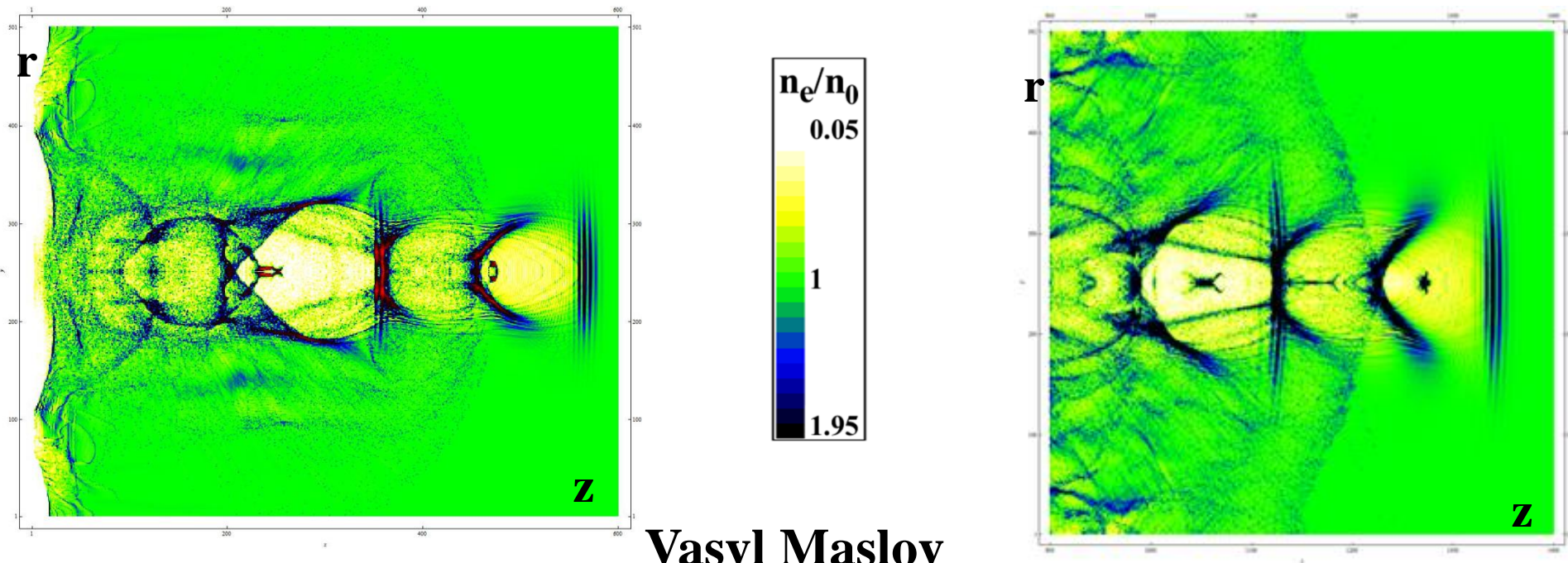


Wakefield Excitation in Plasma by Short Train of Laser Pulses



Vasyl Maslov

NSC Kharkov Institute of Physics and Technology, 61108 Kharkov

vmaslov@kipt.kharkov.ua

**National Science Centre
Kharkov Institute of Physics and Technology,
Kharkov, Ukraine**



Would be happy to collaborate



V.N. Karazin Kharkiv National University is one of the oldest universities in Eastern Europe. It was founded in 1804.

Kharkiv University is the only university in Ukraine that has trained and employed three Nobel Prize laureates: the biologist I. Mechnikov, the economist S. Kuznets, and the physicist L. Landau.

About 12,000 students.

The University is one of the largest research centers in Ukraine: It covers all spheres of modern fundamental research.

Would be happy to collaborate

Large Hadron Collider at CERN



International **L**inear **C**ollider **ILC**

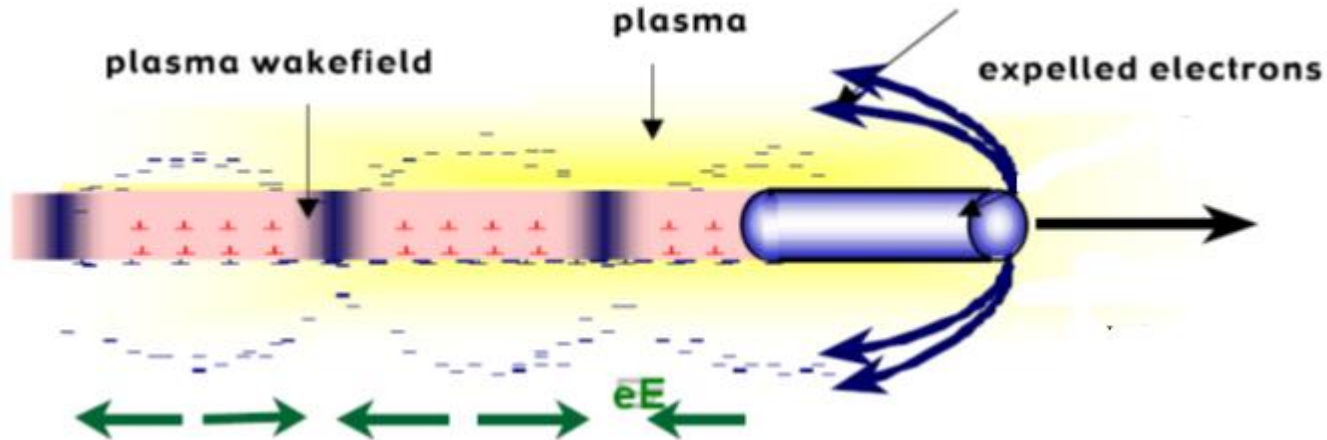
**3 TeV, 50 km length
is planned to be build**

**Modern colliders are large and expensive with
limited accelerating gradient**

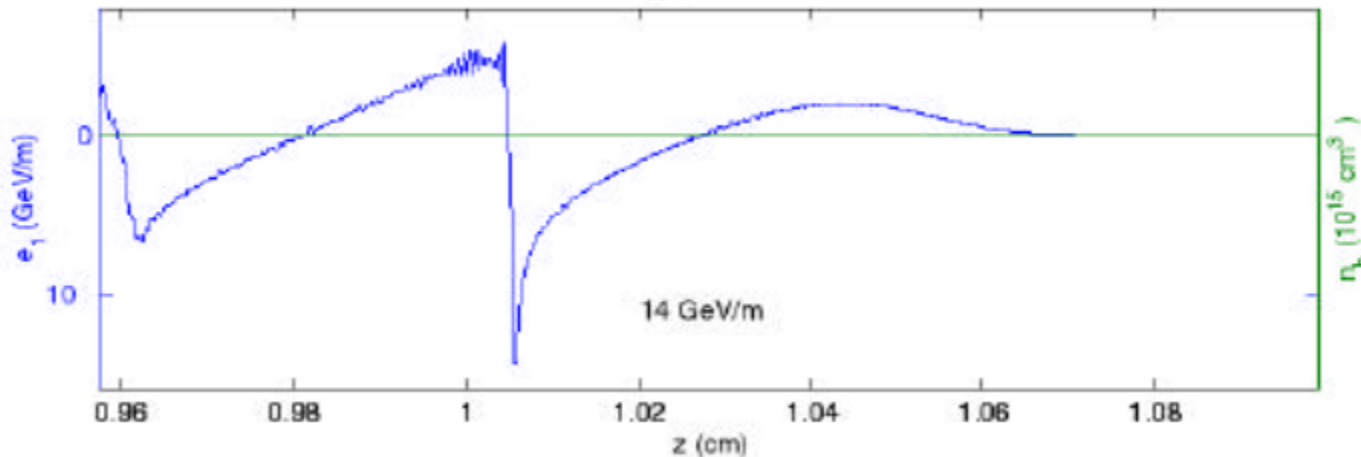
What's next accelerator?

Advanced Accelerators

Advanced Accelerators: by bunches of charged particles and by short laser pulses in plasma, in dielectric and in metallic structure



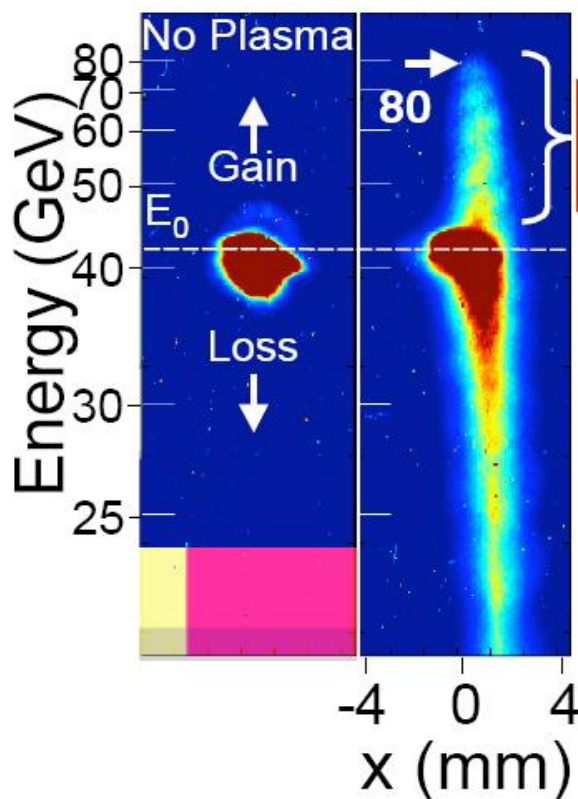
Physical mechanism of wakefield excitation



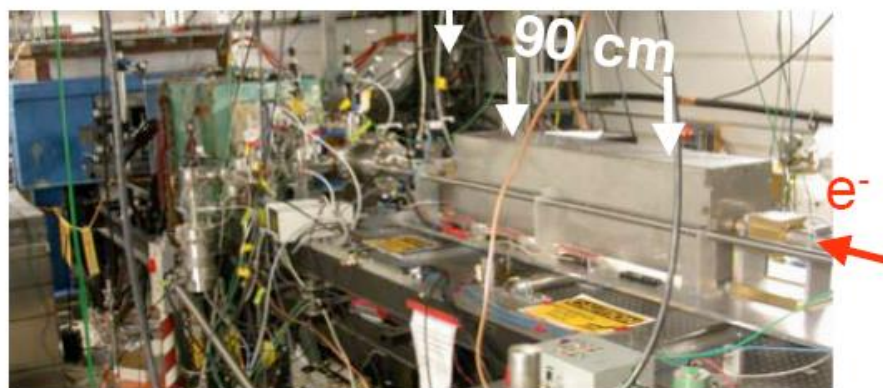
ENERGY GAIN



$$E_0 = 42 \text{ GeV}, N = 1.75 \times 10^{10} e^-, n_e = 2.6 \times 10^{17} \text{ cm}^{-3}, L_p = 90 \text{ cm}$$



$\approx 9.6 \times 10^8 e^-$
 $\approx 154 \text{ pC}$



➡ Energy gain 38 GeV over $\approx 90 \text{ cm}$ of plasma! or 42GV/m!

➡ PWFA = extremely simple and compact accelerator



Beam of relativistic electrons from intense laser-plasma interactions

Short high-power laser pulse is injected through gas jet, produces plasma, excites wakefield in it. Electron beam is accelerated.

