Beam-Beam Interaction

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- Pinch Effect
- Beamstrahlung
- Imperfect Collisions
- Banana Effect
- Secondary Production
- Luminosity Monitoring

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Beam Parameters at Collision

Parameter	Unit	ILC nominal	CLIC
E_{cm}	GeV	500	3000
$\mathcal{L}$	$10^{34}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$	2.0	5.7
N	10^9	20	3.7
σ_x^*	nm	655	40
σ_y^*	nm	5.7	1
σ_z	μm	300	45
n_b		2820	312
f_r	Hz	5	50
Δz	ns	300	0.5
θ_c	mradian	(20)	20
n_γ		1.3	2
$\Delta E/E$	%	2.4	30

In the following, beam sizes are always given at the interaction point

The beams are flat this in order to achieve high luminosity (small $\sigma_x \times \sigma_y$) and low beamstrahlung (large $\sigma_x + \sigma_y$)

The luminosity is given by

$$\mathcal{L} = \frac{N^2 f_r n_b}{4\pi\sigma_x\sigma_y}$$

so what does limit it?

Luminosity

Luminosity is given by (assuming rigid beams, no hour glass effect)

$$\mathcal{L} = H_D \frac{N^2 f_r n_b}{4\pi\sigma_x\sigma_y} \frac{1}{\sqrt{1 + \left(\frac{\sigma_z}{\sigma_x} \tan\frac{\theta_c}{2}\right)^2}}$$

Ignore crossing angle and H_D, yields

$$\mathcal{L} = \frac{N}{4\pi\sigma_x\sigma_y} N f_r n_b \propto \frac{N}{\sigma_x\sigma_y} P_{beam}$$

Can we ignore the crossing angle?

$\Rightarrow$ Need to minimise beam cross section, limits due to

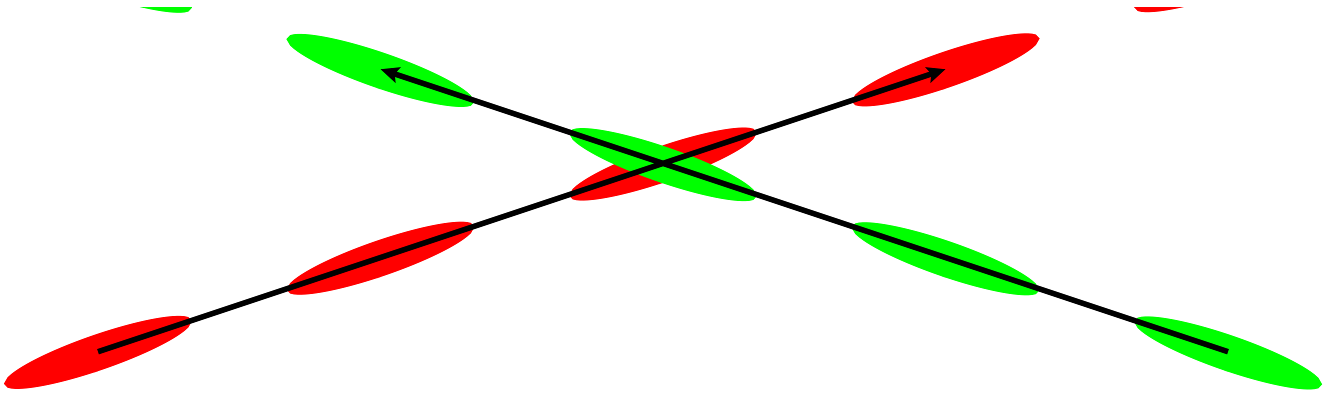
- hour glass effect
- beamstrahlung
- stability

Crossing Angle

A crossing angle between the beams can be required

- to minimise effects of parasitic crossings of bunches

- to be able to cleanly get rid of the spent beam



In a normal conducting machine, the short bunch spacing leaves no choice but to have a crossing angle

In a superconducting machine one can in principle avoid a crossing angle

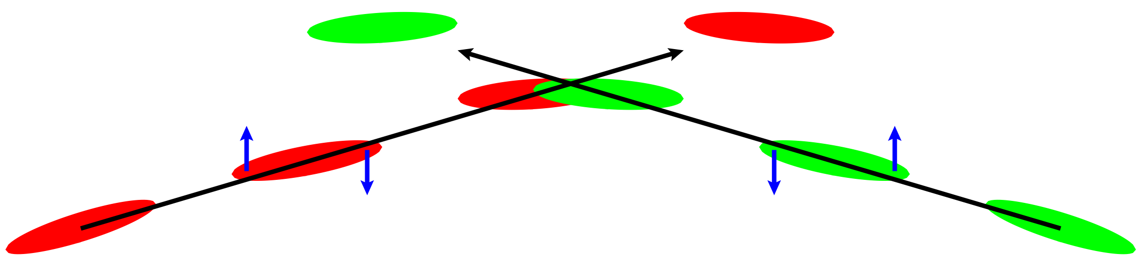
Crab Crossing

The crossing angle θ_c can lead to a luminosity reduction

$$\frac{L}{L_0} = \frac{1}{\sqrt{1 + \left(\frac{\sigma_z}{\sigma_x}\tan\frac{\theta_c}{2}\right)^2}}$$

This can be avoided using the "crab crossing" scheme

- a rotation is introduced into the bunch which makes it straight at collision



From the beam-beam point of view crab crossing can be treated as no crossing angle

need to transform secondaries into laboratory frame

Beam Size Limitation 1: Hour Glass Effect

We can rewrite the beam size at the IP as

$$\mathcal{L} \propto \frac{N}{\sigma_x \sigma_y} P_{beam} = \frac{N}{\sqrt{\beta_x \epsilon_x}\sqrt{\beta_y \epsilon_y}} P_{beam}$$

The emittances $\epsilon_{x,y}$ are beam properties, smaller ϵ is more demanding for the other systems

The beta-functions β are properties of the focusing system

Stronger focusing (lower β) can increase the luminosity

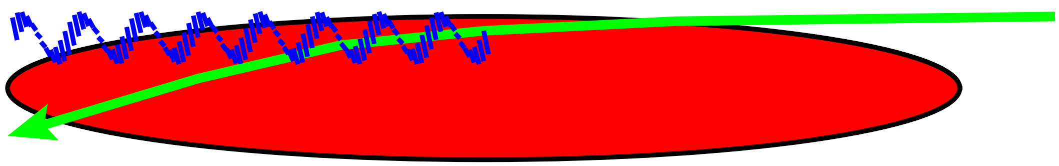
Too low β reduces luminosity due to hour glass effect

$$\sigma_{x,y}(z) = \sqrt{\beta_{x,y}\epsilon_{x,y} + z^2/\beta_{x,y}\epsilon_{x,y}} = \sigma^*_{x,y}\sqrt{1 + z^2/\beta^2_{x,y}}$$

$\Rightarrow$ Lower limit $\beta \geq \sigma_z$

$\Rightarrow$ We will see that this limit is important for the vertical plane, not for the horizontal

Beam Size Limitation 2: Beam-Beam Interaction



The beam is ultra-relativistic

⇒ the fields are almost completely transverse

Due to the high density the electro-magnetic beam fields are high

⇒ focus the incoming beam (electric and magnetic force add)

⇒ reduction of beam crossection leads to more luminosity

⇒ bending of the trajectories leads to emission of beamstrahlung

The increase in luminosity will be expressed by a factor H_D, the luminosity enhancement factor

Disruption Parameter

We consider the motion of one particle in the field of the oncoming bunch and make the following assumptions

- the bunch transverse distribution is Gaussian, with widths σ_x and σ_y
- the particle is close to the beam axis
- the initial particle transverse momentum is zero
- the particle does not move transversely

We obtain for the final particle angle

$$\left.\frac{dx}{dz}\right|_{final} = -\frac{2Nr_e x}{\gamma\sigma_x(\sigma_x+\sigma_y)} \qquad \left.\frac{dy}{dz}\right|_{final} = -\frac{2Nr_e y}{\gamma\sigma_y(\sigma_x+\sigma_y)}$$

$\Rightarrow$ Beam acts as a focusing lens

We introduce the disruption parameter $D_{x,y} = \sigma_z/f_{x,y}$, where $f_{x,y}$ is the focal length

$$D_x = \frac{2Nr_e\sigma_z}{\gamma\sigma_x(\sigma_x+\sigma_y)} \qquad D_y = \frac{2Nr_e\sigma_z}{\gamma\sigma_y(\sigma_x+\sigma_y)}$$

Relevance of the Disruption Parameter

A small disruption parameter $D \ll 1$ indicates that the beam acts as a thin lens on the other beam

A large disruption parameter $D \gg 1$ indicates that the particle oscillates in the field of the oncoming beam

$\Rightarrow$ the notion of the parameter as the ratio of focal length to bunch length is no longer valid, the parameter is still useful

$\Rightarrow$ Since the particles in both beams will start to oscillate, the analytic estimation of the effects becomes tedious

$\Rightarrow$ resort to simulations

In linear colliders one usually finds $D_x \ll 1$ and $D_y \gg 1$

ILC: $D_x \approx 0.15$, $D_y \approx 18$, CLIC: $D_x \approx 0.2$, $D_y \approx 7.6$

Simulation Procedure

Two widely spread codes to simulate the beam-beam interaction are CAIN (K. Yokoya et al.) and GUINEA-PIG (D. Schulte et al.)

- The beam is represented by macro particles
- It is cut longitudinally into slices
- Each slice interacts with one slice of the other beam at a given time
- The slices are cut into cells
- The simulation is performed in a number of time steps in each of them
  - The macro-particle charges are distributed over the cells
  - The forces at the cell locations are calculated
  - The forces are applied to the macro particles
  - The particles are advanced

Beamstrahlung

Particles travel on curved trajectories

$\Rightarrow$ emitt radiation similar to synchrotron radiation

$\Rightarrow$ called beamstrahlung in this context

Beamstrahlung reduces the beam particle energy

$\Rightarrow$ particles collide at energies different from the nominal one

$\Rightarrow$ physics cross section are affected

$\Rightarrow$ threshold scans are affected

Beamstrahlung is not the only relevant process

Synchrotron Radiation vs. Beamstrahlung

Quantum mechanics: particle can scatter in field of individual particles and in collective field of oncoming bunch

Condition for application of synchrotron radiation formulae is that the collective field of the oncoming beam particles is important

- integrate over field of many particles during coherence length

- travel many coherence lengths during bunch passage

Beamstrahlung opening cone is roughly given by $1/\gamma$

$\Rightarrow$ coherence length is the distance traveled while particle is deflected by $1/\gamma$

$\Rightarrow$ Number of coherence lengths

$$\eta = \gamma\theta_x = D_x \frac{\sigma_x}{\sigma_z}\gamma = \frac{2Nr_e}{\sigma_x + \sigma_y}$$

$\Rightarrow$ Usually of the order of several tens or hundreds $\Rightarrow$ OK

Beamstrahlung Description

- Synchrotron radiation is characterised by the critical energy

$$\omega_c = \frac{3}{2}\frac{\gamma^3 c}{\rho}$$

ρ is bending radius

- Beamstrahlung is often characterised using the beamstrahlung parameter Υ

$$\Upsilon = \frac{2}{3}\frac{\hbar\omega_c}{E_0}$$

Υ is the ratio of critical energy to beam energy (times 2/3)

The average value can be estimated as (for Gaussian beams)

$$\langle\Upsilon\rangle = \frac{5}{6}\frac{N r_e^2 \gamma}{\alpha(\sigma_x + \sigma_y)\sigma_z}$$

Emission Spectrum

Sokolov-Ternov spectrum

$$\frac{\mathrm{d}\,\dot{w}}{\mathrm{d}\,\omega} = \frac{\alpha}{\sqrt{3}\pi\gamma^2}\left[\int_x^\infty \mathrm{K}_{\frac{5}{3}}(x')\mathrm{d}x' + \frac{\hbar\omega}{E}\frac{\hbar\omega}{E-\hbar\omega}\mathrm{K}_{\frac{2}{3}}(x)\right]$$

$x = \frac{\omega}{\omega_c}\frac{E}{E-\hbar\omega}$

$K_{5/3}$ and $K_{2/3}$ are the modified Bessel functions

For small Υ

$$\Delta E \propto \Upsilon^2\sigma_z \propto \frac{N}{(\sigma_x+\sigma_y)}\frac{N}{(\sigma_x+\sigma_y)\sigma_z}$$

