((n = 3)\)

\[ \beta \left( \frac{\partial \nu}{\partial a^2} \right) \approx \frac{3}{8} \left\langle \beta^2 b_3 \right\rangle_s \]

\[ B_x = O(s)(y^3 - 3x^2y) \]

\[ B_y = O(s)(x^3 - 3y^2x) \]

Y. Shatunov, SPIN04
Octupole coil and parameters of generator

Coil length: 16 × 2 m
Current: 2.5 kA
Capacitor: 1 μF
Voltage: 1.3 kV
Energy: 1.0 J
Half period: 10 μsec

I(t) injection

μsec

Y. Shatunov, SPIN04
Muon population

CBO damping
Challenges with Octupole

• Eddy currents affecting $B_0$?
  – We can only tolerate effects on $B_d$ at the 0.05 ppm level

• Too many muons lost?
Summary

• **E821 at BNL** achieved 0.54 ppm relative accuracy on $a_\mu$
  – 0.46 ppm statistical
  – 0.28 ppm systematic

• This represents a factor of 14 over the CERN experiment
Where we came from:

![Diagram showing a value of $a_u$ at 116,592,000 and a value of (9.4 ppm) at CERN $\mu^+$ and (10 ppm) at CERN $\mu^-$, with a range suggesting around 1983.]

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Today with $e^+e^-$ based theory:

All E821 results were obtained with a “blind” analysis.

$\sim 2.7 \sigma$

difference with $e^+e^-$ SM value

$$a_\mu = 11659208(6) \times 10^{-10} (0.5 \text{ ppm})$$
Summary

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• **E969 Aims to achieve an additional factor of 2.5**
  – from 0.5 ppm \(\rightarrow\) 0.2 ppm
• Will more than double the physics reach when confronting theory

Please come join us on E969!
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Thank you