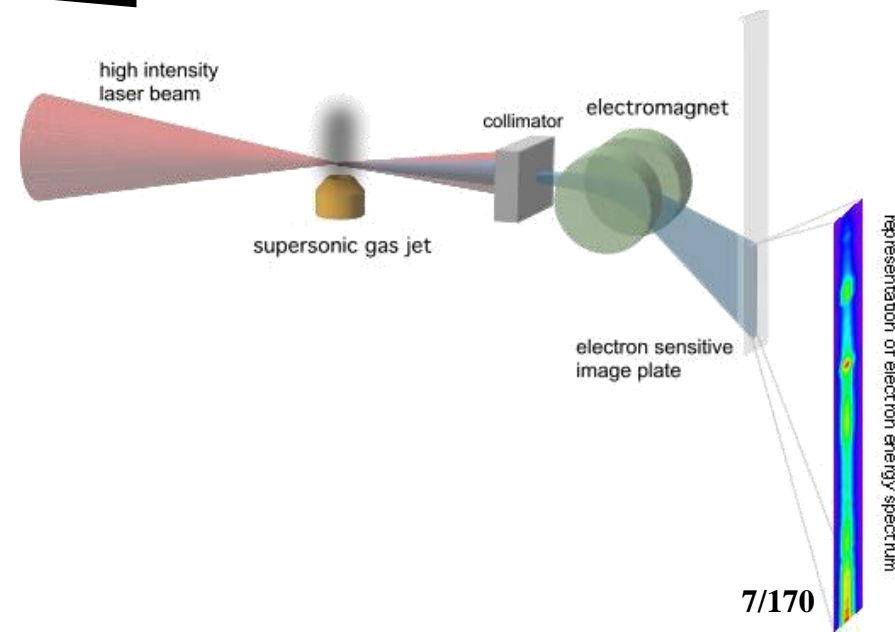
Plasma

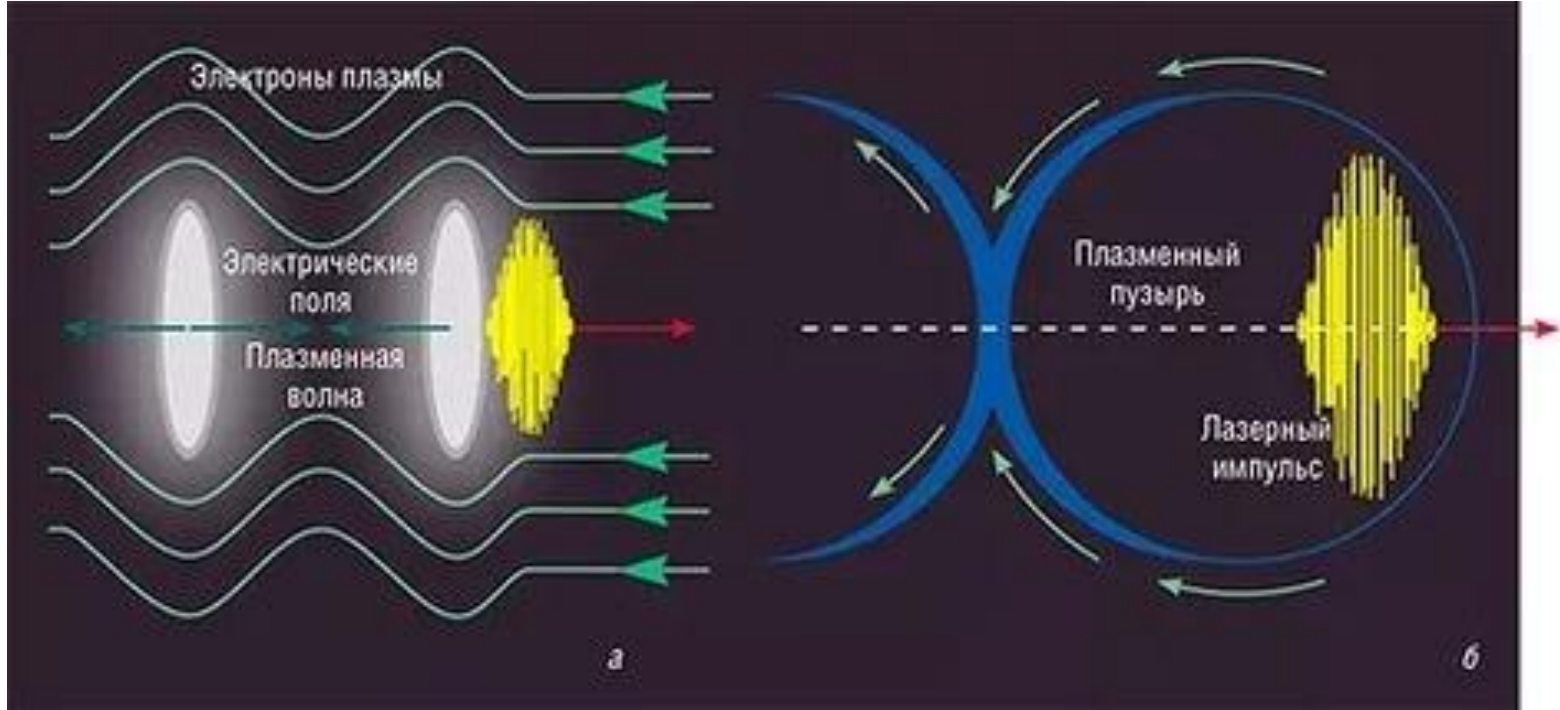
Laser

Accelerated
Electron
Beam

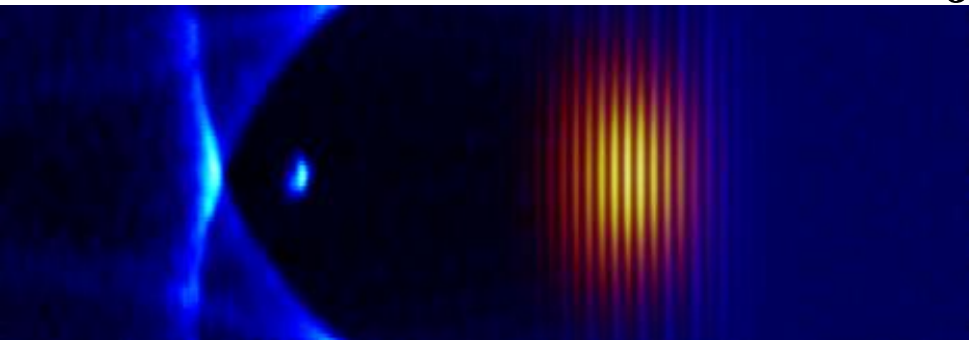
Gas
Jet

$$n_e = 2 \times 10^{19} \text{ cm}^{-3}$$



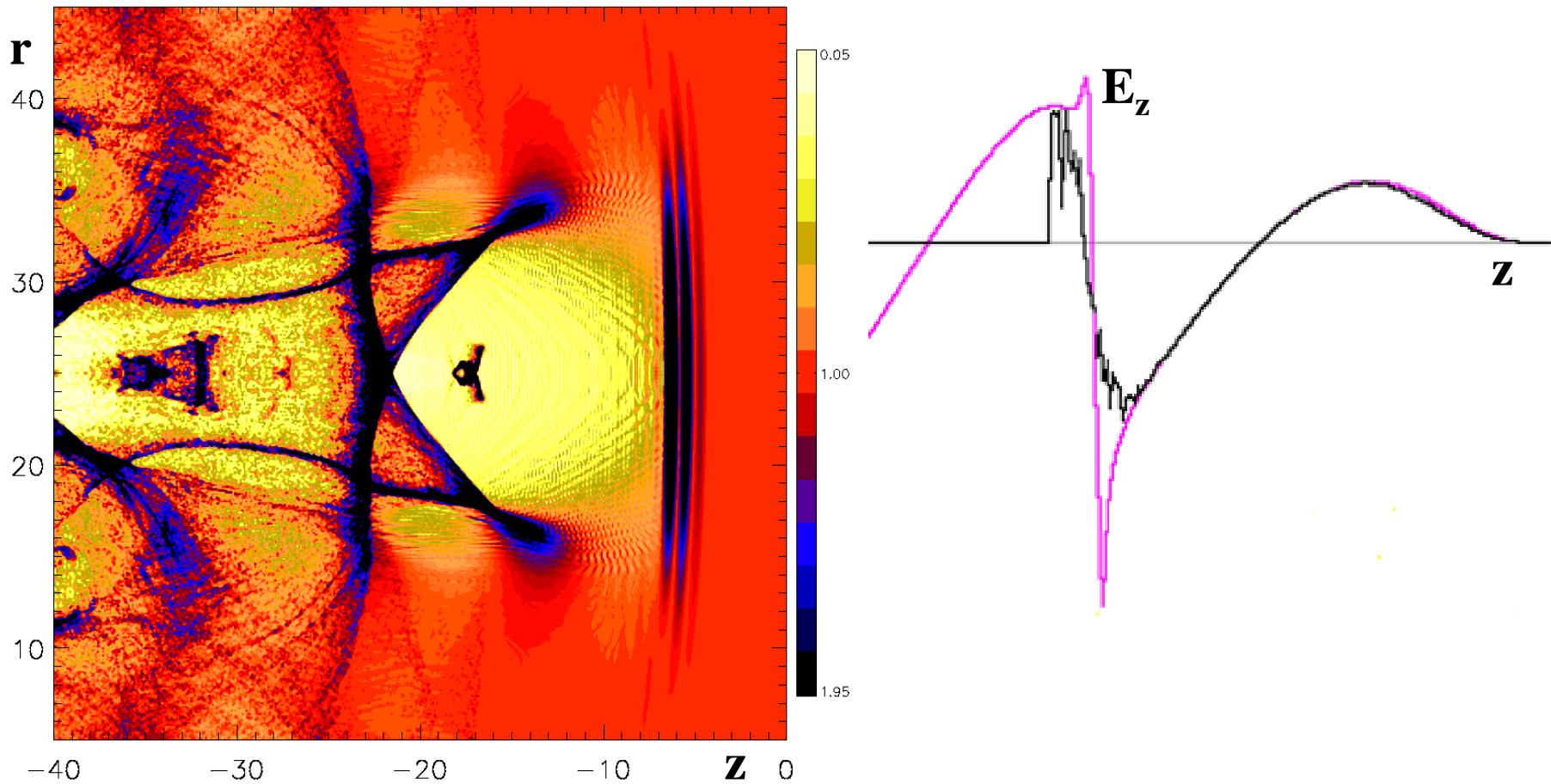


**A short and intense laser pulse pushes the plasma electrons.
 The bubble is formed. The plasma electrons return to the axis.
 The large accelerating electrical field is formed.
 The short electron bunch is self-injected and accelerated.**

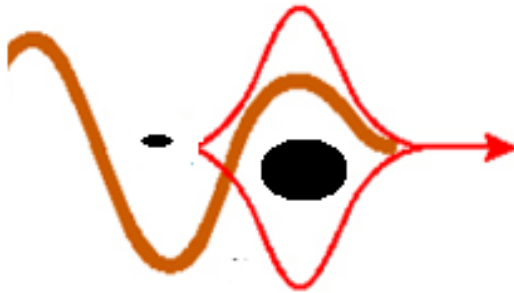


S. Hooker et al.

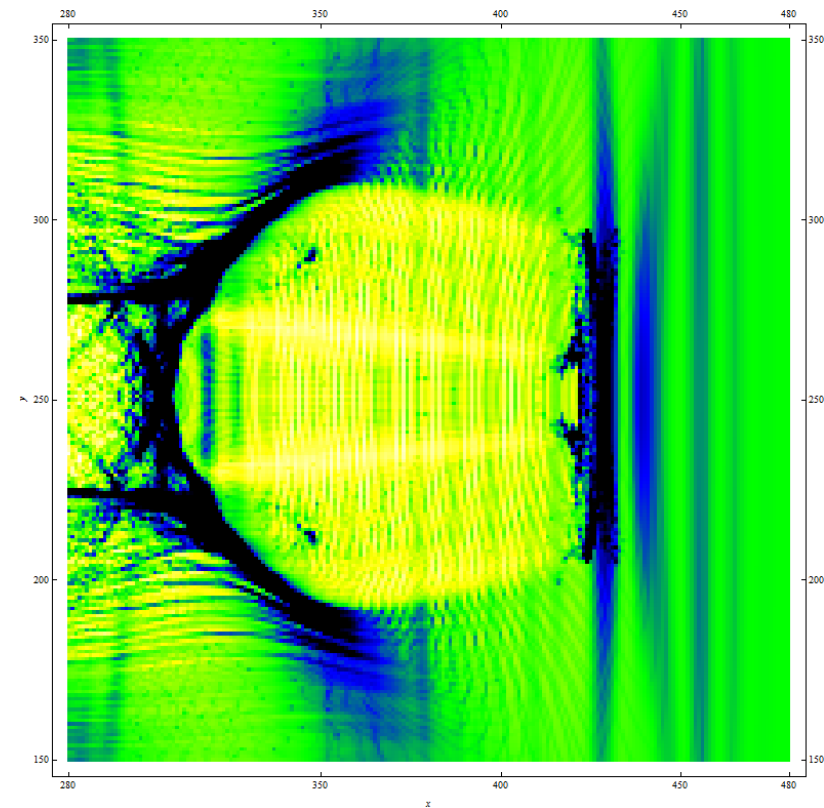
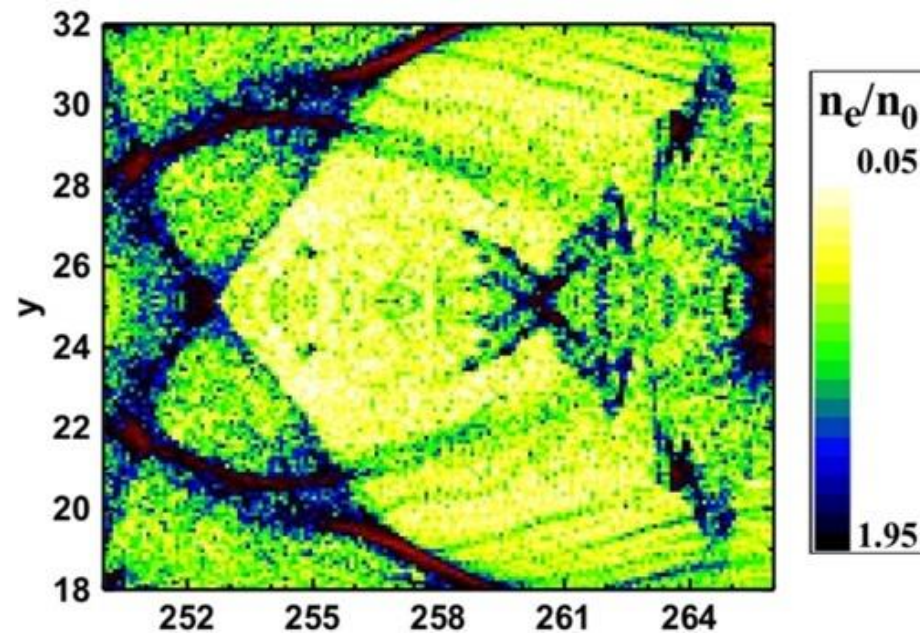
Wakefield excitation in plasma by short train of laser pulses in (nonlinear) bubble/blowout regime



Joint wakefield acceleration by laser pulse and by self-injected electron bunches

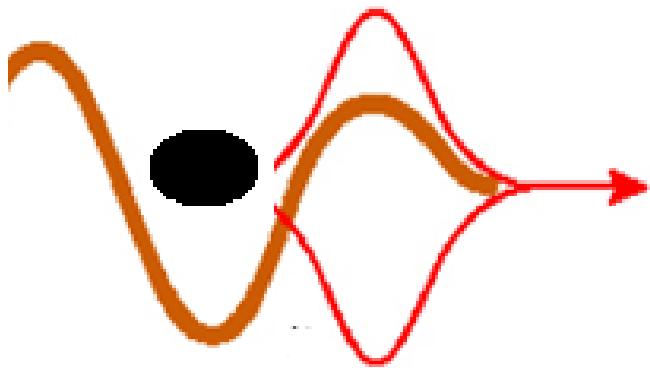


Two scenarios of intensification of electron bunch acceleration by wakefield excited by laser pulses

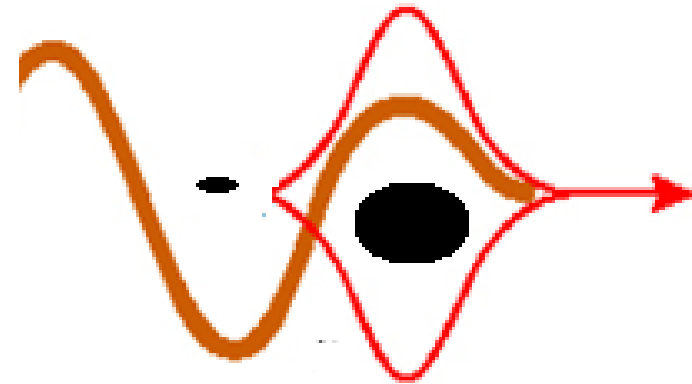


Two scenarios of intensification of electron bunch acceleration by wakefield excited by laser pulse

Two scenarios of transformation of known mechanism of acceleration of self-injected electron witness-bunch by wakefield, excited by a laser pulse, in acceleration of 2nd self-injected electron bunch of small charge by wakefield, excited together by laser pulse and by 1st self-injected electron bunch, which becomes driver.



Laser pulse is driver,
self-injected electron bunch
is witness



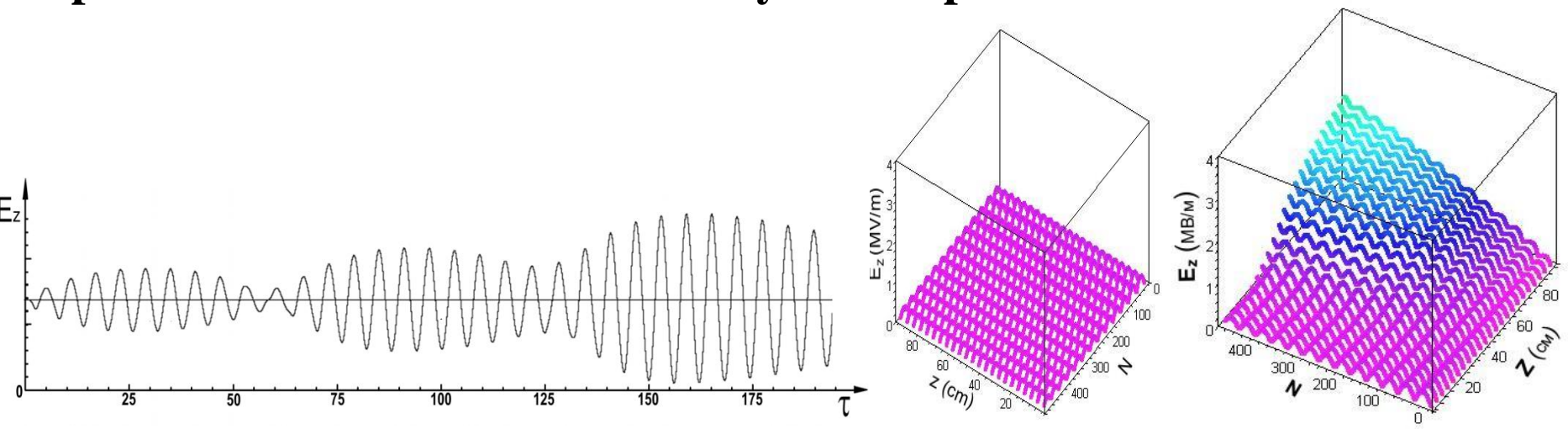
Laser pulse and 1st self-injected
electron bunch are driver,
2nd self-injected electron bunch of
small charge is witness

Importance of radial dynamics of electron bunches

In the paper

K.V.Lotov, V.I. Maslov, I.N. Onishchenko, E.N. Svistun. 2010

interesting phenomenon has been considered. r-dynamics of electron bunches in the conventional metal accelerators is bad but in plasma accelerators it can be good. Namely it is known that at injection of train of ultra-relativistic non-resonant $\omega_d \neq \omega_{pe}$ electron driver-bunches or relativistic bunches near boundary of injection, their radial dynamics is suppressed and they do not excite wakefield in plasma, only beating of small amplitude. But at free radial dynamics for relativistic electron driver-bunches far from the boundary of injection they excite wakefield. We will show that the radial dynamics of electron bunches is also important at wakefield excitation by a laser pulse.

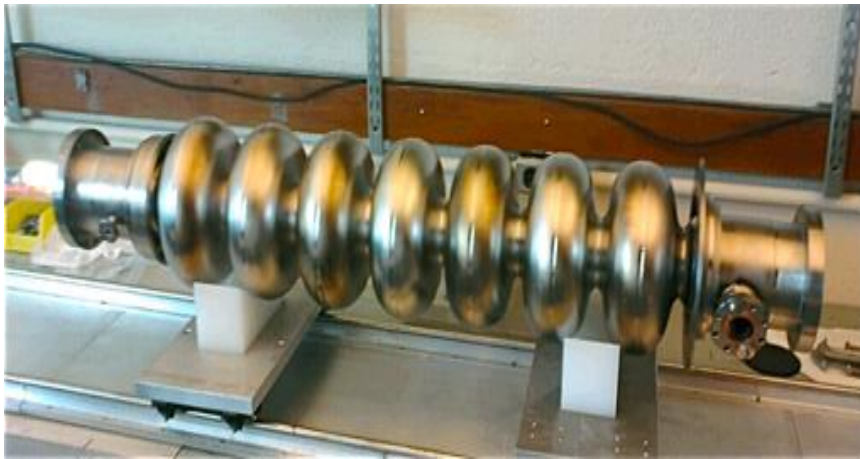


Advantage of electron bunch acceleration by plasma wakefield excited by a laser pulse

In

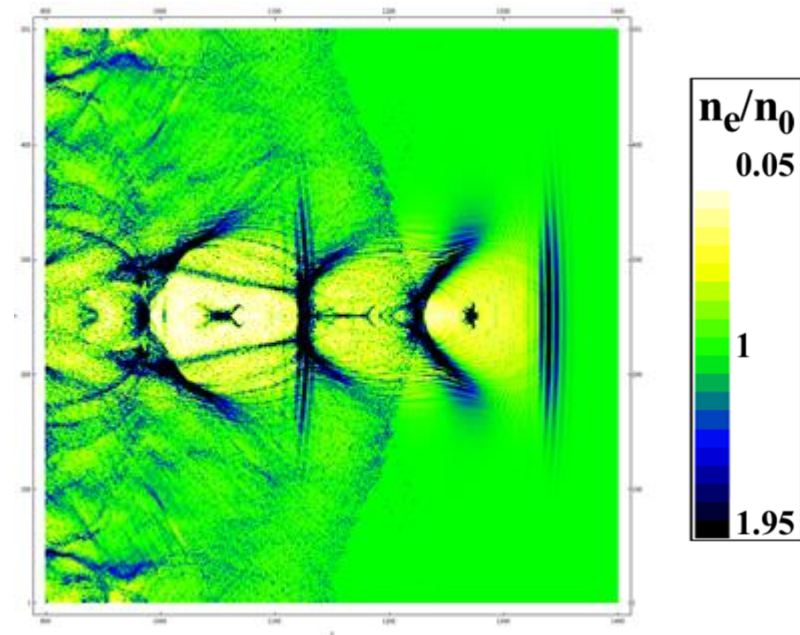
E. Esarey *et al.* 2009; V. Malka *et al.* 2002; W.P. Leemans *et al.* 2010

it has been demonstrated that the accelerating electric field in the plasma may exceed by several orders of electric field in metal structures.