$\Rightarrow$ Use flat beams

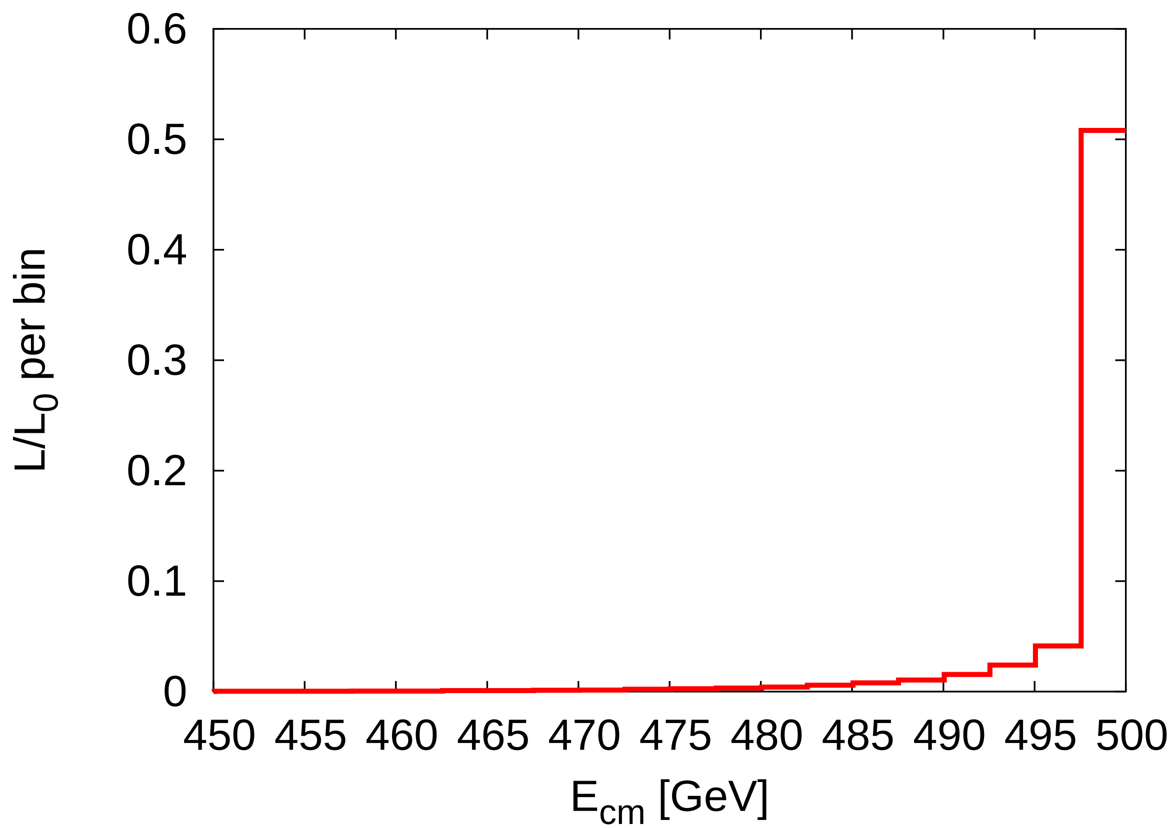
Typically the number of photons per beam particle n_γ is of order unity, $\delta E/E$ is of the order of a few percent

Luminosity Spectrum

The luminosity is still peaked at the nominal centre-of-mass energy

But the reduction is very signififcant

The importance will depend on the phyiscs process you want to measure

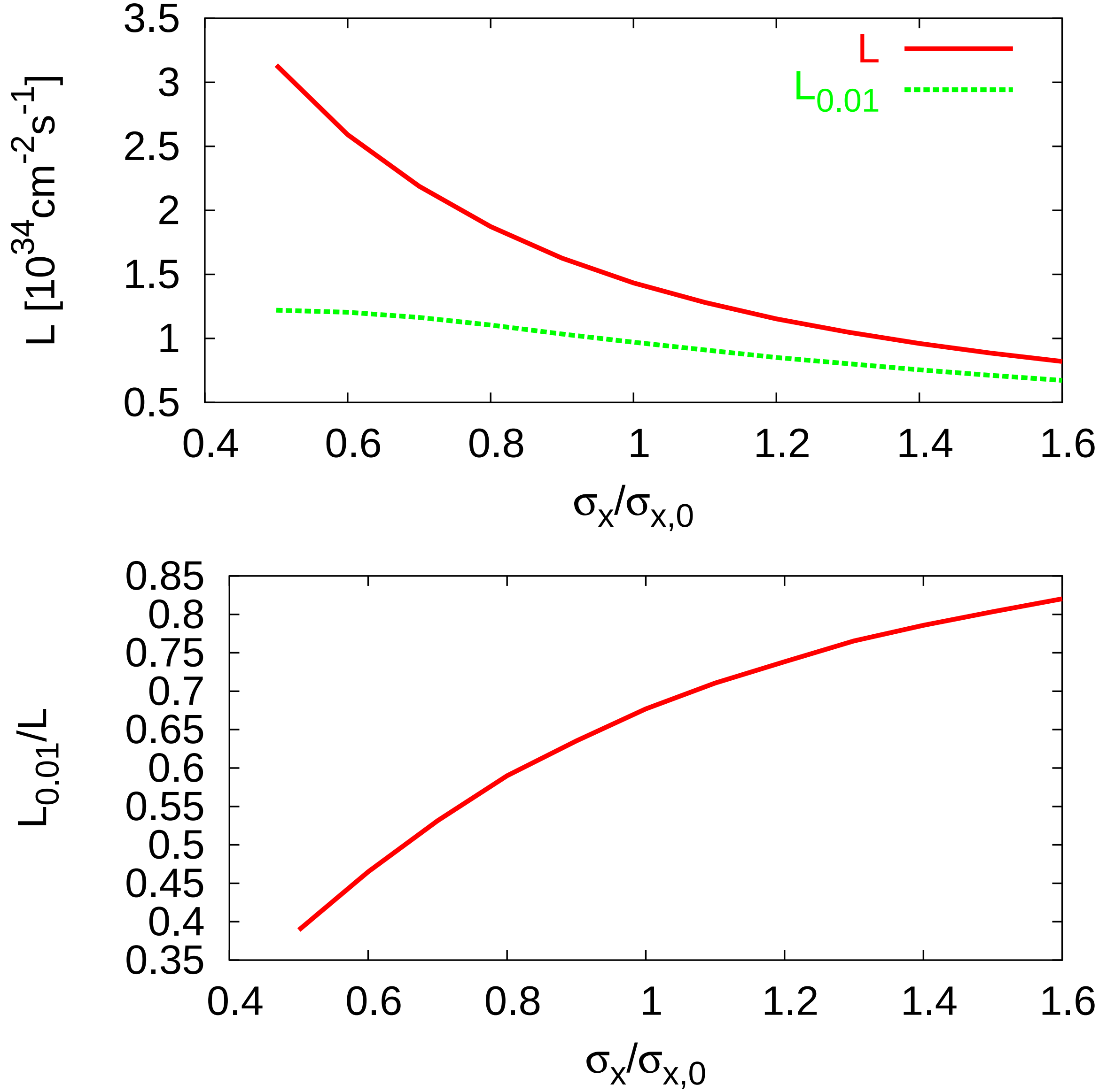


Spectrum Quality vs. Luminosity

By modifying the horizontal beam size one can trade luminosity vs spectrum quality

Variation is around nominal ILC parameter

$\Rightarrow$ Need a way to determine which ΔE is acceptable



Initial State Radiation

Colliding particles can emit photons during the collision

$\Rightarrow$ the collision energies are modified

$\Rightarrow$ e.g. important at LEP

The beam particles can be represented by a spectrum $f_e^e(x, Q^2)$

$\Rightarrow$ the probability that the particle collides with a fraction x of its energy at a scale Q^2

$$f_e^e(x, Q^2) = \frac{\beta}{2}(1-x)^{(\frac{\beta}{2}-1)}\left(1+\frac{3}{8}\beta\right) - \frac{\beta}{4}(1+x)$$

$$\beta = \frac{2\alpha}{\pi}\left(\ln\frac{Q^2}{m^2} - 1\right)$$

The scale Q^2 depends on the actual interaction process of the colliding particles

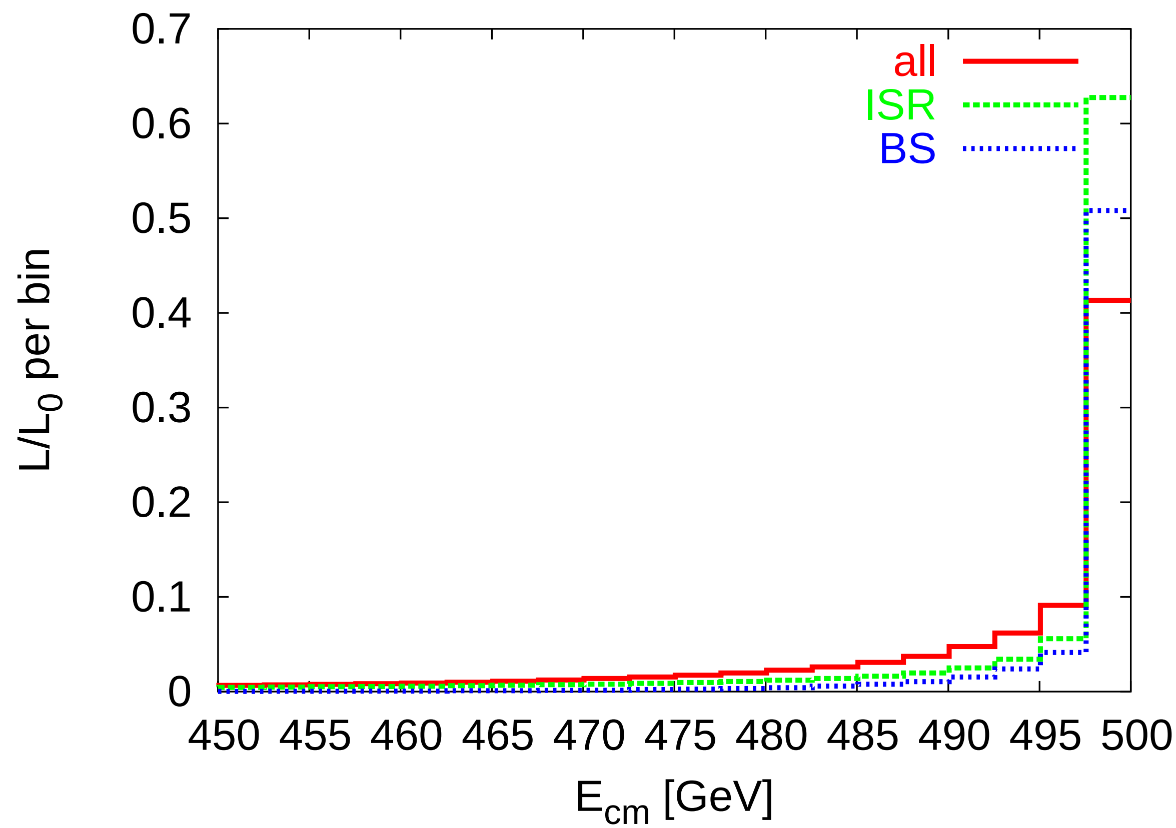
For central production processes $Q^2 = s = 4E_{cm}^2$ can be used

Comparison of Radiation Processes

Initial state radiation and beamstrahlung lead to similar reduction of the luminosity close to the nominal energy

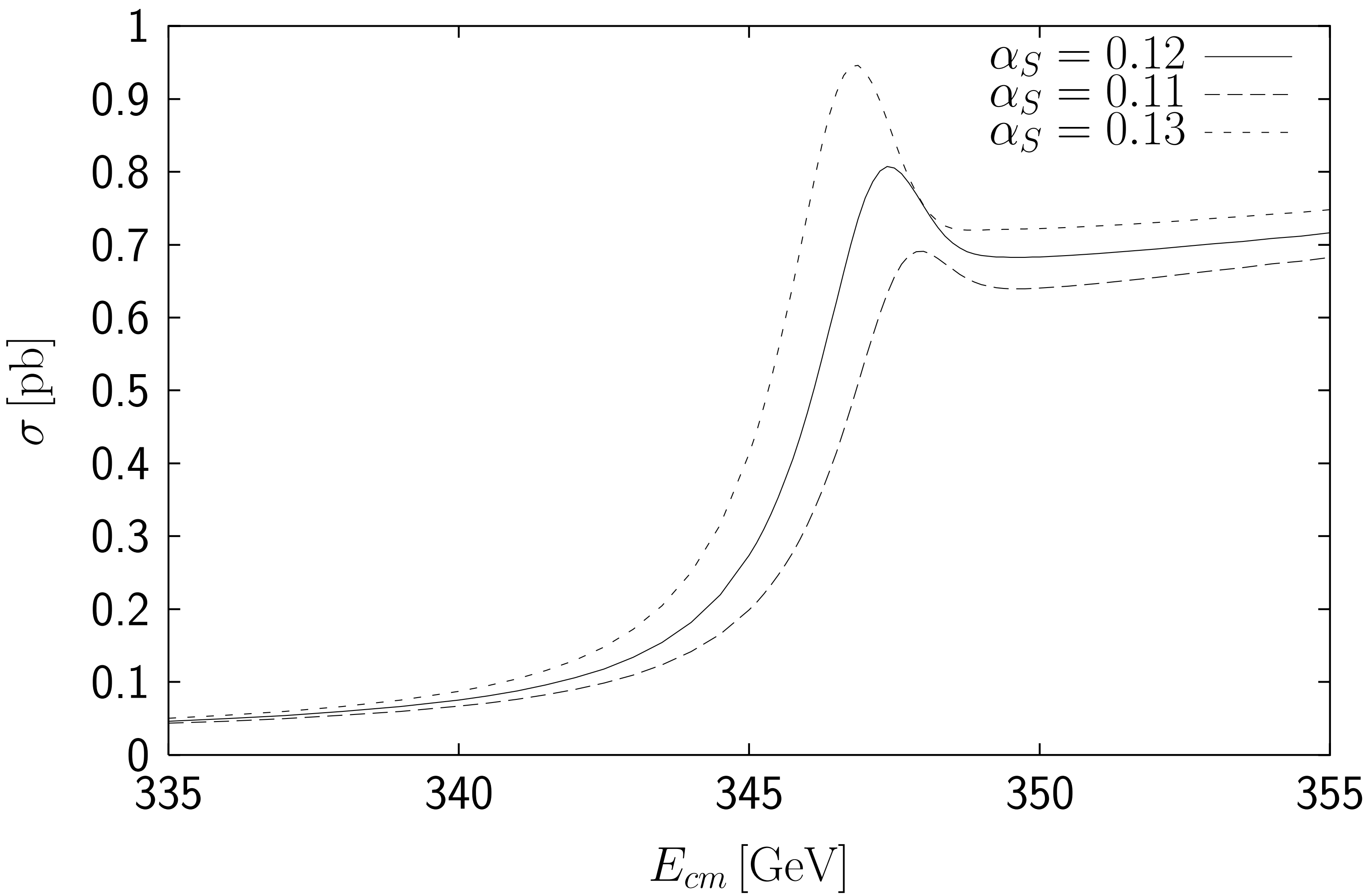
Initial State radiation can be calculated