Metal accelerating structure.

Electric field < 100 MV/m

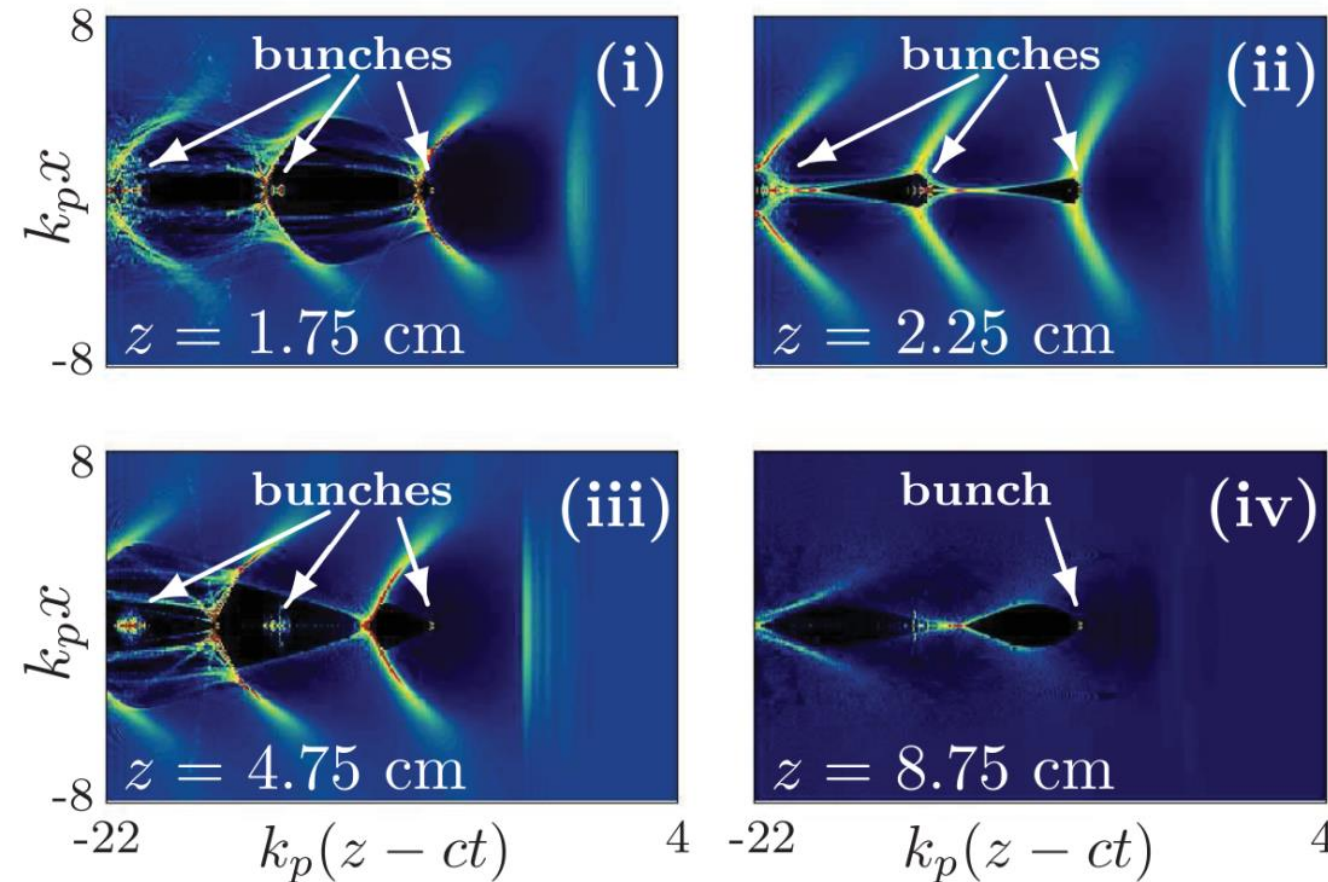


Plasma

Electric field > 100 GV/m

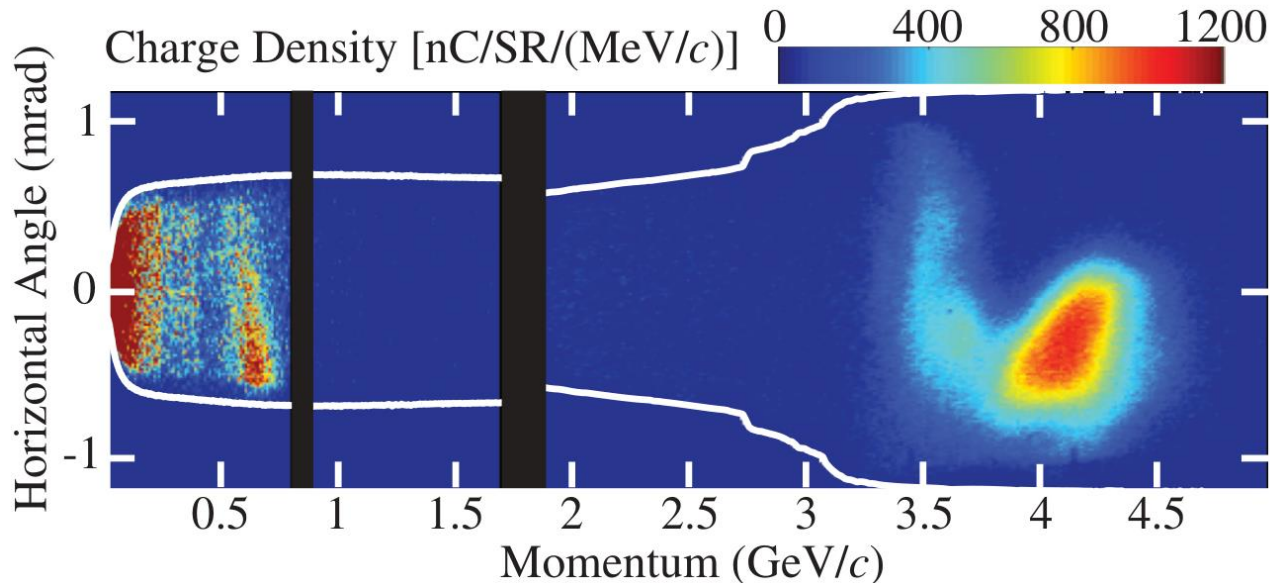
Electron bunch acceleration by wakefield excited by a laser pulse

Progress in development of lasers has led to progress in the creation of good quality bunches accelerated to several GeV.



Numerical simulation of plasma wake perturbation of the electron density at different depths of penetration of the laser pulse in a plasma of length 9cm

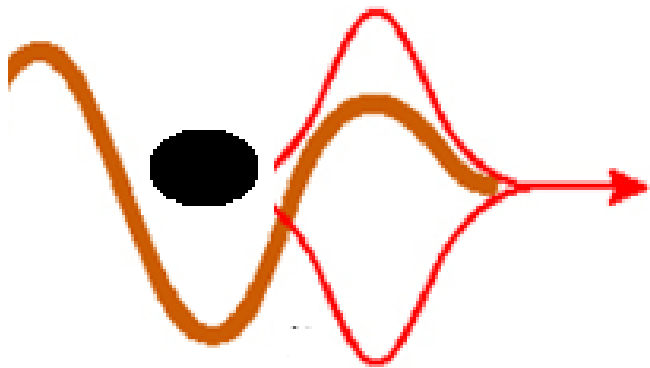
electron bunches of energy 4.2 GeV have been obtained with 6% energy spread, with charge of 6 pC, with the angular spread of 0.3 mrad in the plasma waveguide, obtained by capillary discharge of length 9 cm with the plasma density $\approx 7 \times 10^{17} \text{ cm}^{-3}$, using laser pulses with peak power 0.3 PW. $\langle E_z \rangle = 47 \text{ GV/m}$.



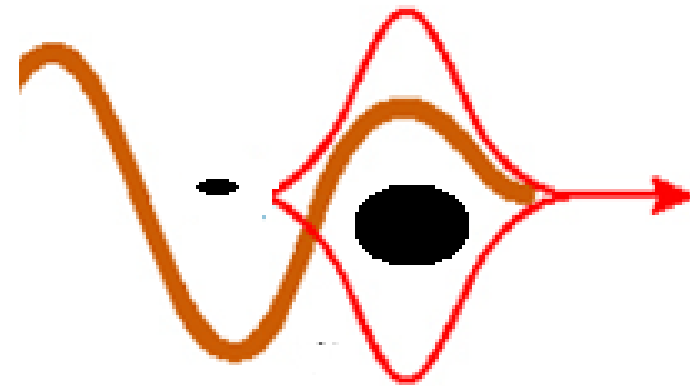
measured energy
spectrum of 4.2 GeV
electron bunch

X. Wang, R. Zgadzaj, N. Fazel et al, 2013
2 GeV quasimonoenergetic electrons, accelerated by a laser pulse in a plasma

There is the problem at laser wakefield acceleration. The laser pulse is quickly destroyed because of its expansion. One way to solve this problem is to use a capillary as a waveguide for laser pulse. The second way or the additional way to solve this problem is to transfer pulse energy to the electron bunches, which become drivers and which can accelerate witness. A transition from a laser wakefield accelerator to (beam-) plasma wakefield accelerator can occur in some cases at laser plasma interaction.



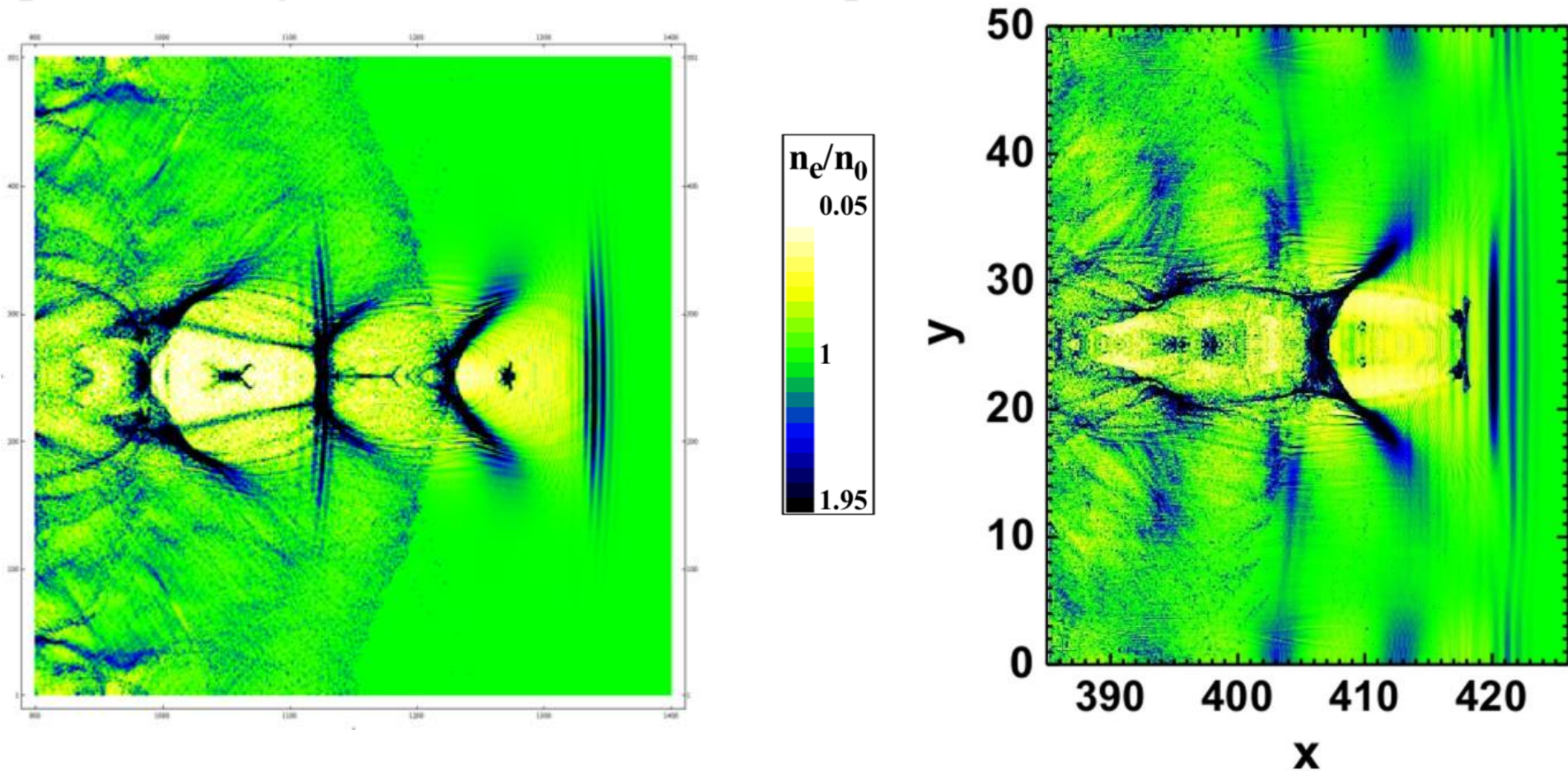
**Laser pulse is driver,
self-injected electron bunch
is witness**



**Laser pulse and 1st self-injected
electron bunch are driver,
2nd self-injected electron bunch of
small charge is witness**

Numerical simulation of the electron bunch dynamics

Blowout or bubble regime of wakefield excitation by single laser pulse and by short train of two laser pulses.



We use a relativistic electromagnetic code

S.V. Bulanov *et al.*, 1997.

Plasma parameters in numerical simulation

Plasma particles are modeled by micro-particles.

Simulated area (x, y) : $0 < x < 800\lambda$, $0 < y < 50\lambda$, λ is the laser wavelength. Time step $\tau = 0.05t_0$.

Number of particles in the cell 8, total number of particles $15.96 \cdot 10^6$. Time of simulation : up to 800 laser periods. Period of laser pulse $t_0 = 2\pi/\omega_0$, ω_0 is the frequency of laser pulse.

Laser pulse is injected in homogeneous plasma perpendicular to the border.

Plasma density: $n_0 = 0.01016n_c = 1.8 \times 10^{19} \text{ cm}^{-3}$, $n_c = m_e \omega_0^2 / (4\pi e^2)$ is the critical plasma density.

Laser parameters in numerical simulation

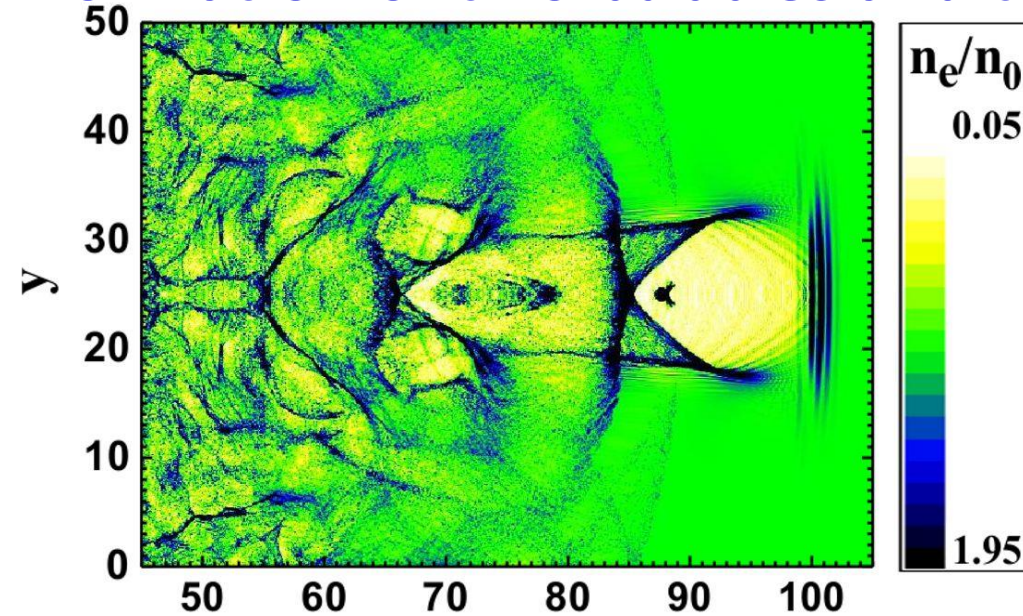
Laser pulse in the longitudinal direction is approximately Gaussian, " \cos^2 ", and Gaussian in transversal direction. Longitudinal half-length of laser pulse $= 2\lambda$. Radius of laser pulse $= 8\lambda$.

Normalized amplitude E_0 of laser pulse $b_0 = eE_0/m_e c \omega_0 = 5$.

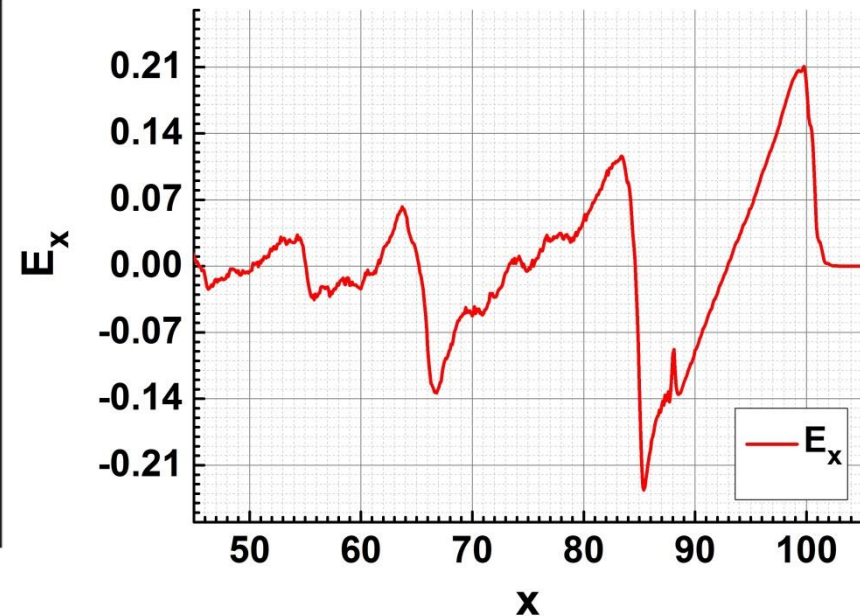
Intensity of laser pulse: $I = 5.3 \times 10^{19} \text{ W/cm}^2$.

Coordinates x and y , time t , amplitude of electric field E_x and plasma electron density n_0 are normalized on λ , $2\pi/\omega_0$, $m_e c \omega_0 / 2\pi e$, $m_e \omega_0^2 / 16\pi^3 e^2$.

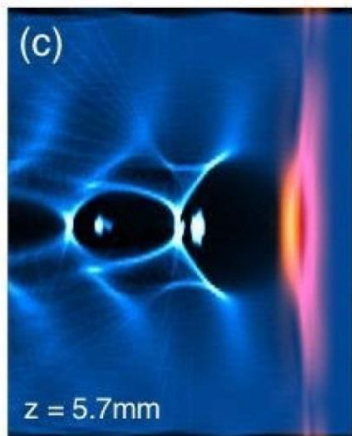
Formation of two bubbles and three short electron bunches



Wake perturbation of plasma electron density, excited by single laser pulse, at time $t = 105t_0$



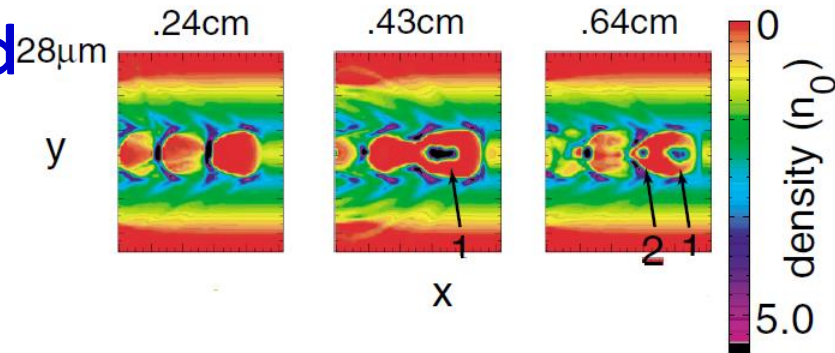
Longitudinal wakefield E_x , excited by single laser pulse, at time $t = 105t_0$



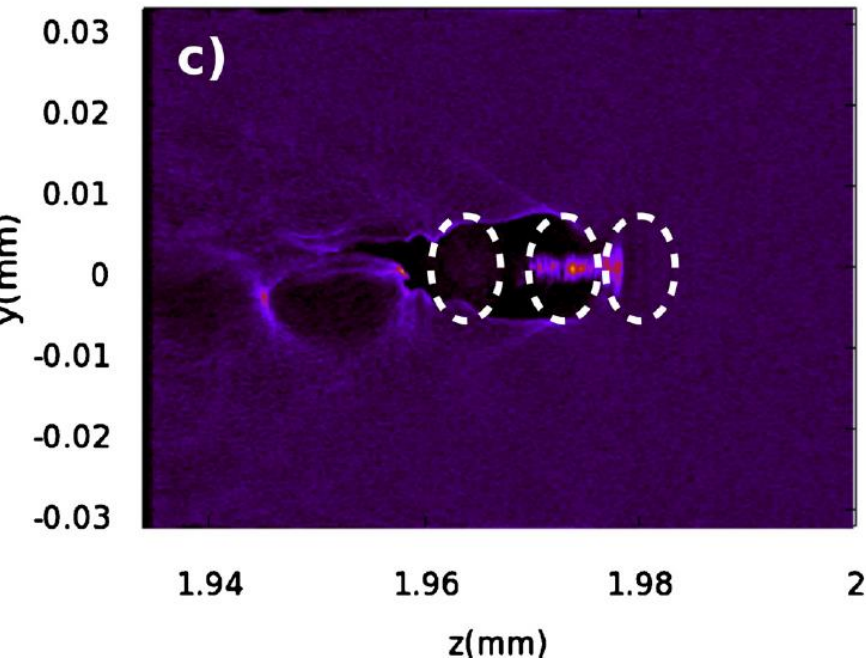
Multi-bunch self-injection at wakefield excitation by a laser pulse

W.Lu et al. 2007

F.S.Tsung et al. 2004



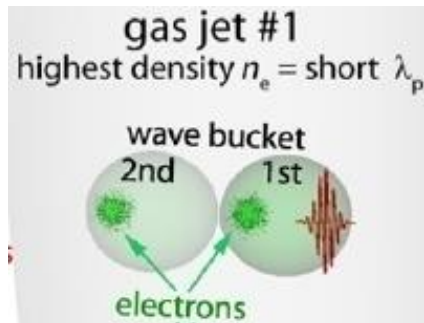
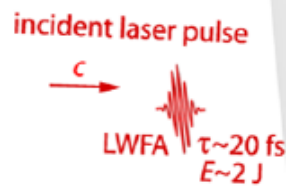
Multi-bunch self-injection at wakefield excitation by a laser pulse



K.H.Pae et al. 2010

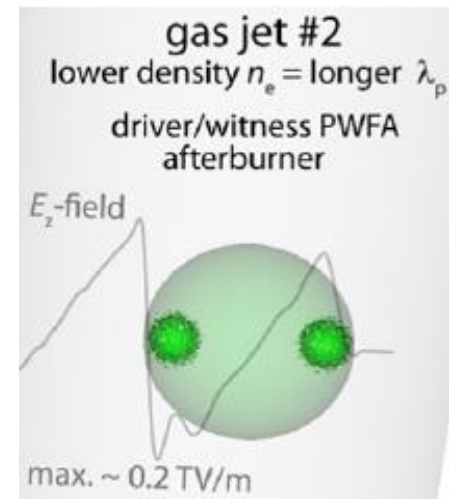
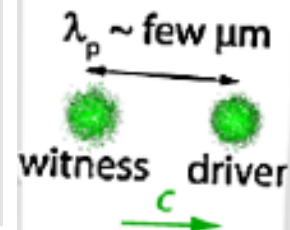
B.Hidding
et al. 2010

1st stage:

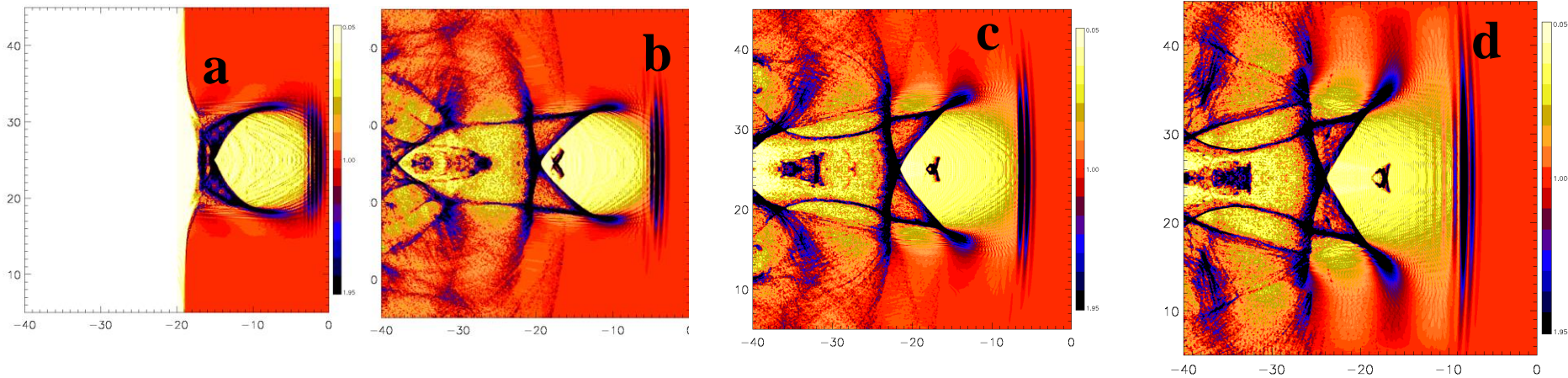


2nd stage:

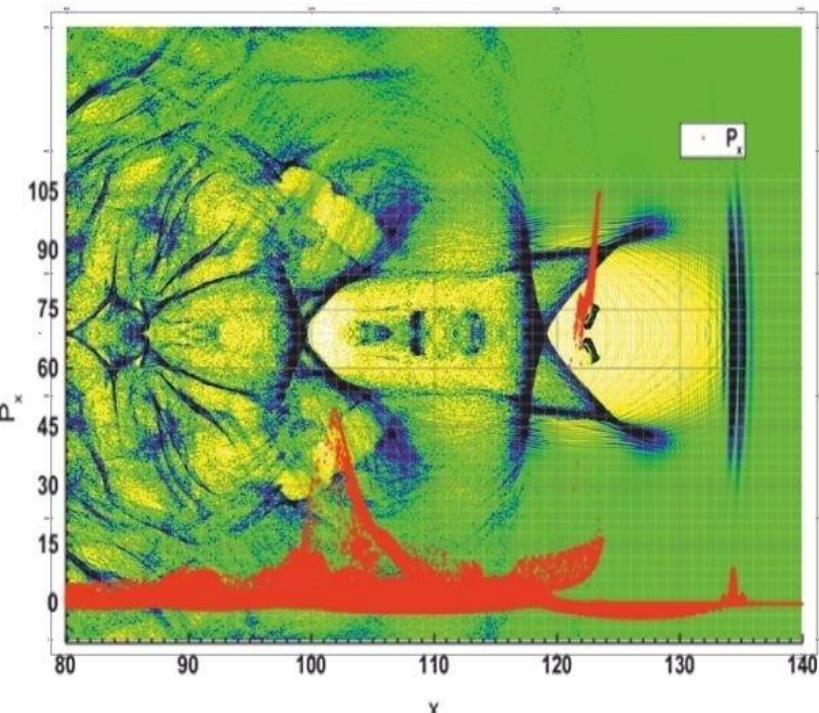
double bunch
in vacuum



Wakefield excitation by single laser pulse

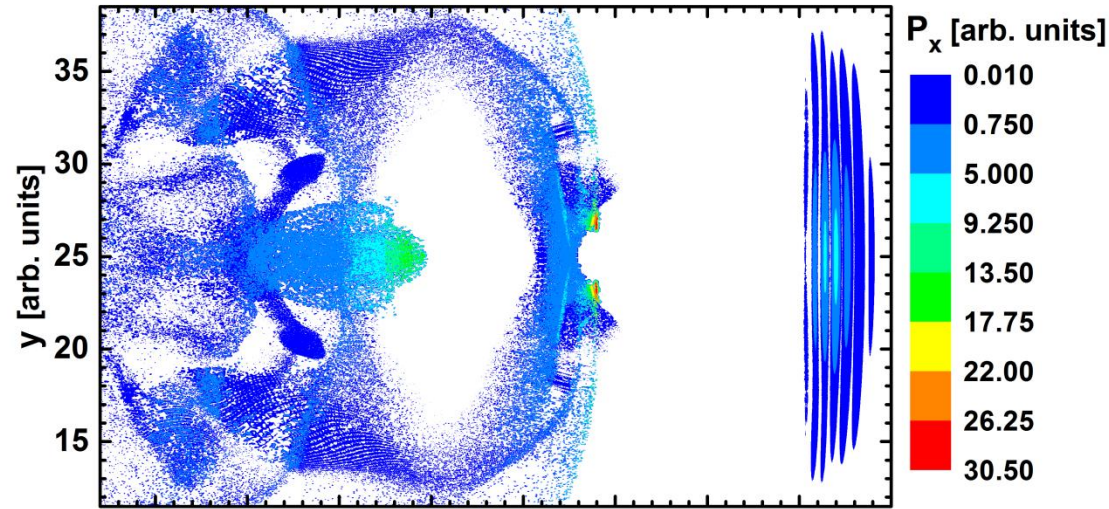


**Two plasma electron bubbles and several electron bunches,
accelerated by one laser pulse**

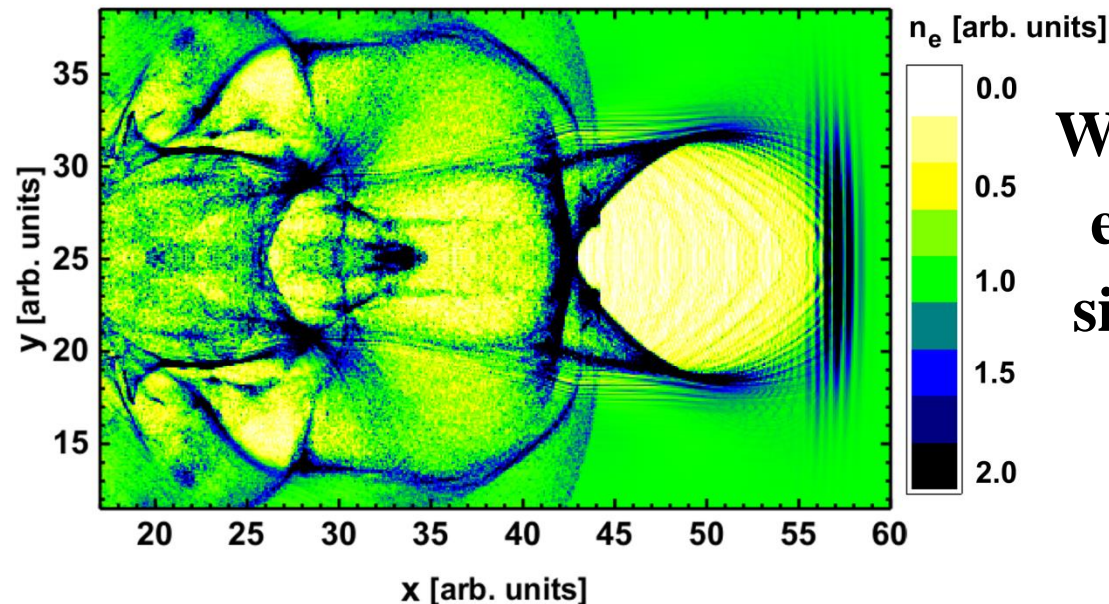


**Longitudinal distribution of the
longitudinal momentum of the
plasma electrons**

After about $60t_0$ from the beginning of the interaction of laser pulse with plasma laser wakefield acceleration scheme is transformed in the second bubble in the combined scheme of laser wakefield acceleration and beam wakefield acceleration.