Beamstrahlung depends on beam parameters, requires careful measurement of relevant parameters

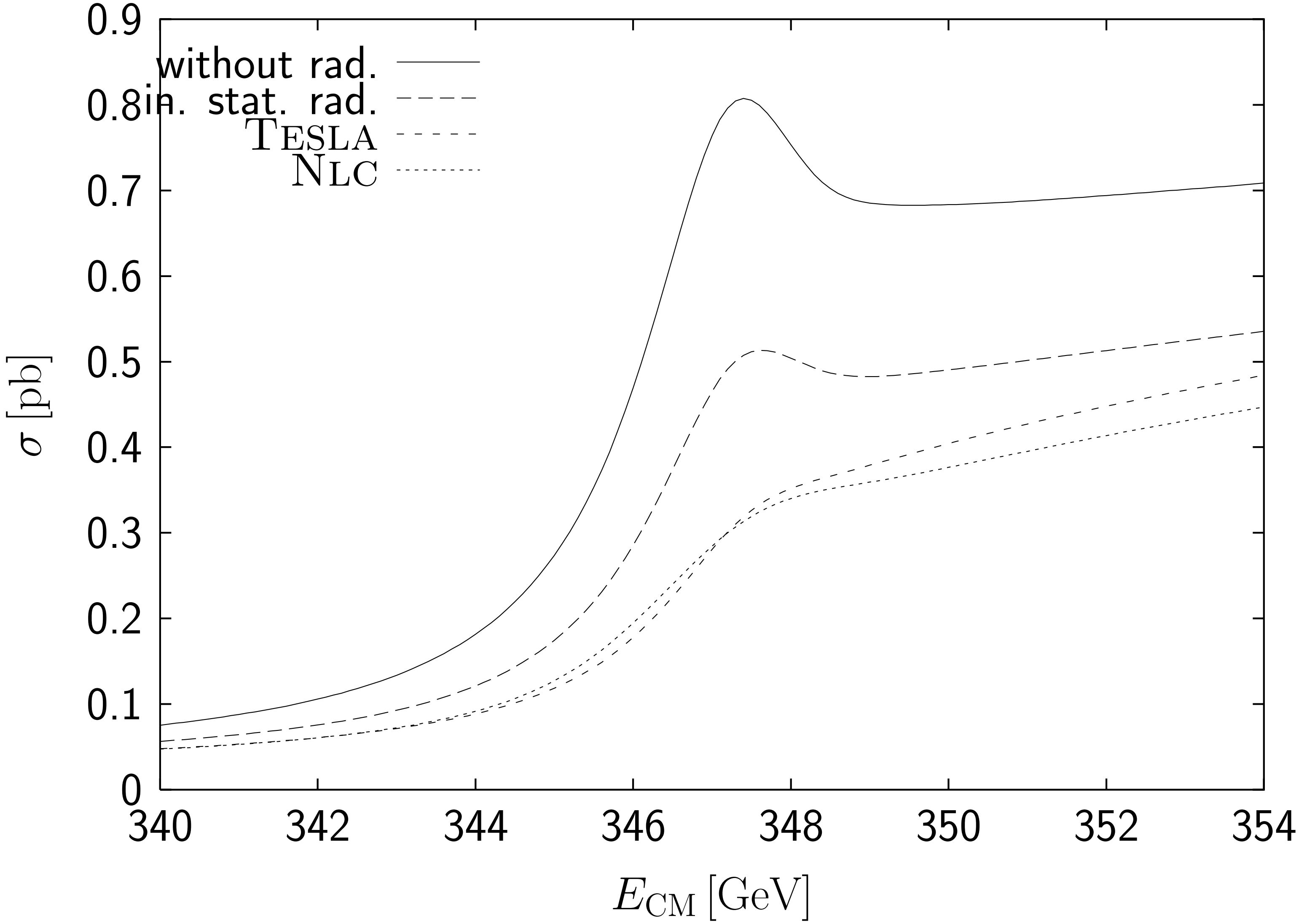


Relative luminosity spectrum, considering beamstrahlung (BS), initial state radiation (ISR) and both (all)

Example of Impact of Beamstrahlung: Top Threshold Scan



Example of Impact of Beamstrahlung



Keeping the Beams in Collision

The vertical beam size is very small (few nm)

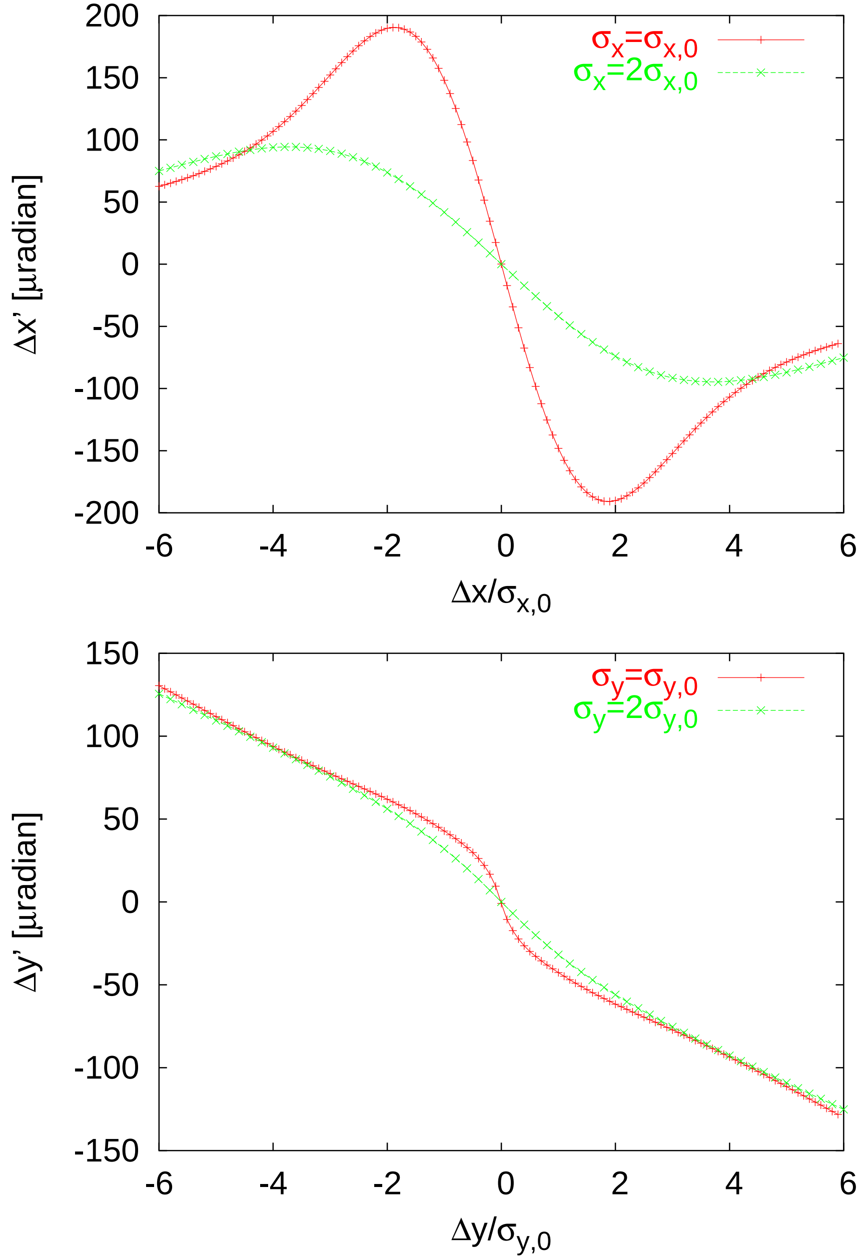
Even ground motion effects become important at this level

nm-offsets lead to tens of μradian deflection angles

$\Rightarrow$ can be measured with BPM and used for feedback

$\Rightarrow$ in ILC intra-pulse feedback is possible

$\Rightarrow$ in CLIC this will be tough



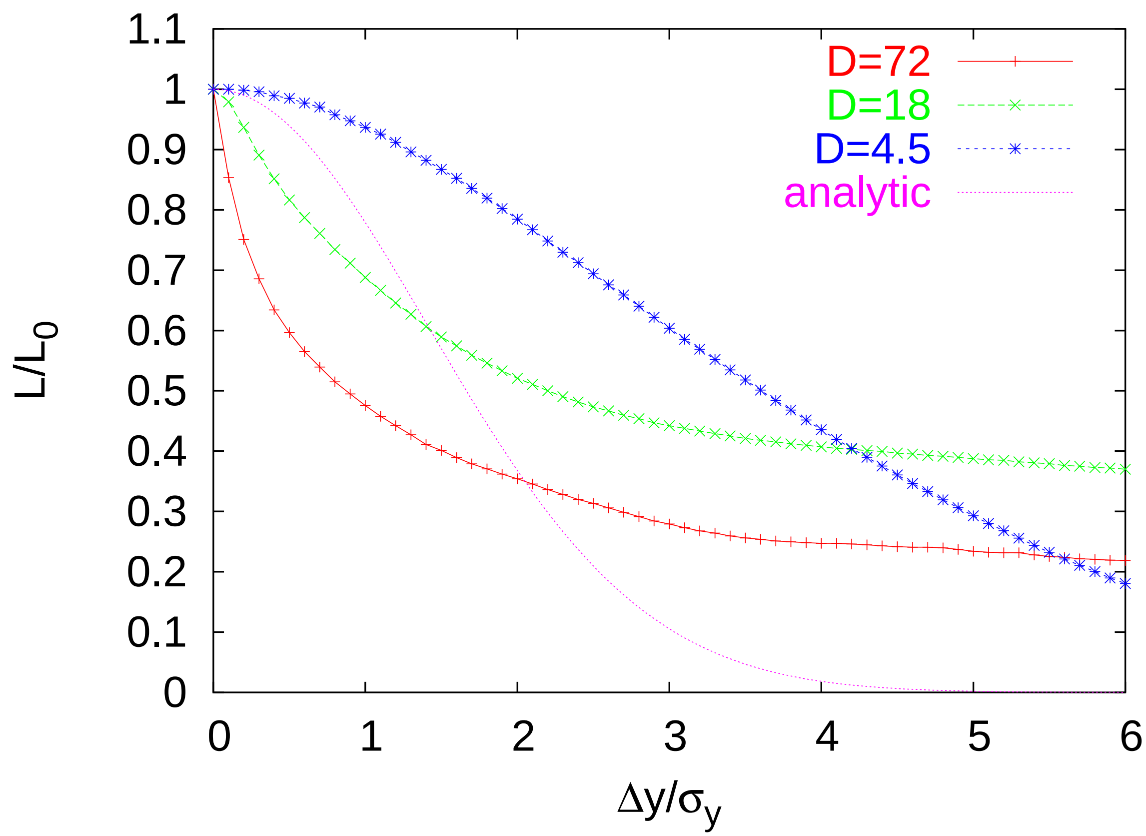
Luminosity as a Function of Offset

Neglecting beam waist one can estimate for rigid bunches from the overlap of Gaussian distributions

$$\frac{\mathcal{L}}{\mathcal{L}_0} = \exp\left(-\frac{\Delta y^2}{4\sigma_y^2}\right)$$