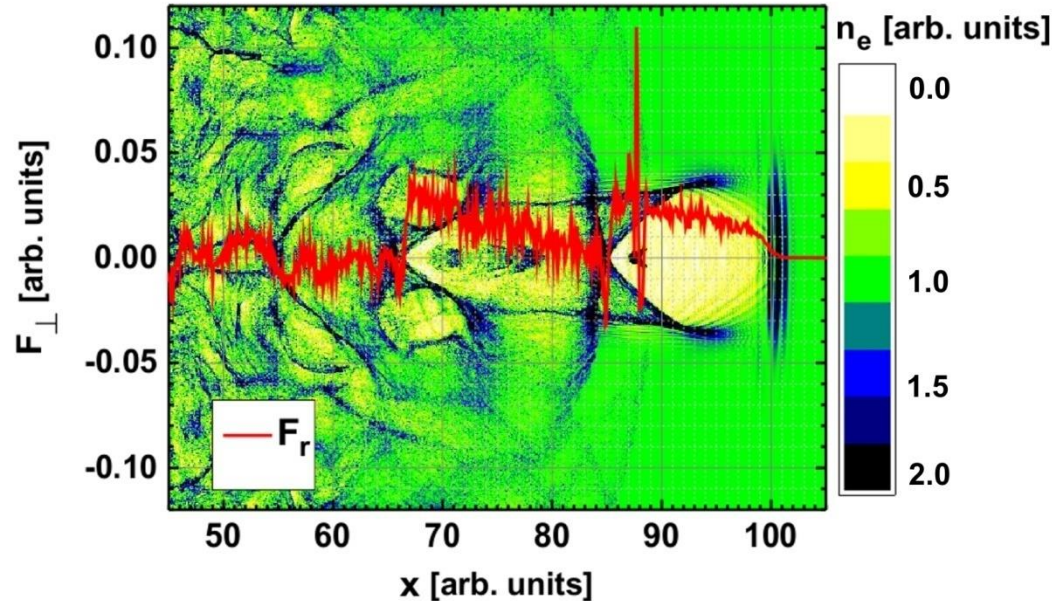


The longitudinal
momentum of the electrons
 P_x at time $t = 60t_0$



Wake perturbation of plasma
electron density, excited by
single laser pulse, at time $t =$
 $60t_0$

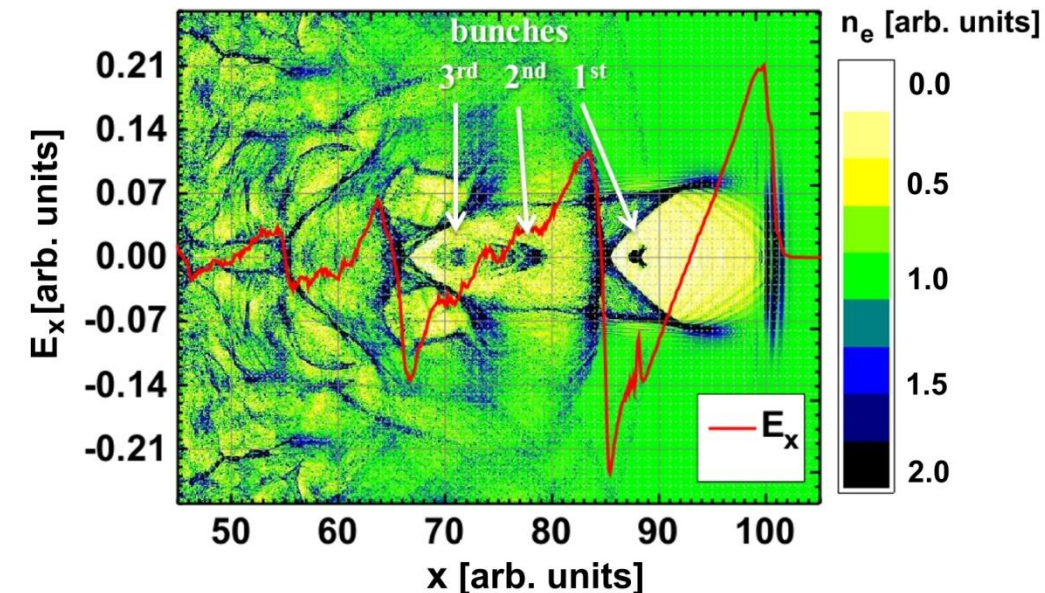
Self-injection of three short electron bunches



Wake perturbation of plasma electron density and radial wake force F_r (red line) is out of axis (at = 25.5), excited by single laser pulse, at time $t = 105t_0$

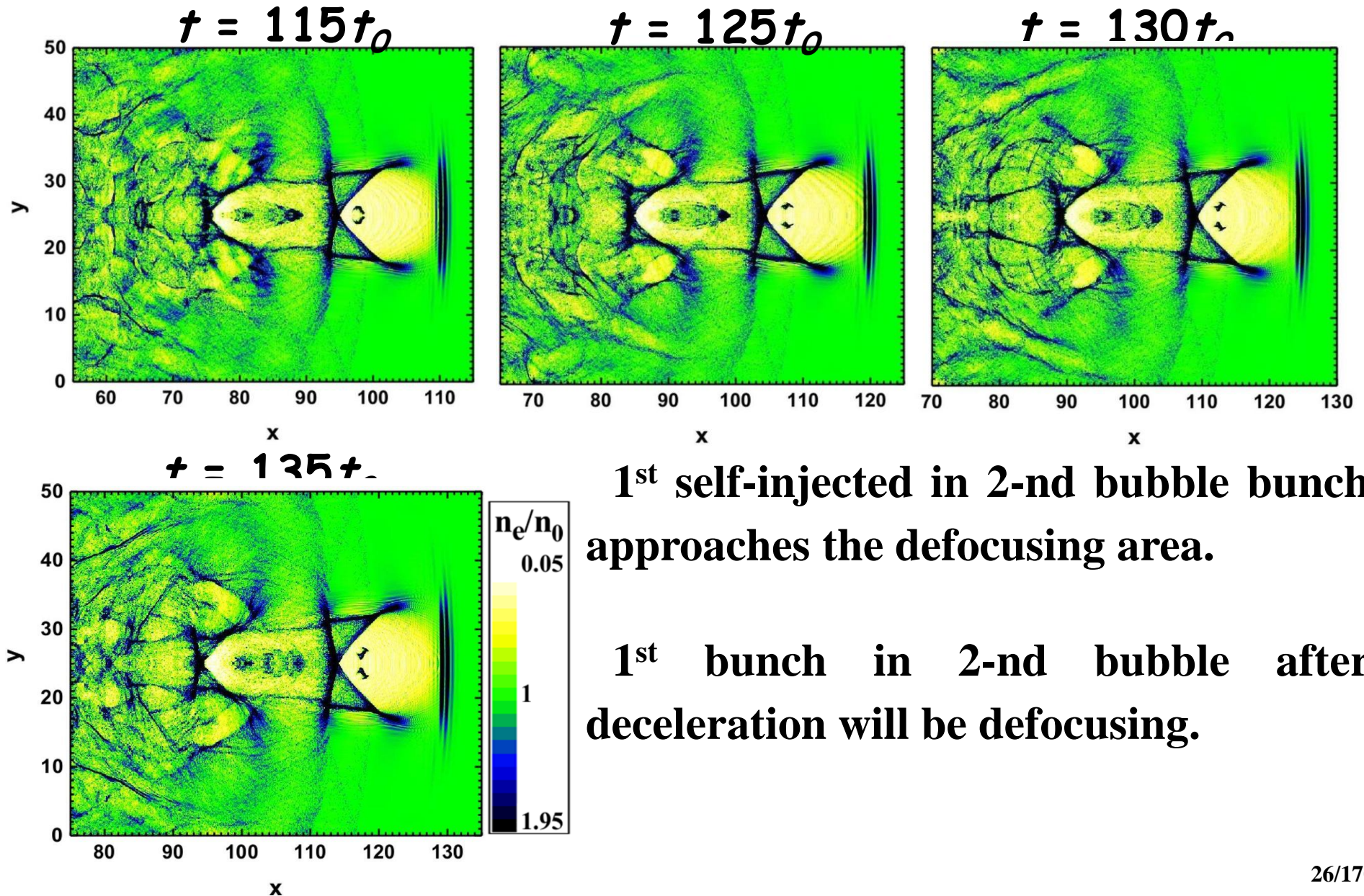
1st bunch in 2-nd bubble is decelerated, it is close to region with strong defocusing field.

2nd bunch in 2-nd bubble is accelerated.

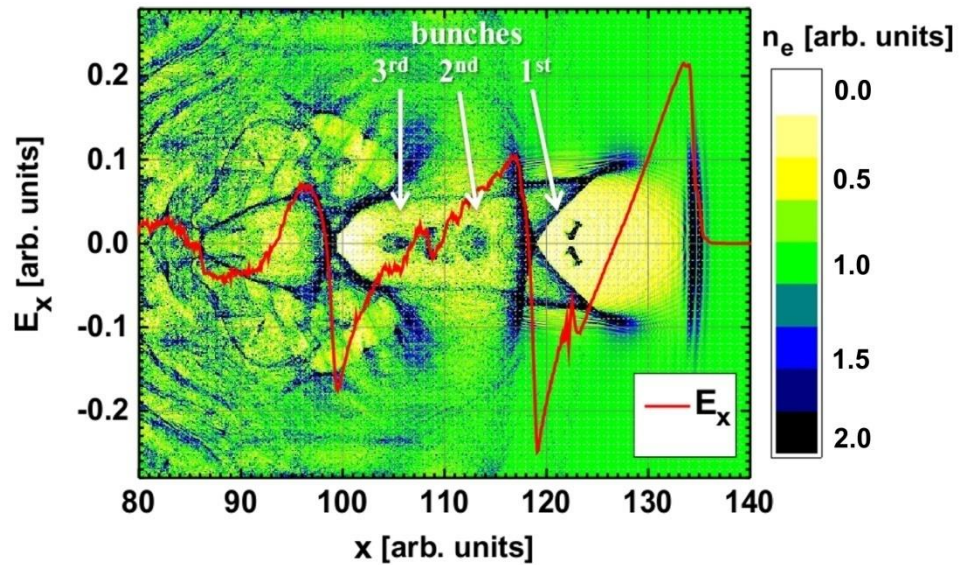


Wake perturbation of plasma electron density and longitudinal wakefield E_x (red line), excited by single laser pulse, at time $t = 105t_0$

1st bunch in 2-nd bubble approaches the defocusing area

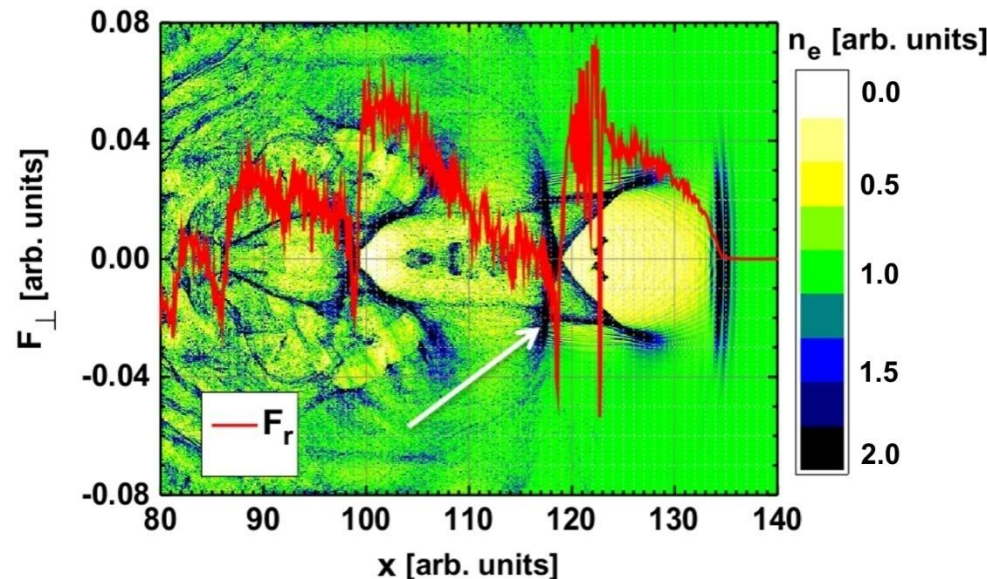


Defocusing 1st bunch self-injected in 2-nd bubble



Wake perturbation of plasma electron density and longitudinal wakefield E_x (**red line**), excited by single laser pulse, at time $t = 140t_0$

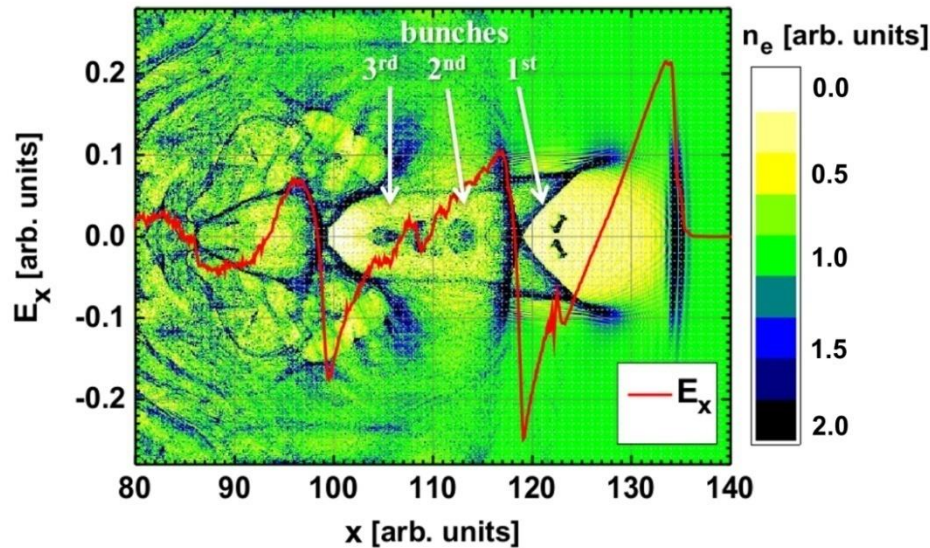
1st bunch in 2-nd bubble is in the region with defocusing field and 1st bunch in 2-nd bubble is approaching the area with more defocusing field.



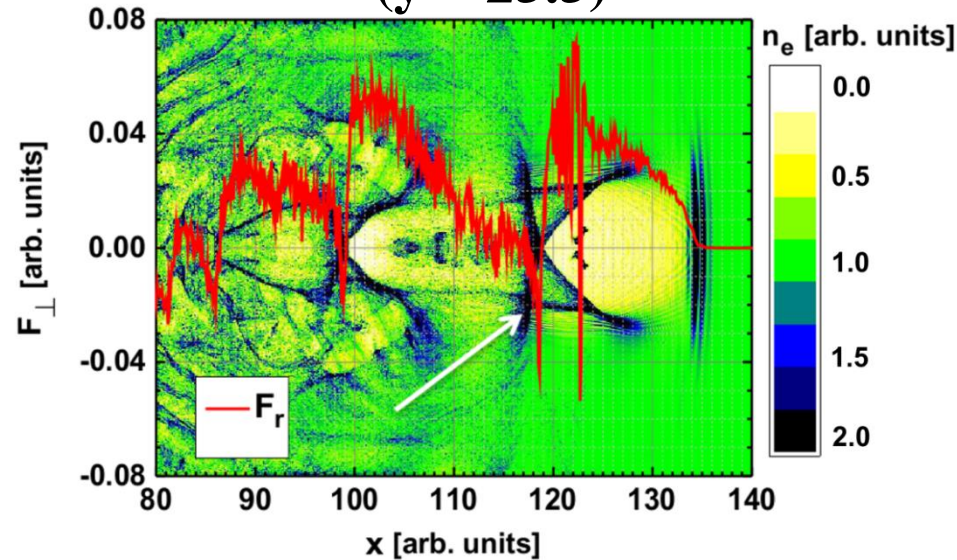
Wake perturbation of plasma electron density and radial wake force F_r (**red line**) is out of axis (at = 25.5), excited by single laser pulse, at time $t = 140t_0$

Wake perturbation, excited by single laser pulse, at time $t = 140t_0$

Longitudinal wakefield E_x



Radial wake force F_r out of axis ($y = 25.5$)

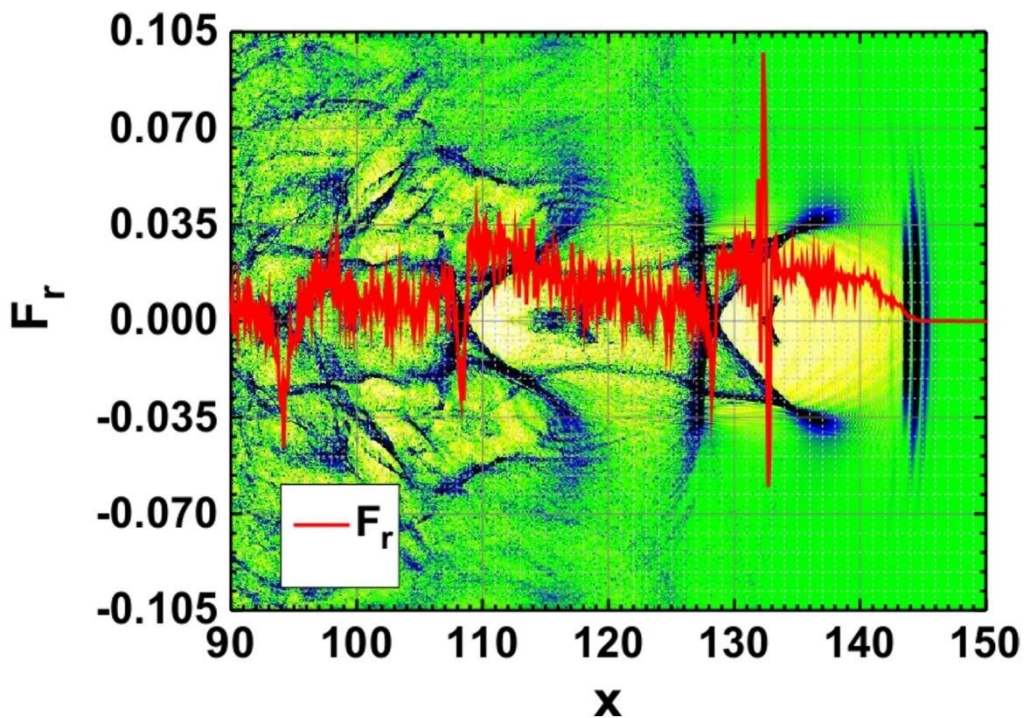
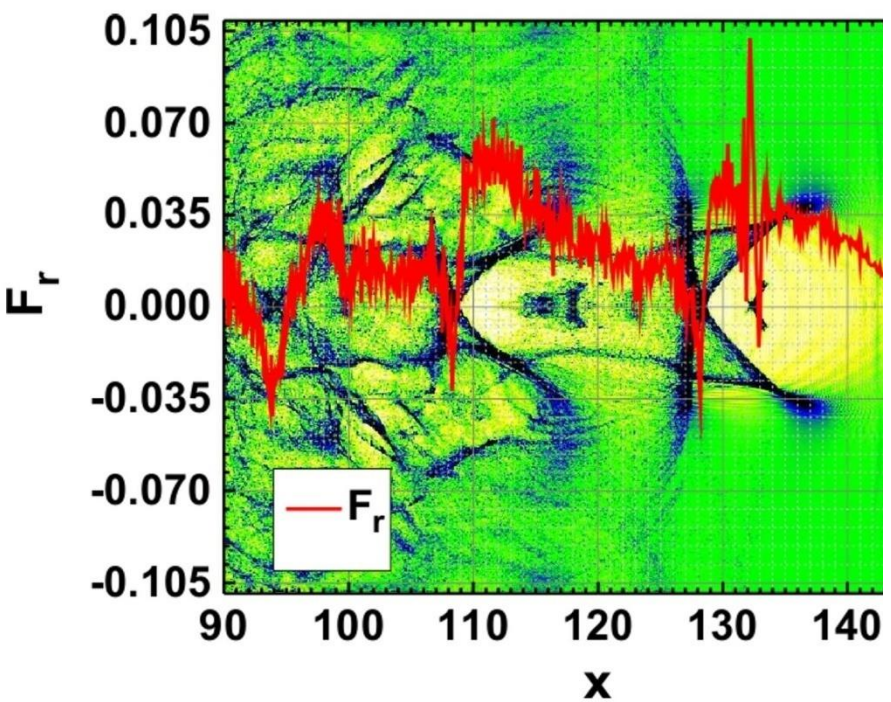
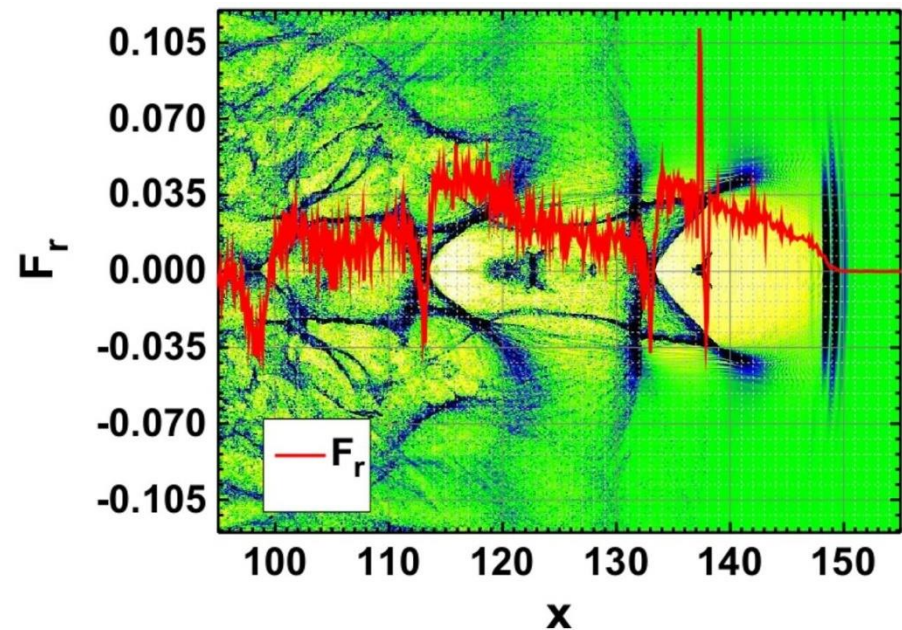
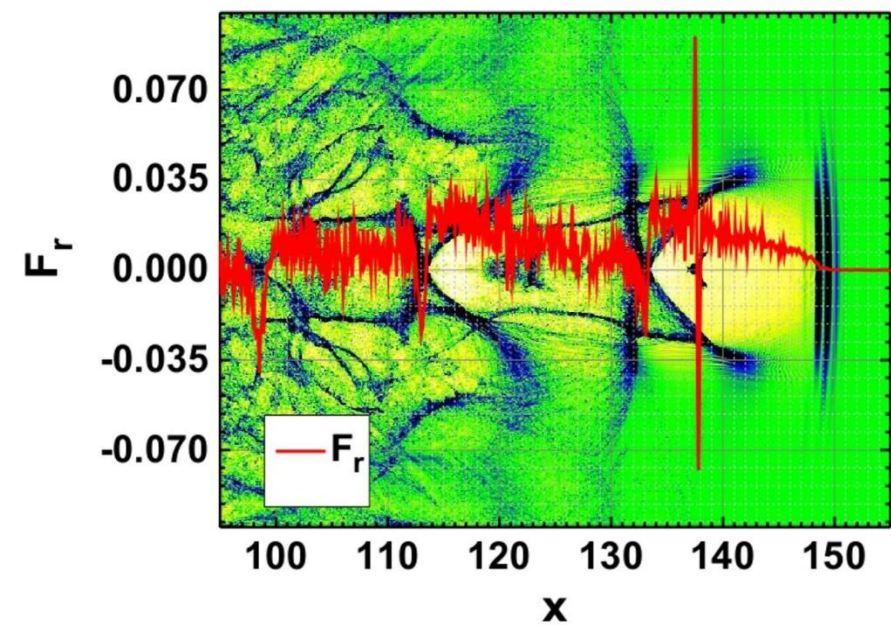


1st bunch in 2-nd bubble is in the area of defocusing . Therefore, with further approaching to 1-st wake steepening (the back edge of the 1st bubble) 1st bunch in 2-nd bubble will be quickly self-cleaned due to defocusing.

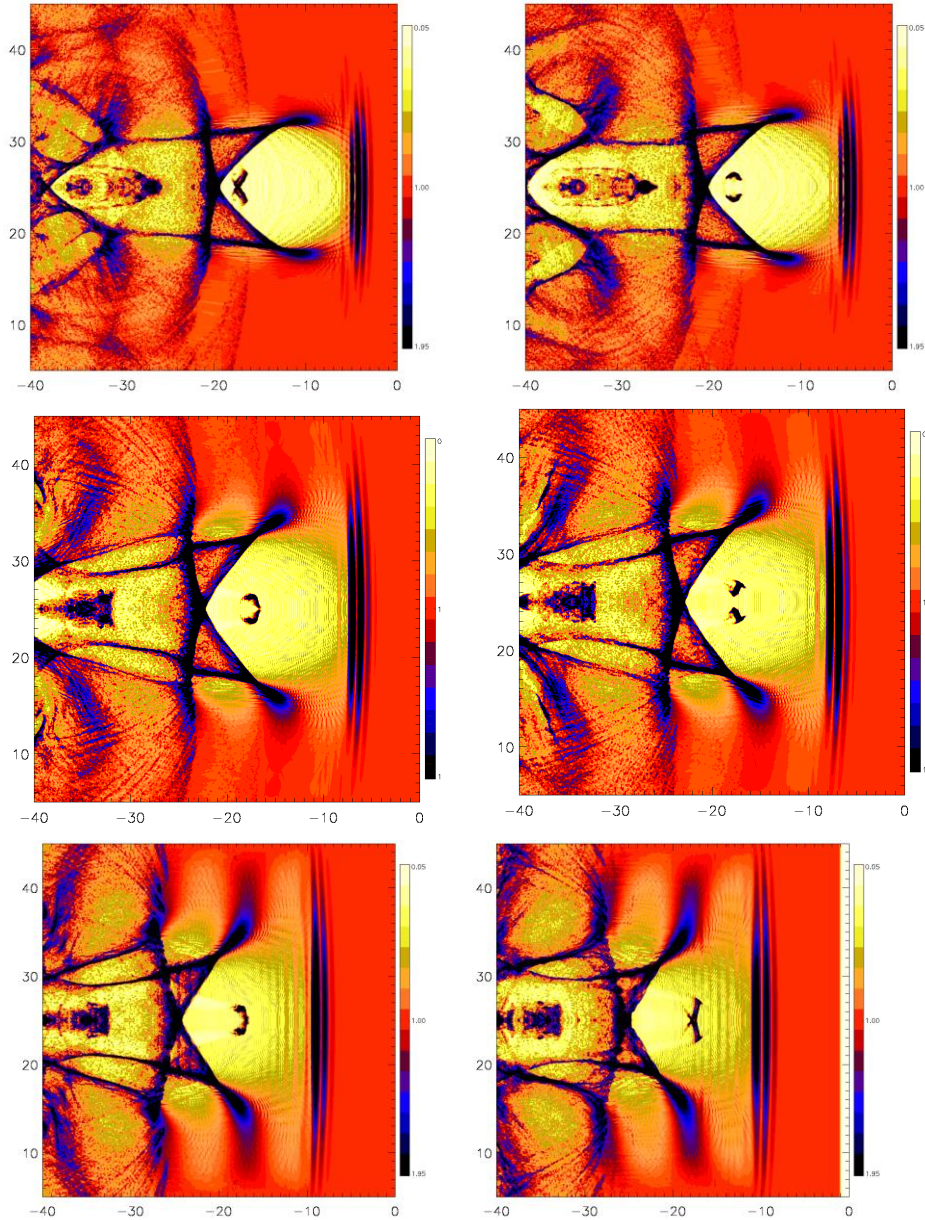
Other bunches are in the focusing fields and oscillate along the radius.

Part of 1st accelerated bunch in 1-st bubble is focused to the axis so that its field of space charge is larger than the bubble field. Consequently, at the following times this part of the 1st bunch is periodically oscillated along radius.

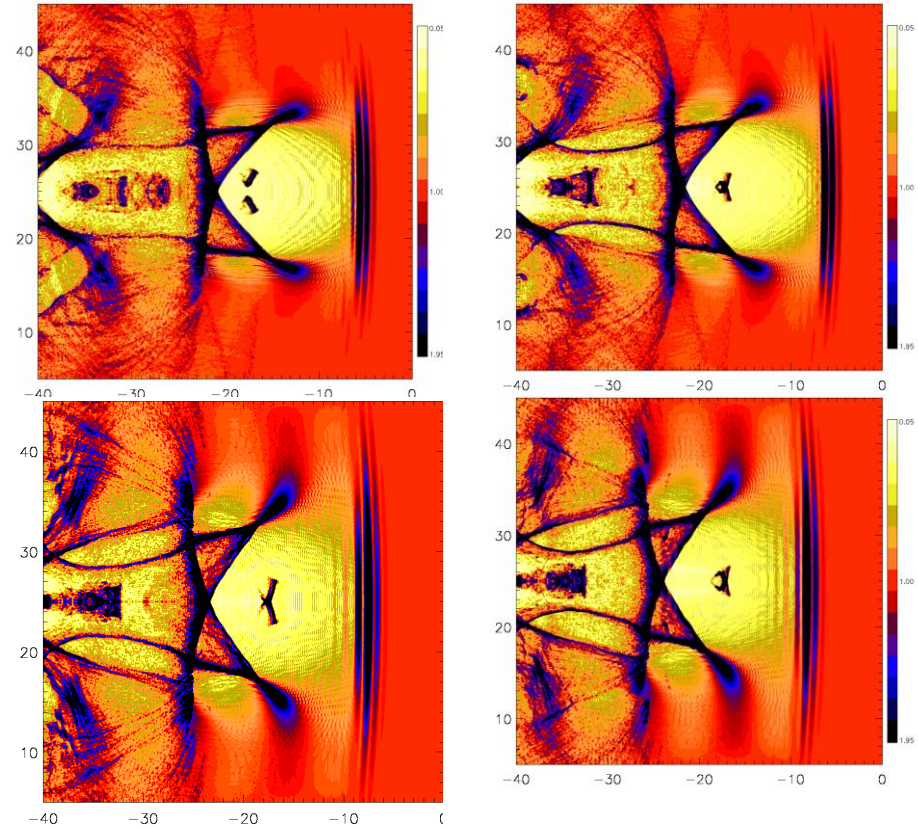
Betatron oscillations



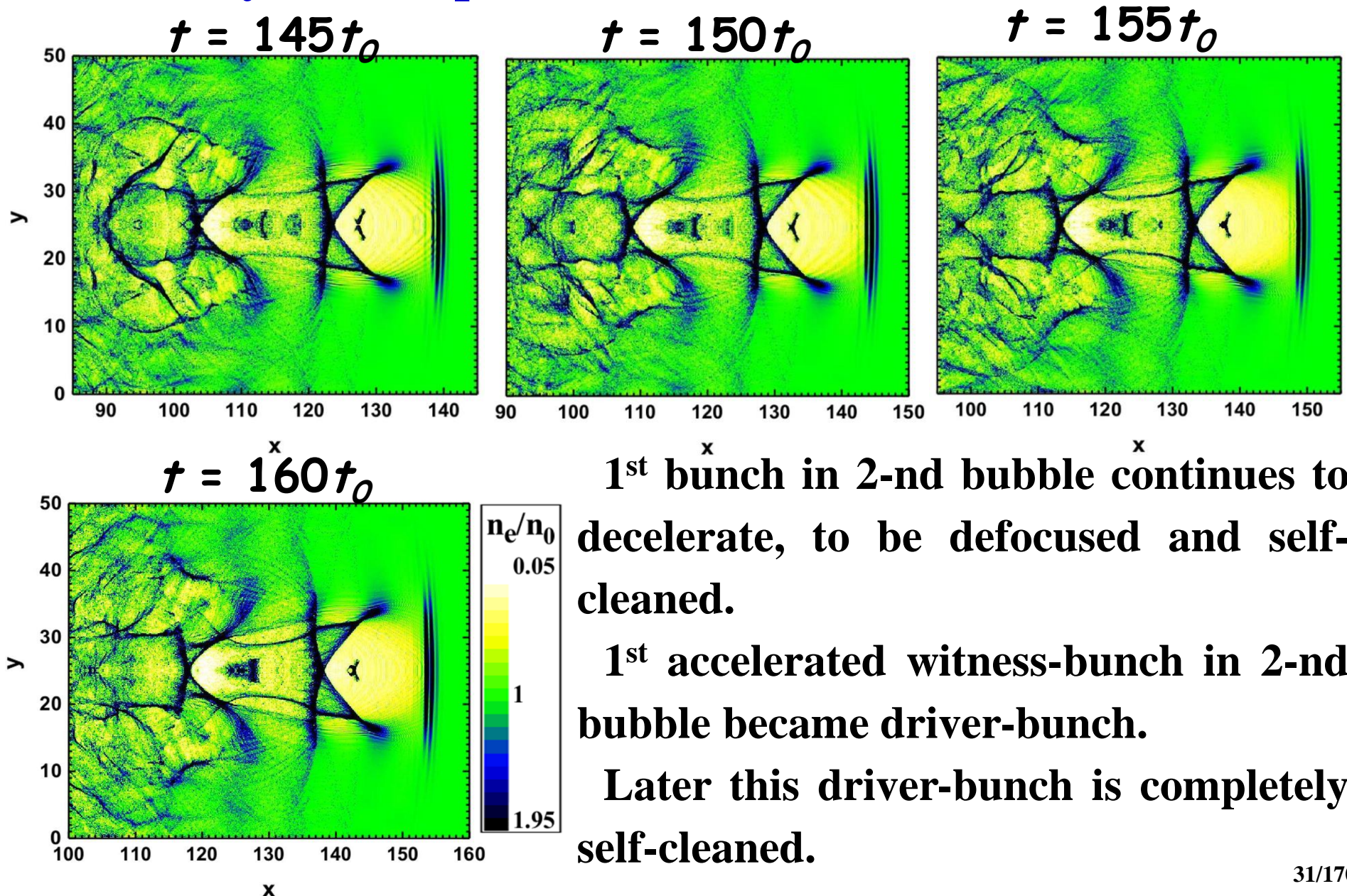
Betatron oscillations



Plasma undulator



1-st mechanism of amplify the laser wakefield acceleration by bunch (plasma) wakefield acceleration

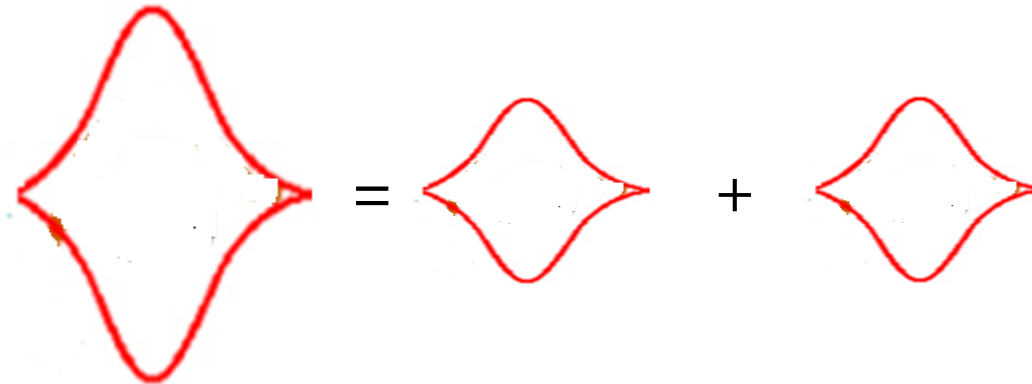


Benefit of injection of train of laser pulses

Multi-pulse laser wakefield acceleration: a new route to efficient, high-repetition-rate plasma accelerators and high flux radiation sources

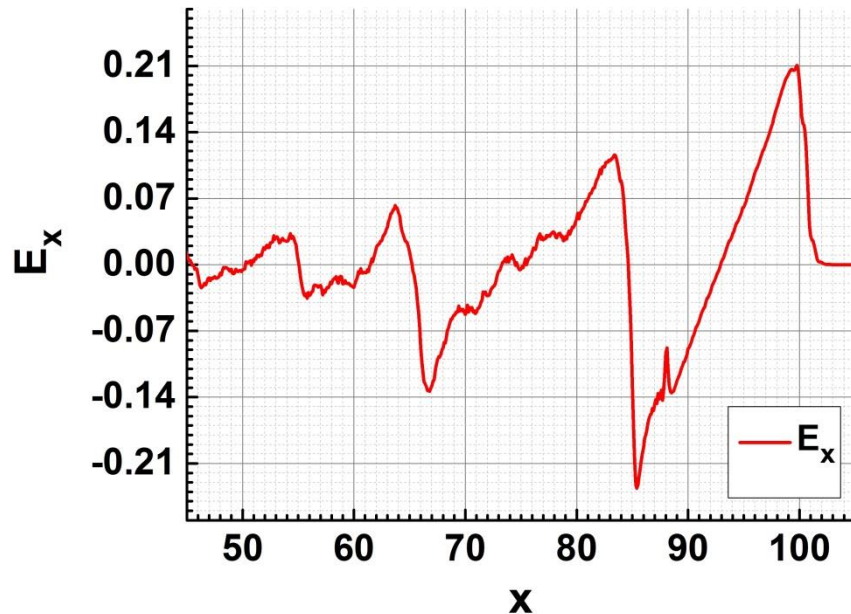
S.M. Hooker

Really since the best results on electron bunch acceleration by laser pulse have been achieved not at the maximum parameters of the laser pulse, it is advantageous to convert the laser pulse with the maximum parameters in a train of several pulses and to receive increased current of accelerated electrons.