The beam-beam forces modify this

$\Rightarrow$ the beams attract each other



$\Rightarrow$ If the disruption parameter is very large, we are more sensitive to beam offsets

Choice of Disruption Parameter

Evidently a large disruption parameter makes the beam more sensitive to offsets

$\Rightarrow$ one should limit the disruption

But, the vertical disruption parameter is a function of the luminosity

Assuming $\sigma_x \gg \sigma_y$ one can calculate

$$\mathcal{L} = H_D \frac{N}{4\pi\sigma_x\sigma_y} P_{beam} = a \frac{N}{\sigma_x\sigma_y}$$

$$\mathcal{L} = a \frac{2Nr_e\sigma_z}{\gamma\sigma_y(\sigma_x + \sigma_y)} \frac{\gamma}{2r_e\sigma_z} \frac{\sigma_x + \sigma_y}{\sigma_x}$$

$$\boxed{\mathcal{L} \approx b \frac{D_y}{\sigma_z}}$$

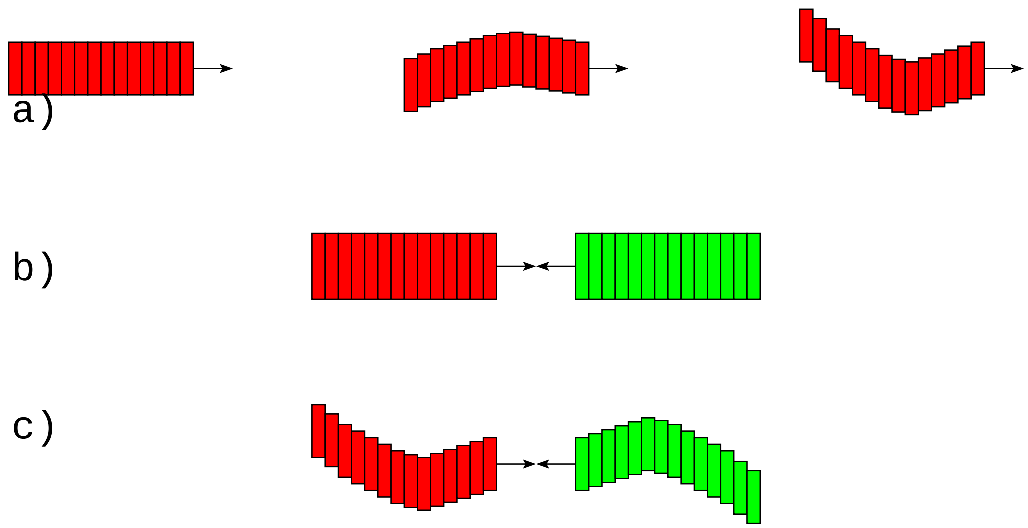
$\Rightarrow$ A long bunch requires a high vertical disruption parameter to reach high luminosity

The Banana Effect

At large disruption, correlated offsets in the beam are important

⇒ offsets of parts of the beam lead to instability

The emittance growth in the beam leads to correlation of the mean y position to z



a) shows development of beam in the main linac

b) simplified beam-beam calculation using projected emittances

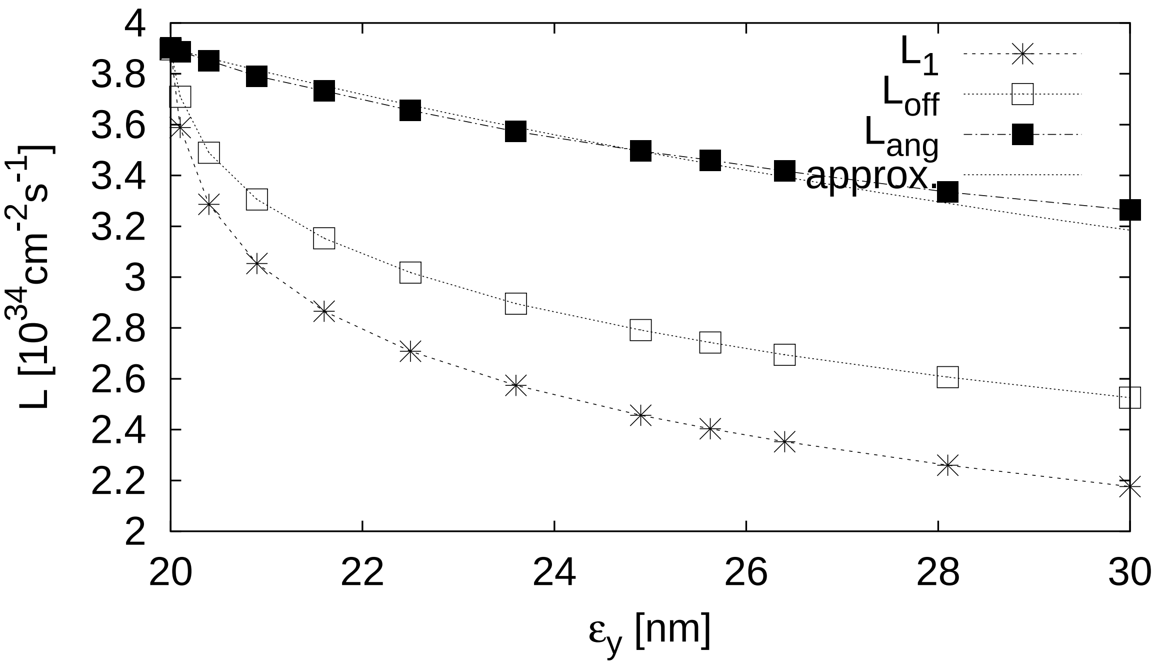
c) beam-beam calculation with full correlation

Mitigation of the Effect

Example with TESLA parameters is shown

The effect can be cured by using luminosity optimisation in a pulse

While the effect can be cured by performing luminosity optimisation, this leads to a more complex tuning scheme



First angle and offset are corrected

Then luminosity is optimised

Approximate analytical scaling is $\mathcal{L} \propto 1/\sqrt{\epsilon_y}$

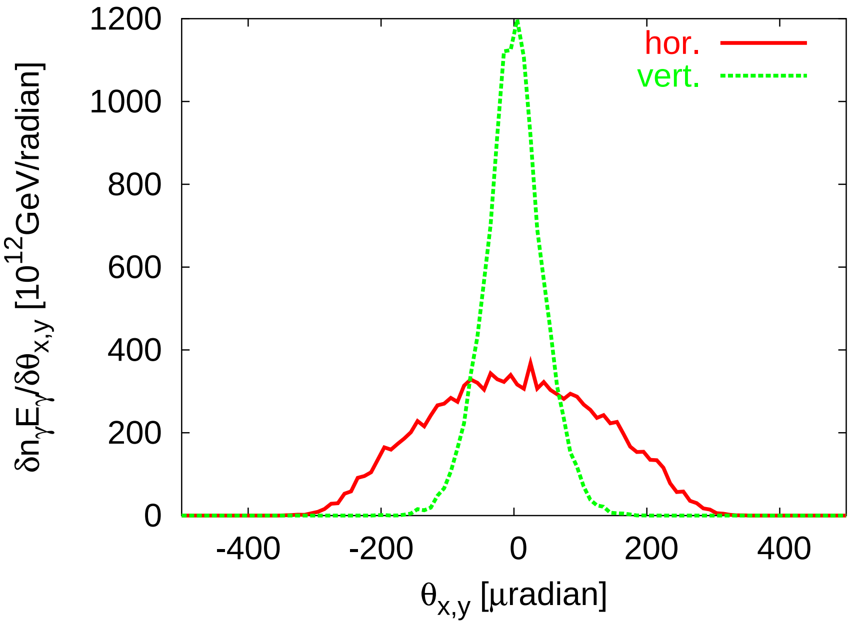
Secondary Production

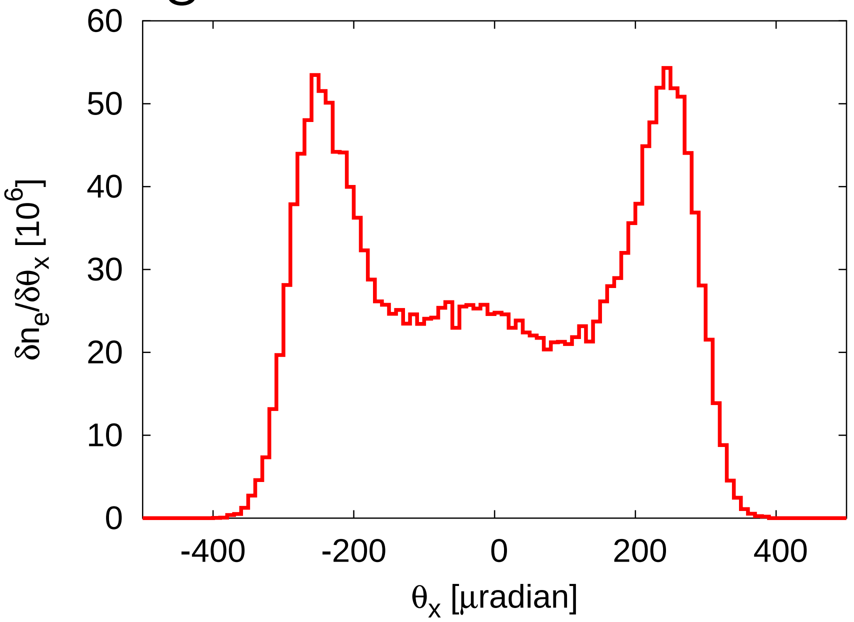
We will focus on

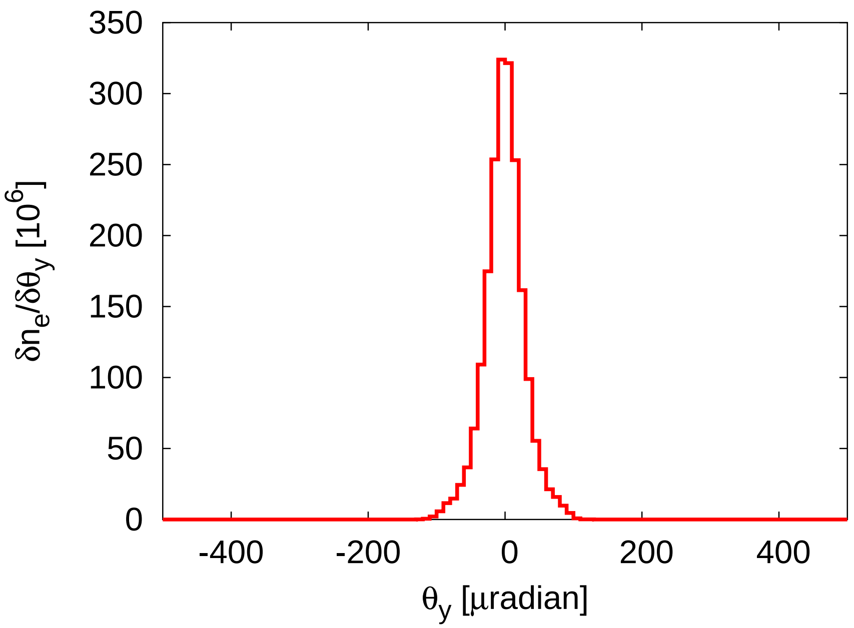
- beamstrahlung (see above)
- incoherent pair production
- coherent pair production
- bremsstrahlung
- hadron production

Spent Beam and Beamstrahlung

Spent beam particles have relatively small angles







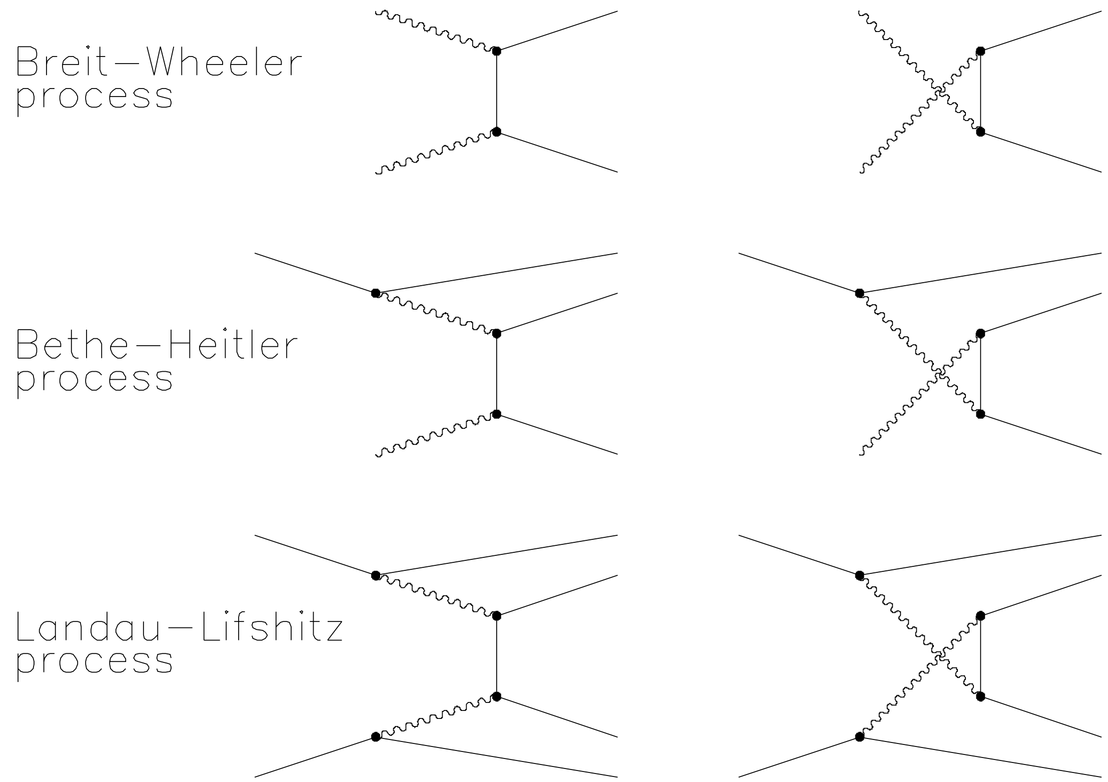
Incoherent Pair Production

Three different processes are important

- Breit-Wheeler
- Bethe-Heitler
- Landau-Lifshitz

The real photons are beamstrahlung photons

The processes with virtual photons can be calculated using the equivalent photon approximation and the Breit-Wheeler cross section



Breit-Wheeler Process

Collisions of two photons can produce electron positron pairs

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = \frac{2\pi r_e^2 m^2}{s^2}\left[\left(\frac{t-m^2}{u-m^2}+\frac{u-m^2}{t-m^2}\right)-4\left(\frac{m^2}{t-m^2}+\frac{m^2}{u-m^2}\right)\right.$$
$$\left.-4\left(\frac{m^2}{t-m^2}+\frac{m^2}{u-m^2}\right)^2\right] \quad (1)$$

$s = (k_1 + k_2)^2$, $t = (k_1 - p_1)^2$ and $u = (k_1 - p_2)^2$ are Mandelstam variables

In centre-of-mass system

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} \propto \frac{1+\beta\cos\theta}{1-\beta\cos\theta}+\frac{1-\beta\cos\theta}{1+\beta\cos\theta} = 2\frac{1+(\beta\cos\theta)^2}{1-(\beta\cos\theta)^2}$$

Cross section is peaked in forward and backward direction ($\cos\theta \approx 1$)

$\Rightarrow$ pairs are usually produced with small transverse momentum

Equivalent Photon Approximation

In the equivalent photon (or Weizäcker-Williams) approximation the virtual photon in a process is treated as real and an equivalent photon flux is used

The photon spectrum is given by

$$\frac{d^2 f_e^\gamma(x,Q^2)}{dx dQ^2} = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\frac{1}{Q^2}$$

Since we neglect the virtuality we can integrate over Q^2

$$\frac{dn_e^\gamma(x)}{dx} = \frac{\alpha}{2\pi}\frac{1+(1-x)^2}{x}\ln\frac{\hat{Q}^2}{\check{Q}^2}$$

The lower boundary is given by

$$\check{Q}^2 = \frac{x^2 m^2}{1-x}$$

The upper boundary depends on the process, we use $\hat{Q}^2 = m^2 + p_\perp^2$

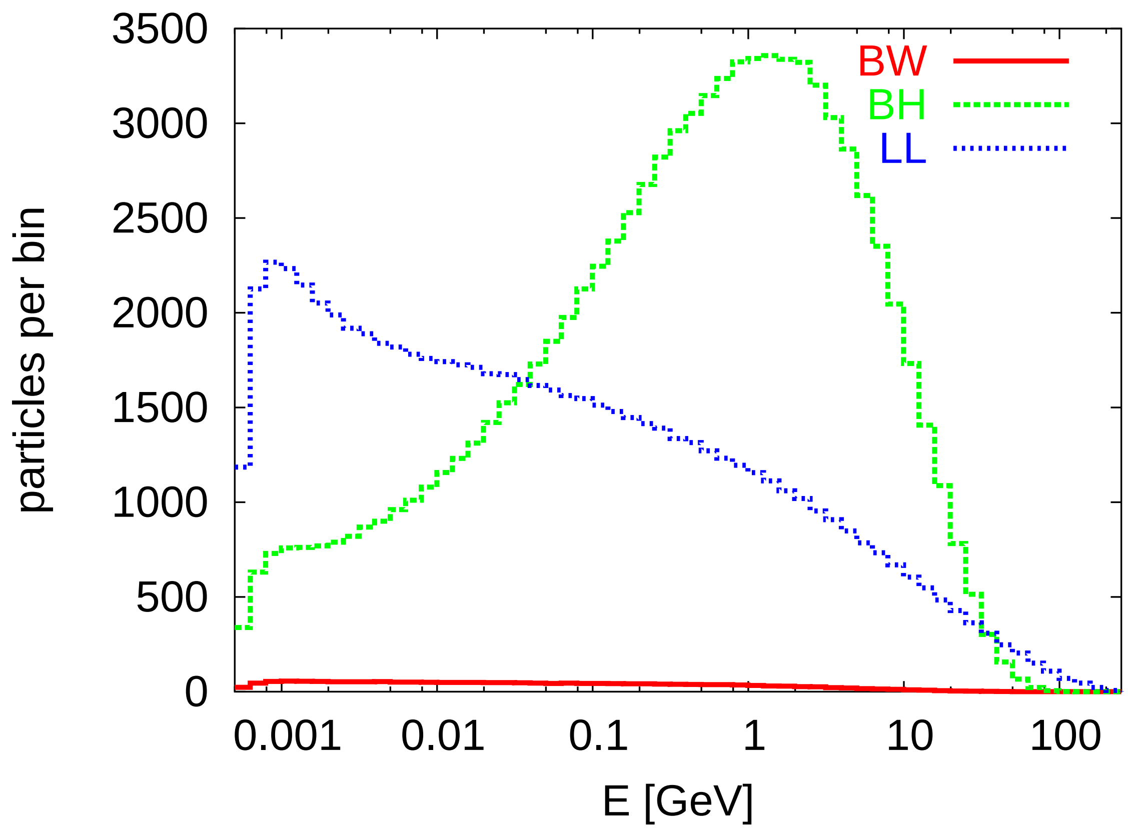
Spectrum of the Pairs

The Breit-Wheeler process produces the smallest amount of particles

- but they often have larger angles

The Landau-Lifshitz process is produces more soft and hard particles than the Bethe-Heitler process

In the Bethe-heitler process usually the beamstrahlung photon is the hard photon



Beam Size Effect

The virtual photons with low transverse momentum are not well localised

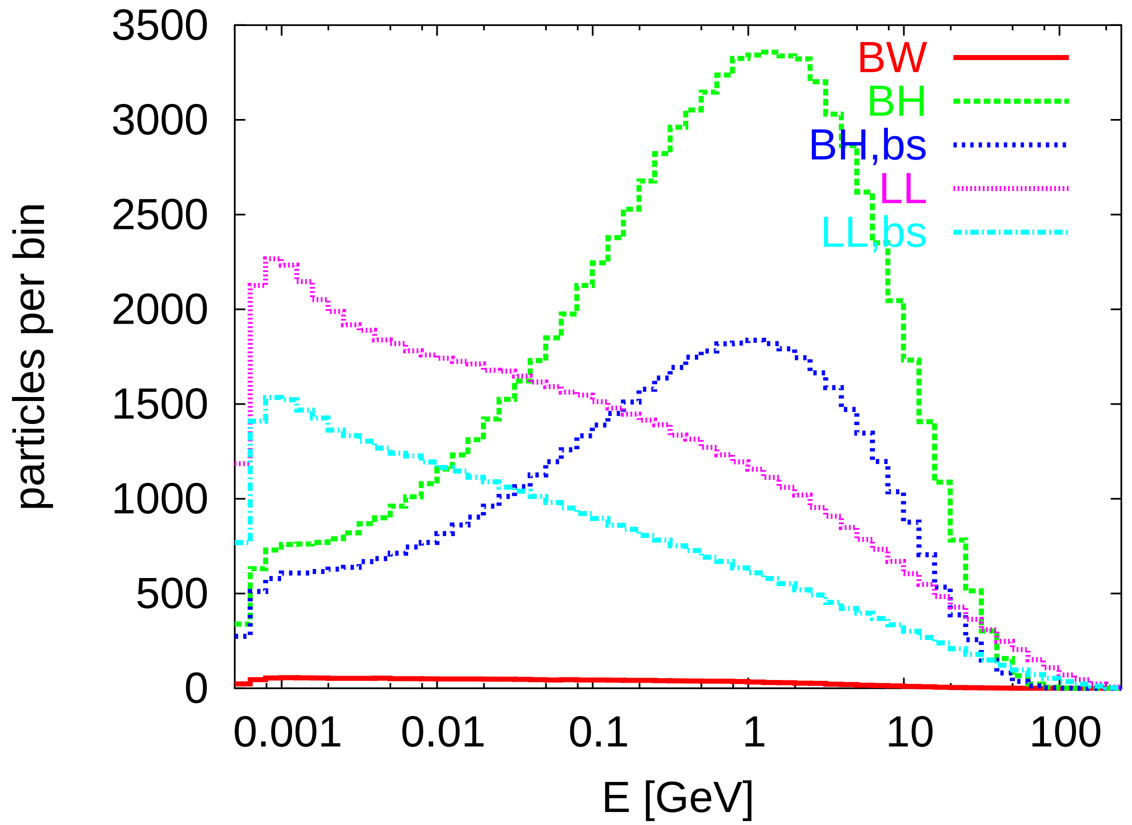
The beams are very small

$\Rightarrow$ need to correct the cross section

Model is to use the typical impact parameter $b \approx \hbar/q_\perp$

If $b > \sigma_y$ the process is suppressed

Typical reduction of the production rate is a factor two



Deflection by the Beams

Most of the produced particles have small angles

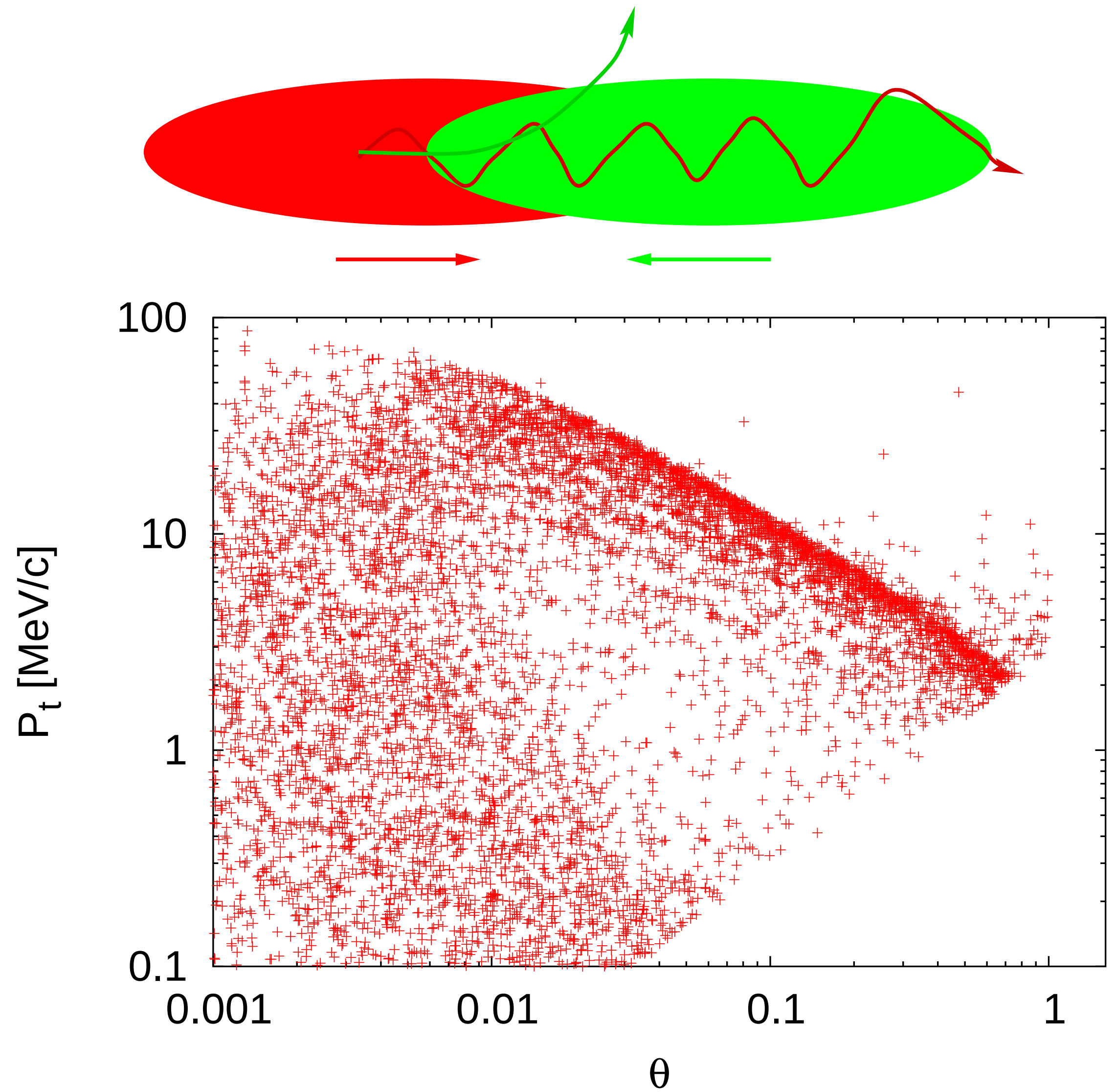
The forward or backward direction is random