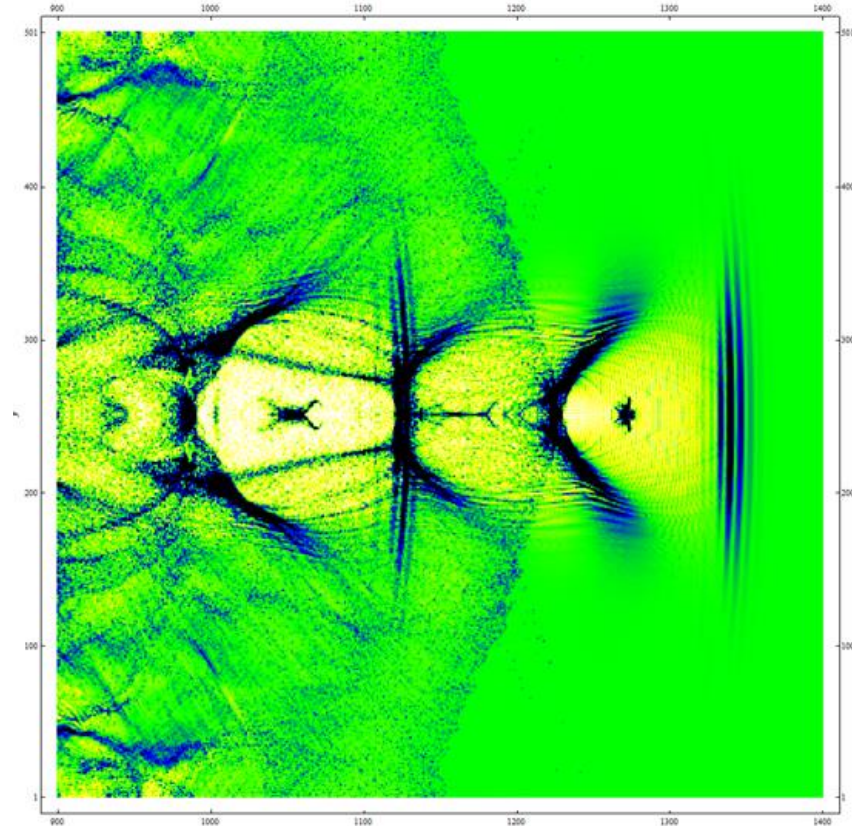
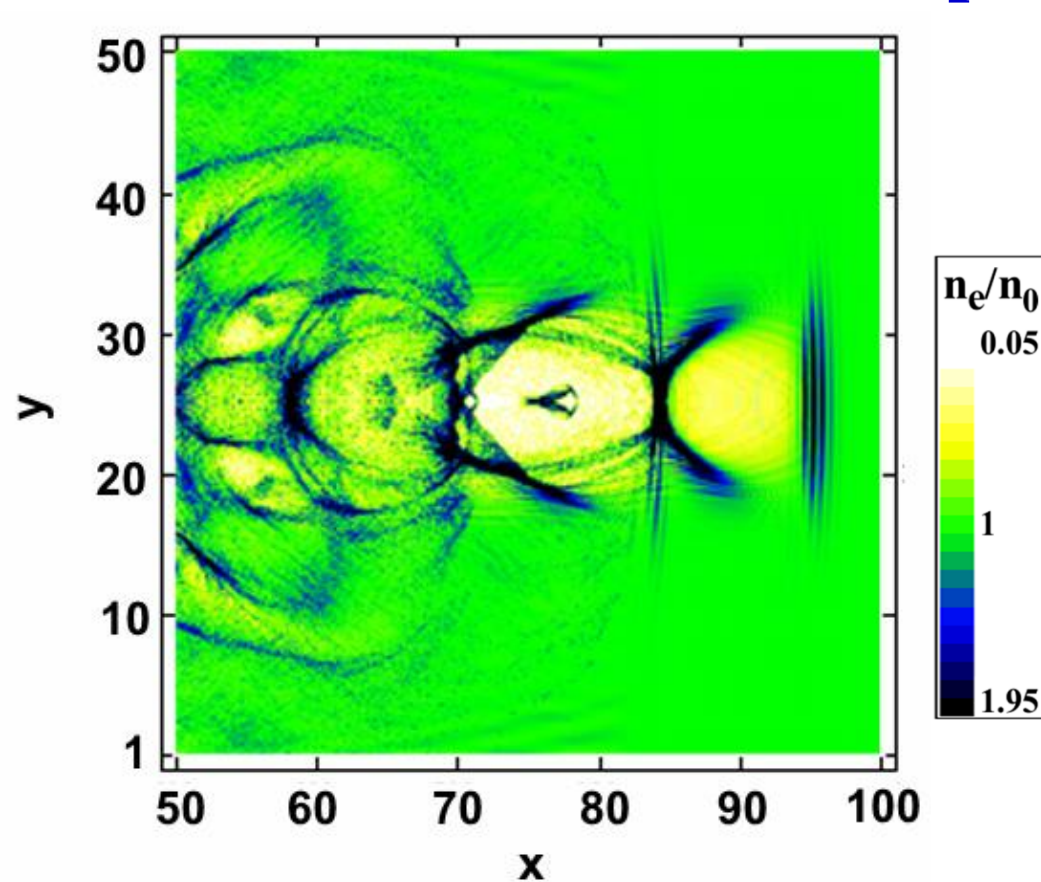
Benefit of injection of train of laser pulses

Also, because after the first bubble there is wake it is useful for increase efficiency to enhance its by next laser pulse and to use for acceleration of next electron bunches. Therefore, we consider both cases: injection of single laser pulse and injection of a short train of two laser pulses.



Longitudinal wakefield E_x ,
excited by single laser pulse

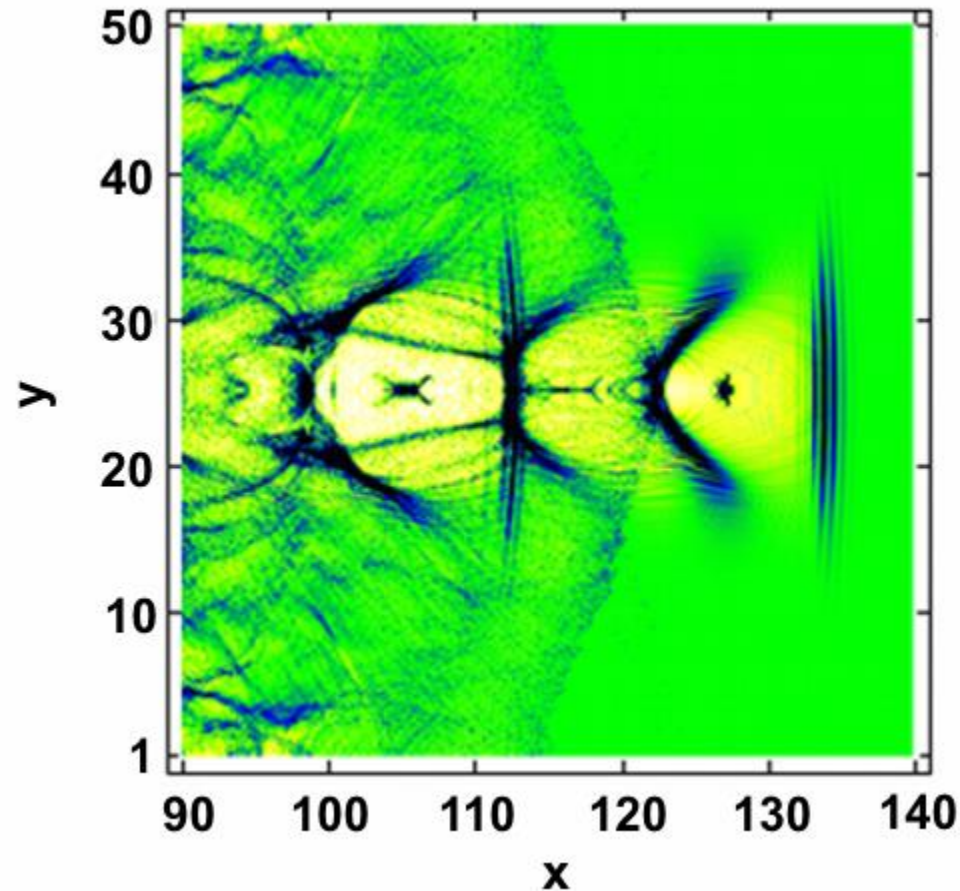
Self- injection of electron bunches at the injection of two laser pulses



In the case of two laser pulses, distributed through one λ , bunch of the accelerated electrons is formed only after the last pulse.

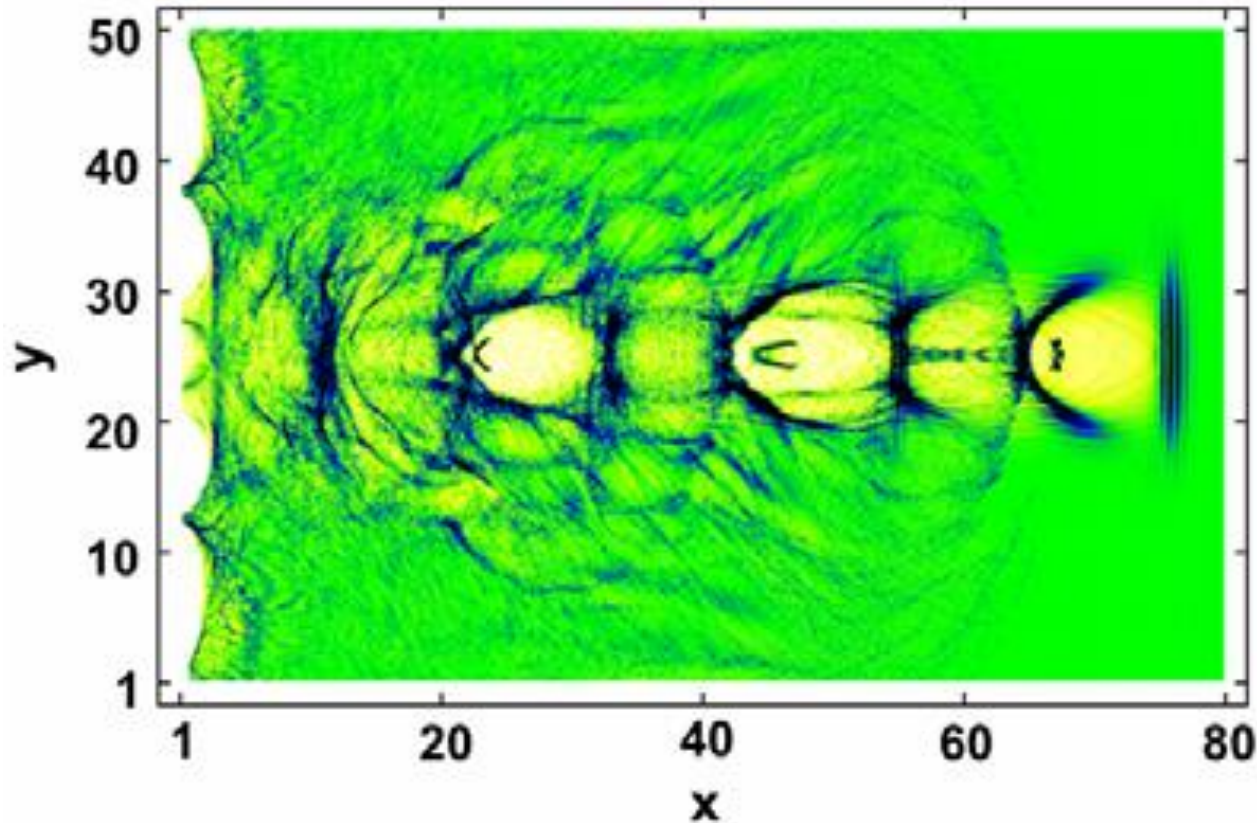
If two laser pulses are distributed through two λ , the electron bunch is accelerated after every pulse.

Self- injection of electron bunches at the injection of two laser pulses



Train of two electron bunches, accelerated by a train of two laser pulses, distributed through two wavelengths of plasma oscillations

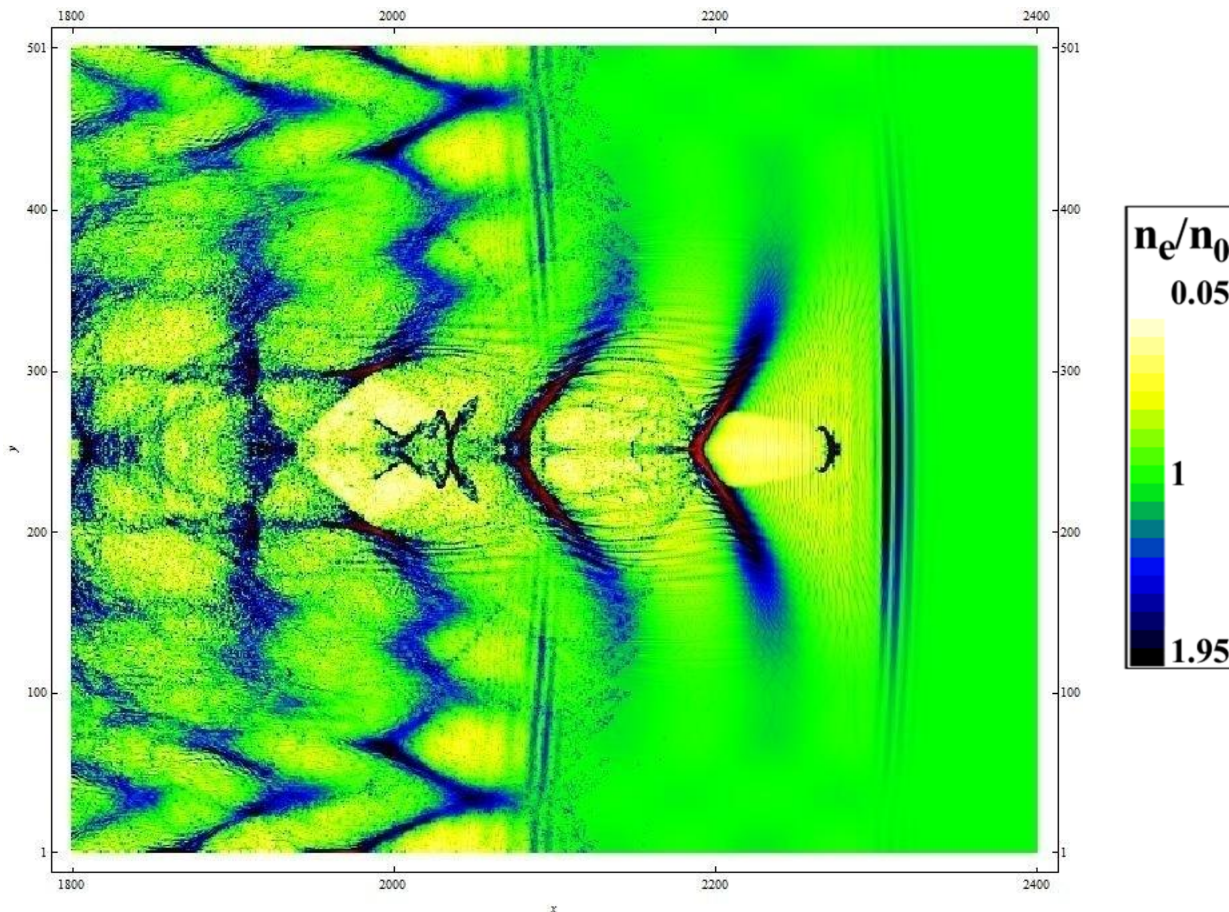
Self- injection of electron bunches at the injection of three laser pulses