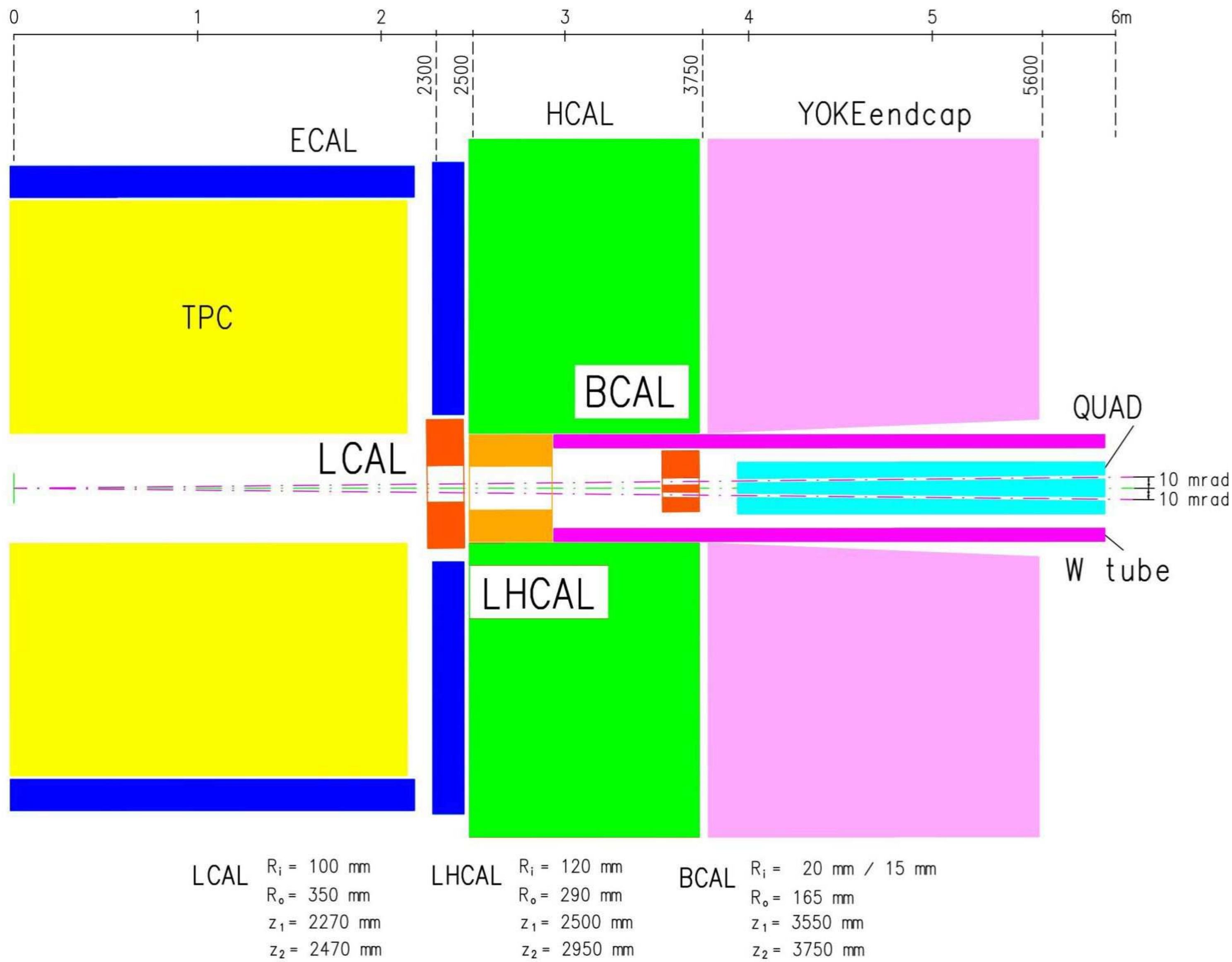
The pairs are affected by the beam

$\Rightarrow$ some are focused
some are defocused

Maximum deflection

$$\theta_m = \sqrt{4 \frac{\ln\left(\frac{D}{\epsilon}+1\right) D\sigma_x^2}{\sqrt{3}\epsilon\sigma_z^2}}$$




0
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TPC
BCAL
LCAL
LHCAL
QUAD
W tube
10 mrad
10 mrad
LCAL
R_i = 100 mm
R_o = 350 mm
z_1 = 2270 mm
z_2 = 2470 mm
LHCAL
R_i = 120 mm
R_o = 290 mm
z_1 = 2500 mm
z_2 = 2950 mm
BCAL
R_i = 20 mm / 15 mm
R_o = 165 mm
z_1 = 3550 mm
z_2 = 3750 mm

Vertex Detector

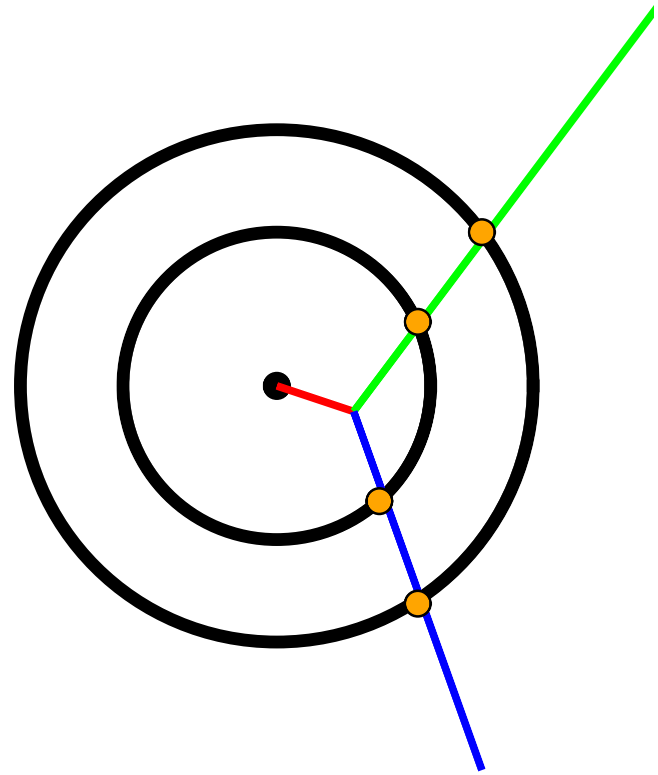
The vertex detector is the detector component closest to the interaction point

It should measure the production vertex

- e.g. if in an event a b-quark is produced it will decay after a short time into lighter quarks

- the tracks of these lighter particles will orign from the point where the b-quark decayed, not from the beam position

The closer the vertex detector to the IP the better the resolution

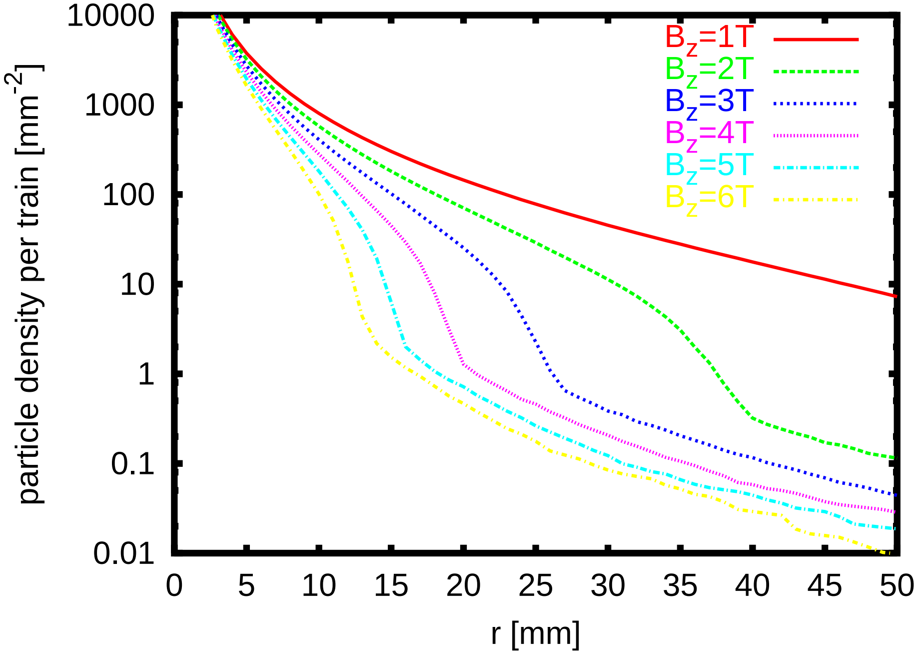


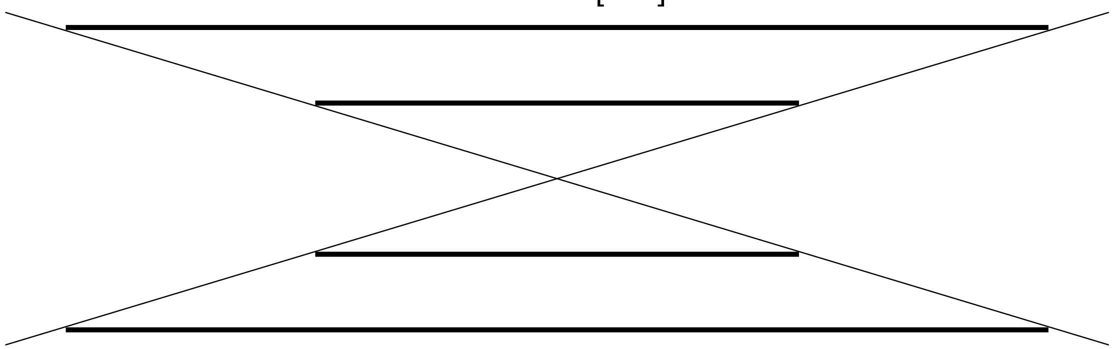
Impact of the Pairs on the Vertex Detector

Hits of the pairs in the vertex detector can confuse the reconstruction of tracks

Can avoid this problem by combination of two means

- use sufficient opening angle of the vertex detector
- confine pairs to small radii by use of longitudinal magnetic field this exists in the detector anyway



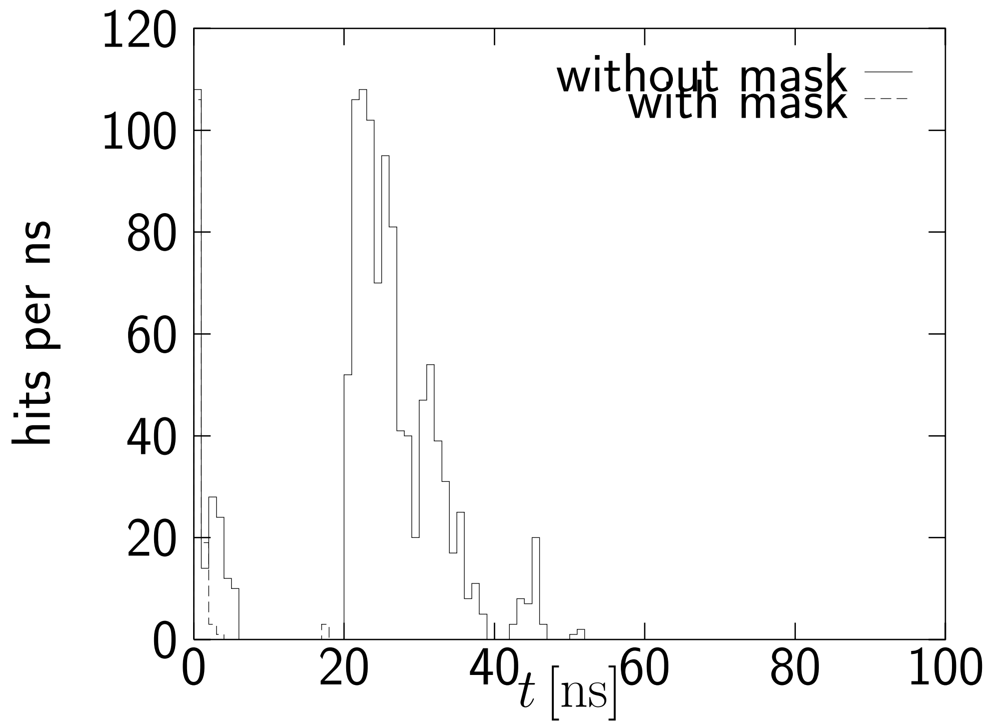


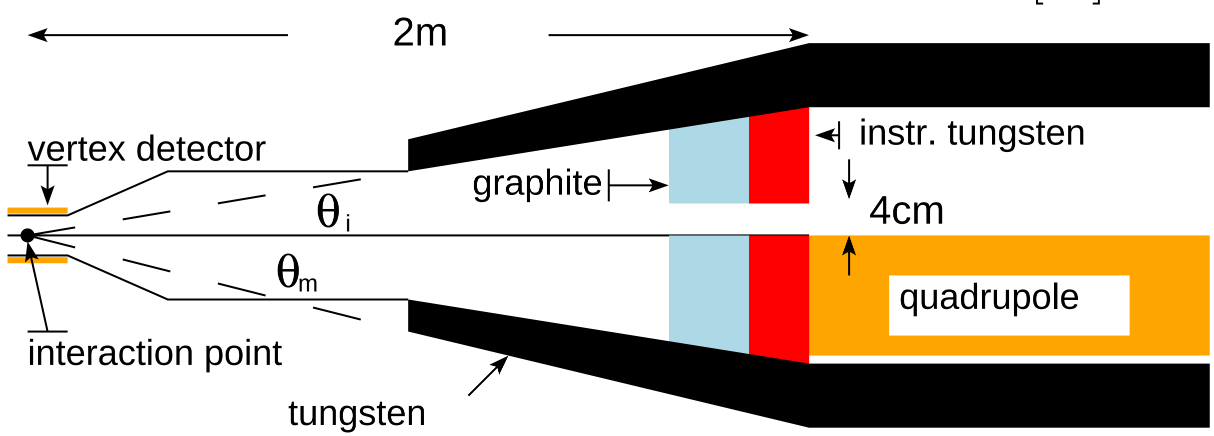
Impact on the Detector Design

A significant number of the pair particles can be hit something in the detector

⇒ their secondaries can be a problem

Example: the old TESLA design





Bremsstrahlung

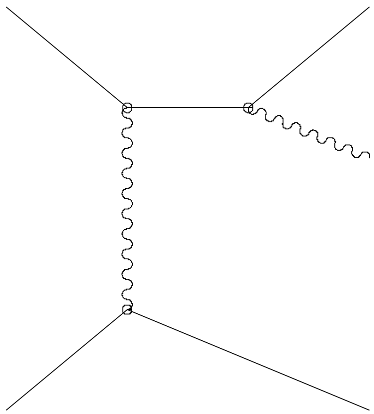
Interaction of particle with individual particle of other beam

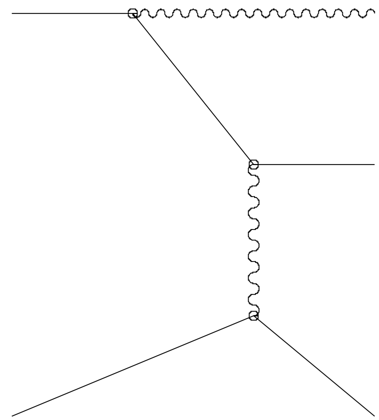
Also called "radiative Bhabha"

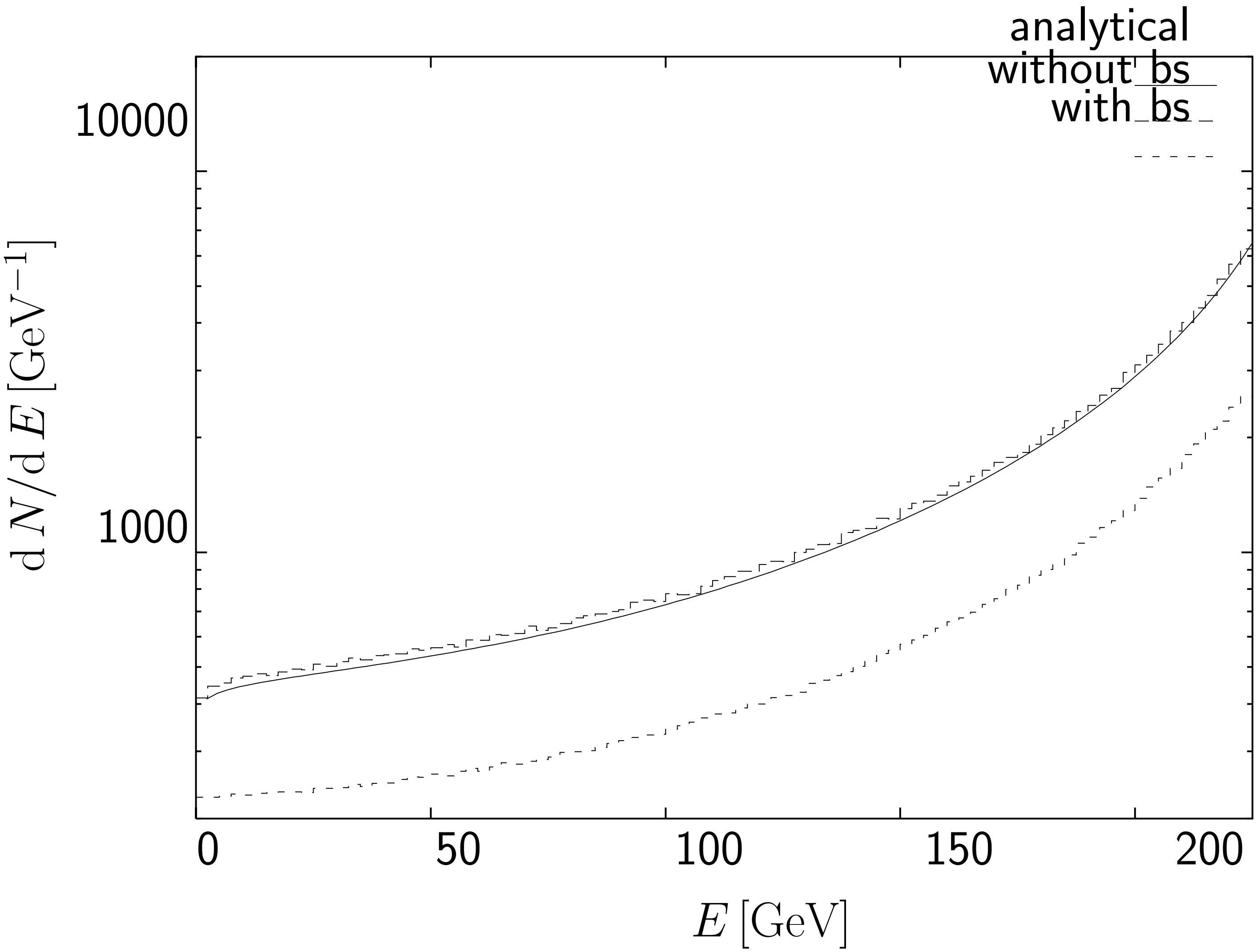
Soft scatter between two particles with emission of inital state radiation

Can be calculated as Compton scattering of vitual photon spectrum with beam particle

Yields a relatively flat spectrum






analytical
without bs
with bs
10000
1000
d N/d E [GeV−1]
0
50
100
150
200
E [GeV]

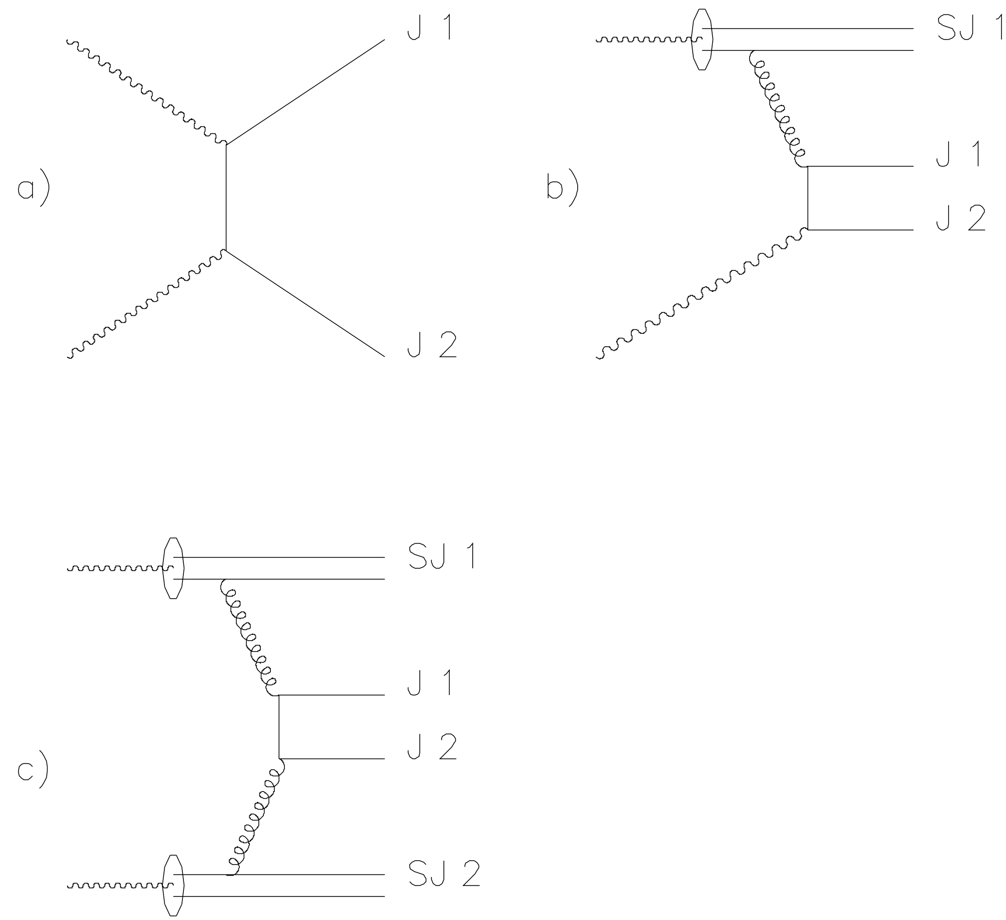
Hadronic Background

A photon can contribute to hadron production in two ways

- direct production, the photon is a real photon
- resolved production, the photon is a bag full of partons

Hard and soft events exist

e.g. “minijets”



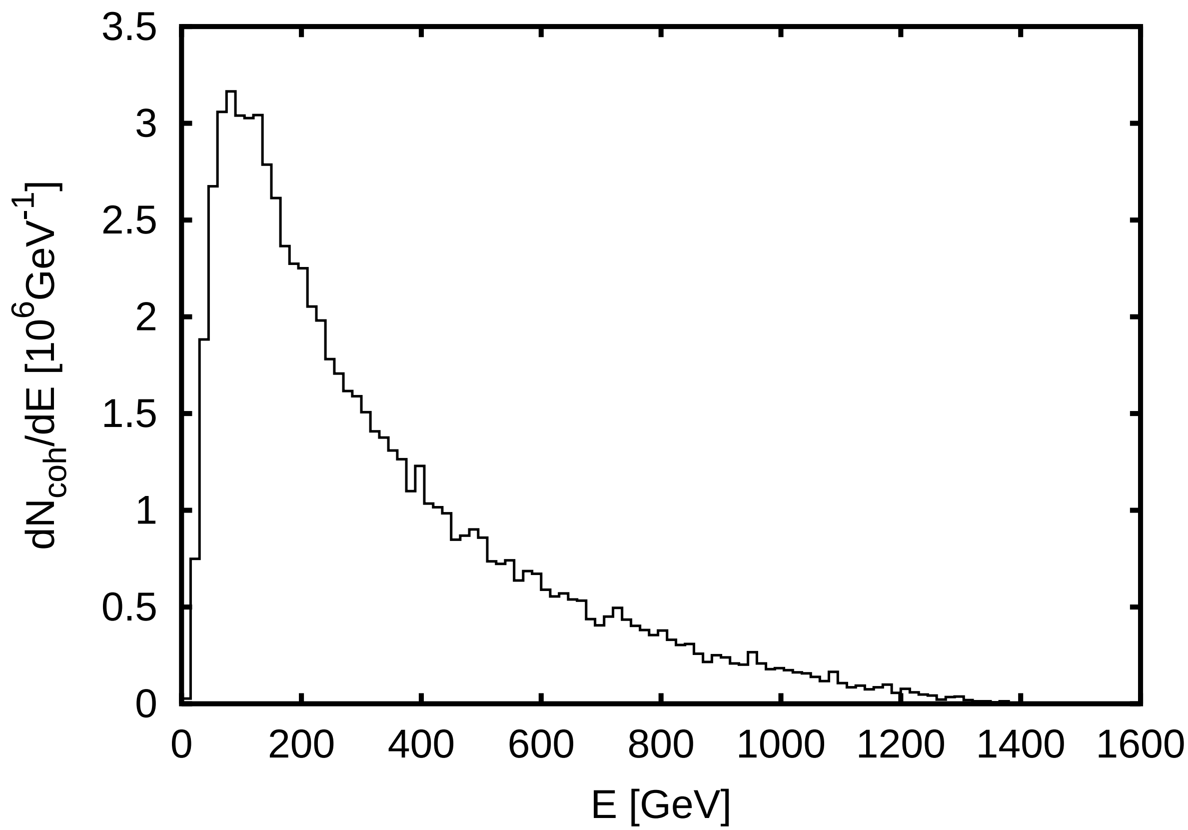
Coherent Pairs

Coherent pairs are generated by a photon in a strong electro-magnetic field

Cross section depends exponentially on the field

⇒ Rate of pairs is small for centre-of-mass energies below $1\,\mathrm{TeV}$

⇒ In CLIC, rate is substantial



Need to foresee large enough exit hole (about 10mradian)

Luminosity Monitoring

Fast luminosity measurement is crucial for machine tuning

The detector will use Bhabha scattering

$\Rightarrow$ very good signal for accurate measurement

$\Rightarrow$ this signal is too slow for our luminosity optimisation

Need to use other signals, e.g.

- beamstrahlung/beam energy loss
- incoherent pairs
- bremsstrahlung

Two approaches

- try to reconstruct beam parameters from observables
- optimise one tuning knob after the other

Use of Bremsstrahlung

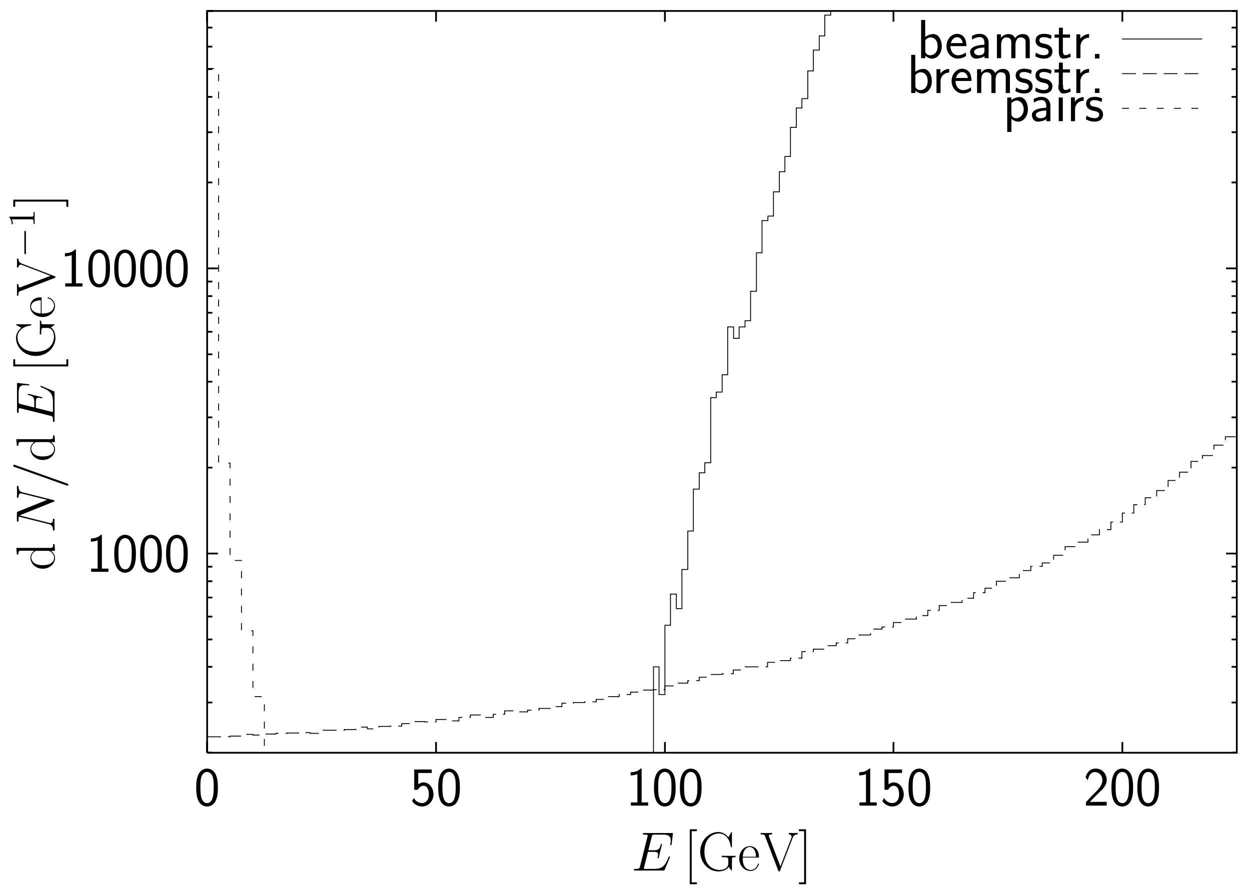
Bremsstrahlung is emitted into small angles

⇒ quite proportional to luminosity

⇒ the emitting particle is not scattered much

⇒ it cannot be seperated from the beam by its angle

⇒ one needs to seperate it by it's energy

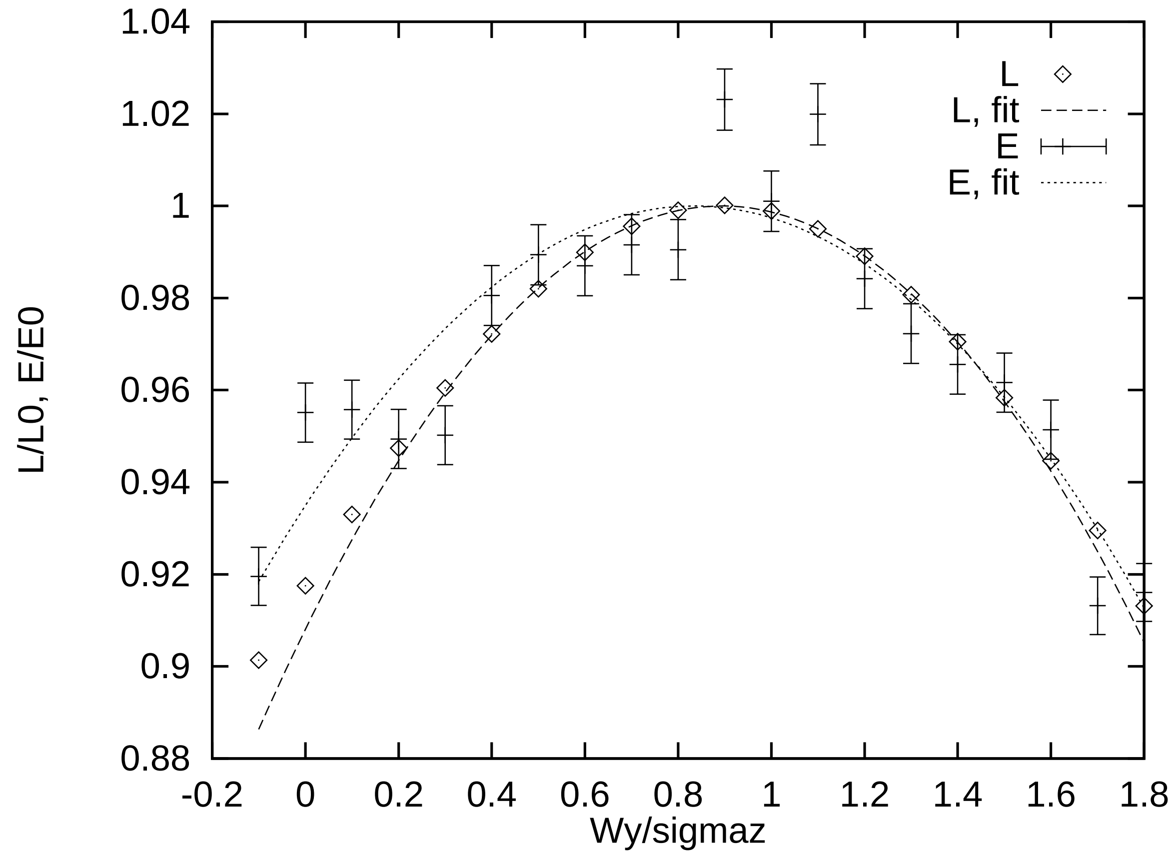


Needs careful design of the spent beam line and it's instrumentation

Use of Incoherent Pairs

The total energy of incoherent pairs is proportional to luminosity time some function of the beam parameters

Example shown is a scan of the vertical waist, i.e. the longitudinal position of the vertical focal point



Use of Beamstrahlung

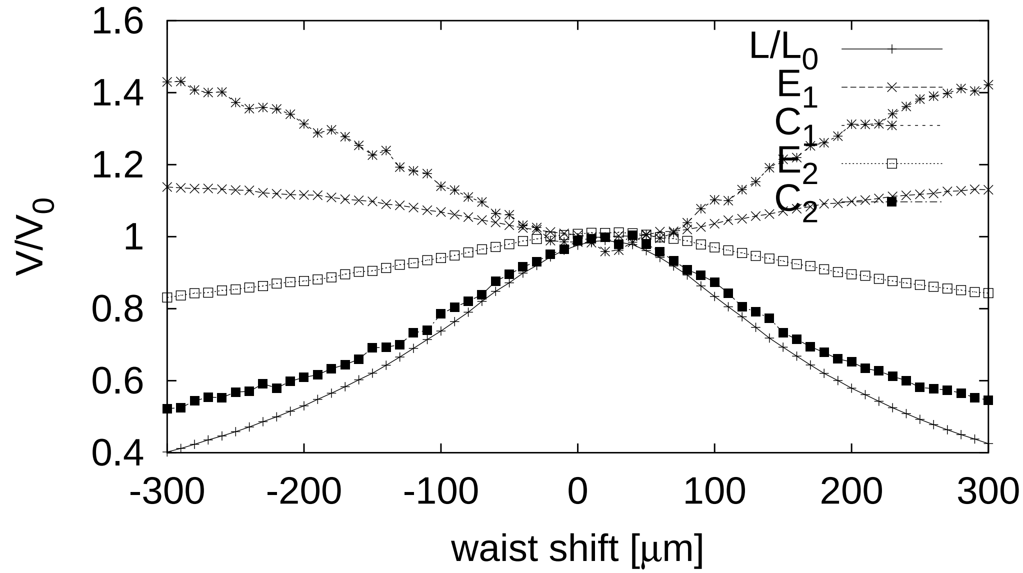
Beamstrahlung is not proportional to luminosity at all

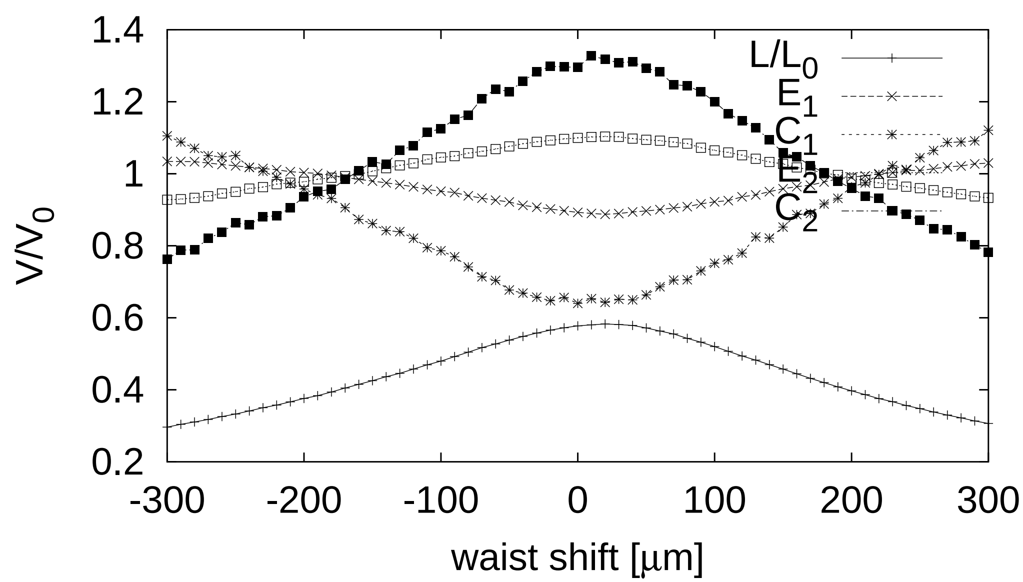
Can use beamstrahlung to optimise knobs which modify one parameter at a time

Need to identify correct combination of beamstrahlung

- sum of radiation of both beams

- difference of radiation of both beams





Conclusion

- High luminosity with limited beamstrahlung requires flat beams
- Beamstrahlung can significantly affect the experiments
- Beam-beam effects can generate background
  - most important for the vertex detector
- Beam-beam background can be used for luminosity monitoring