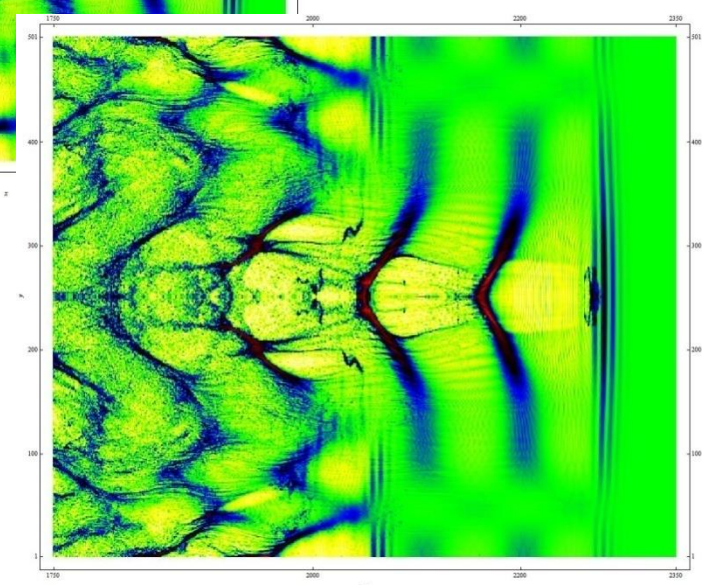
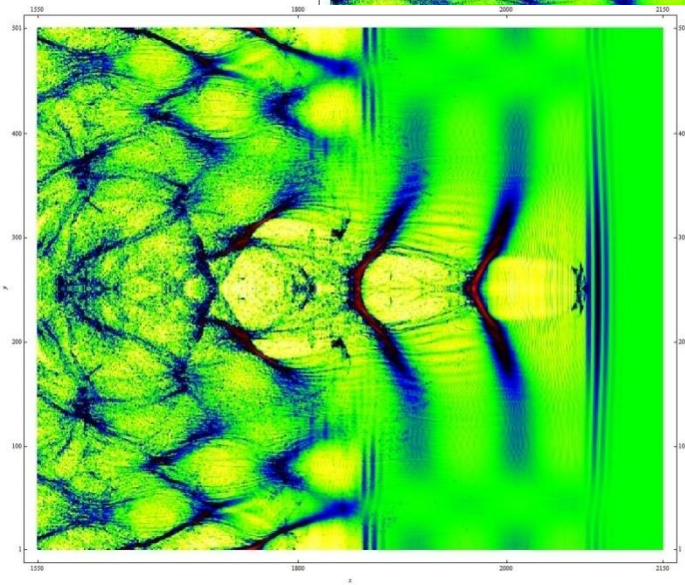
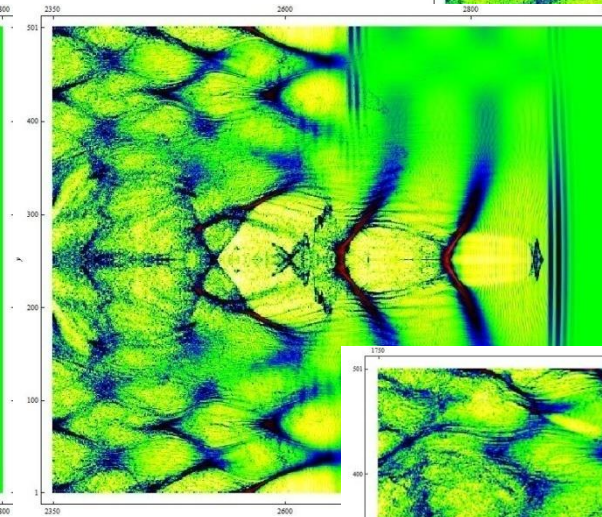
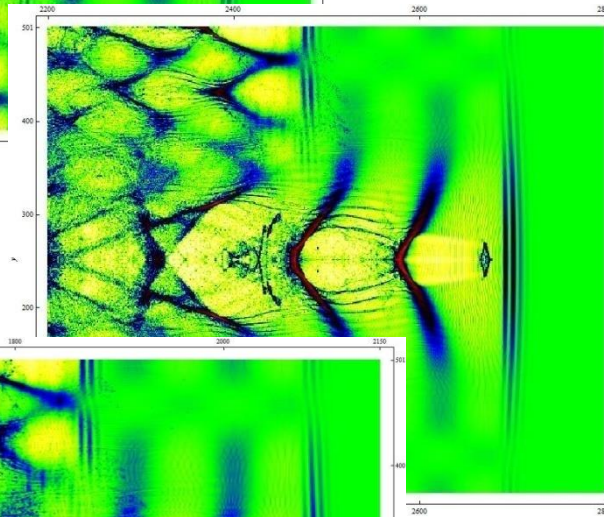
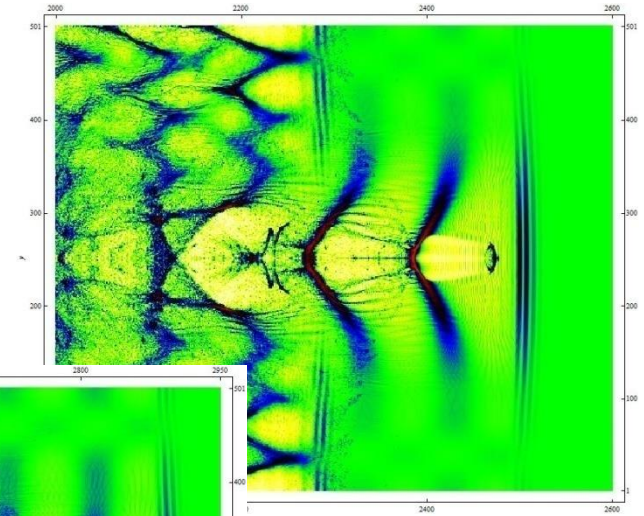
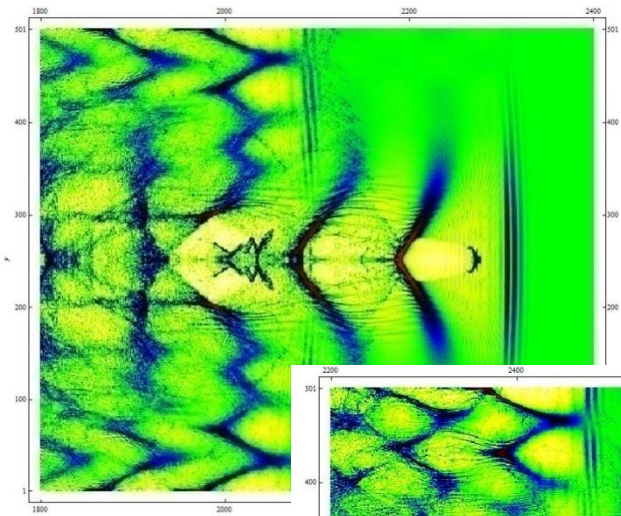
Wake perturbation of plasma electron density, excited by a train of three laser pulses of large intensity, distributed through two wavelengths of plasma oscillations, and train of three accelerated electron bunches

Self-cleaning due to defocusing at the injection of two laser pulses

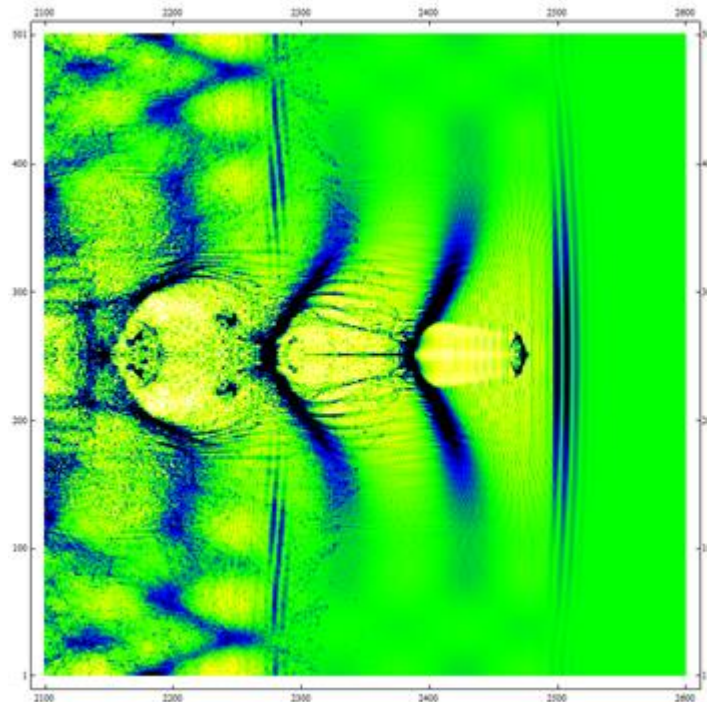
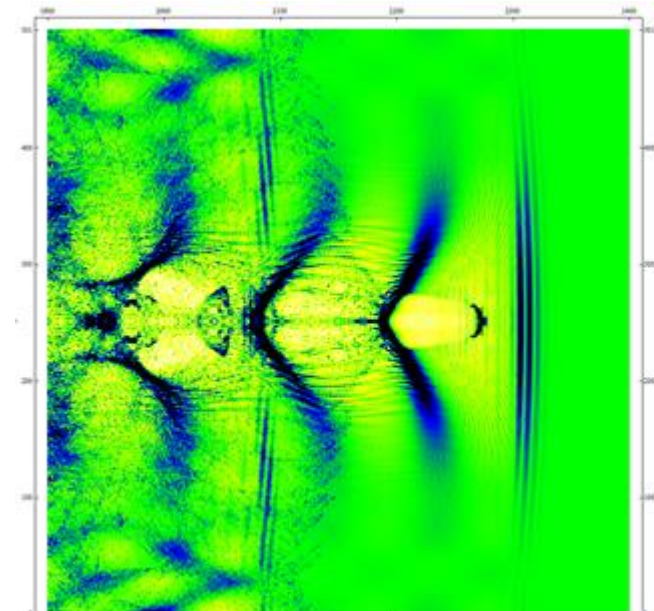
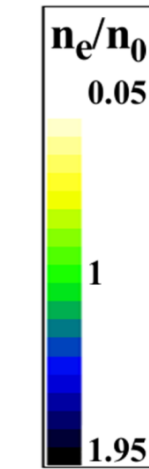
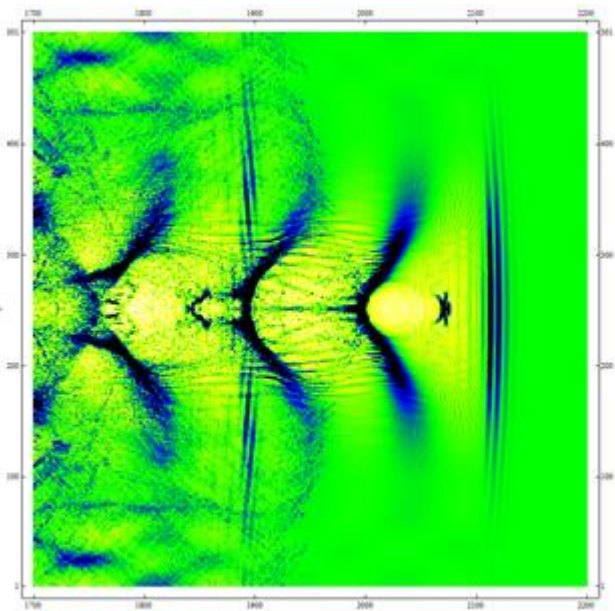
In the case of injection of two laser pulses 1st witness-bunch in 3-rd bubble, which became driver-bunch, after deceleration is defocused.



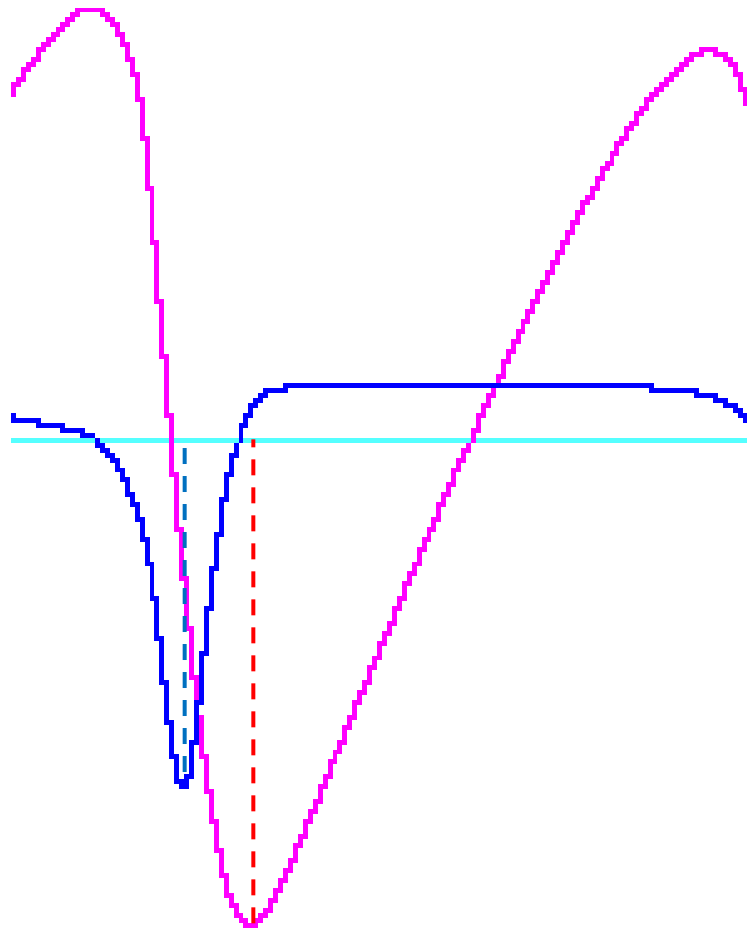
Self-cleaning due to defocusing at the injection of two laser pulses

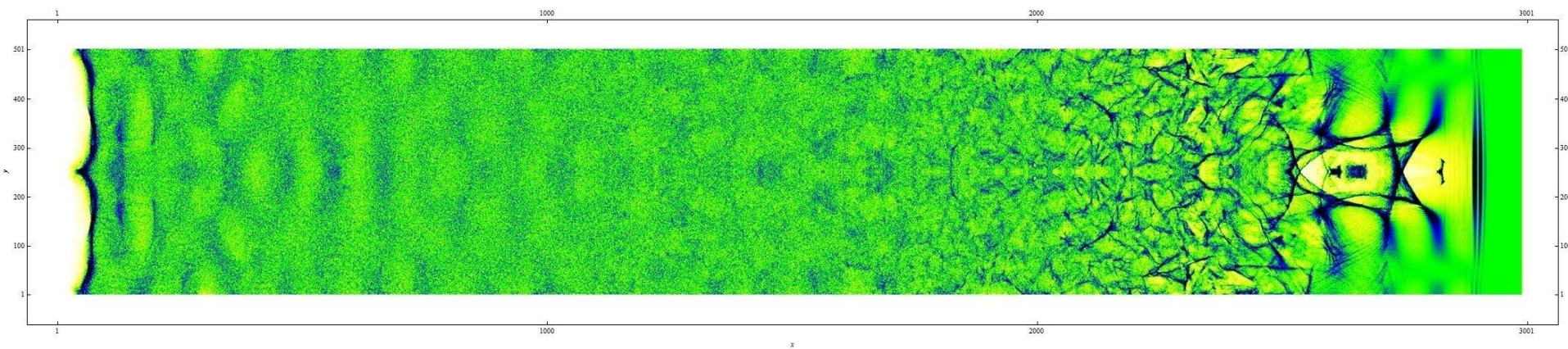
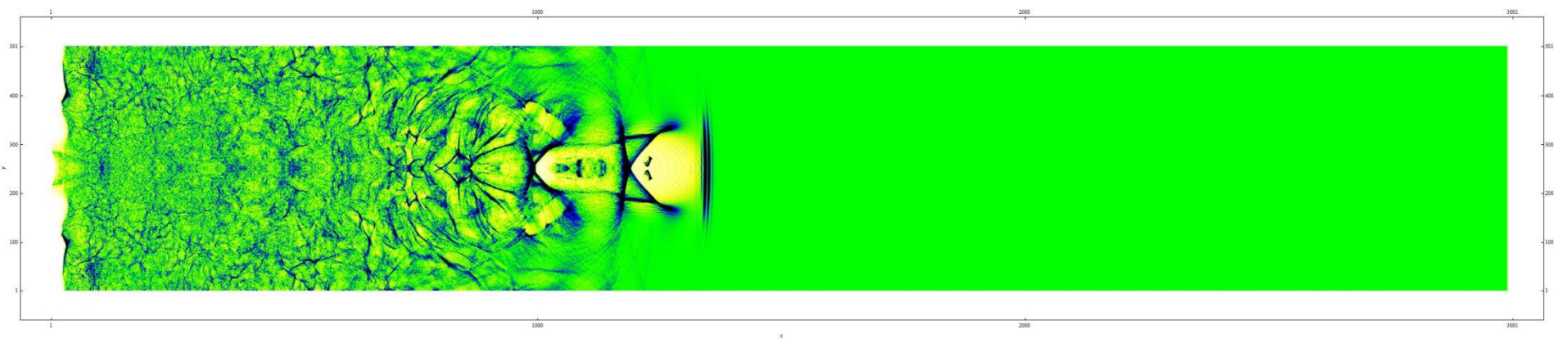
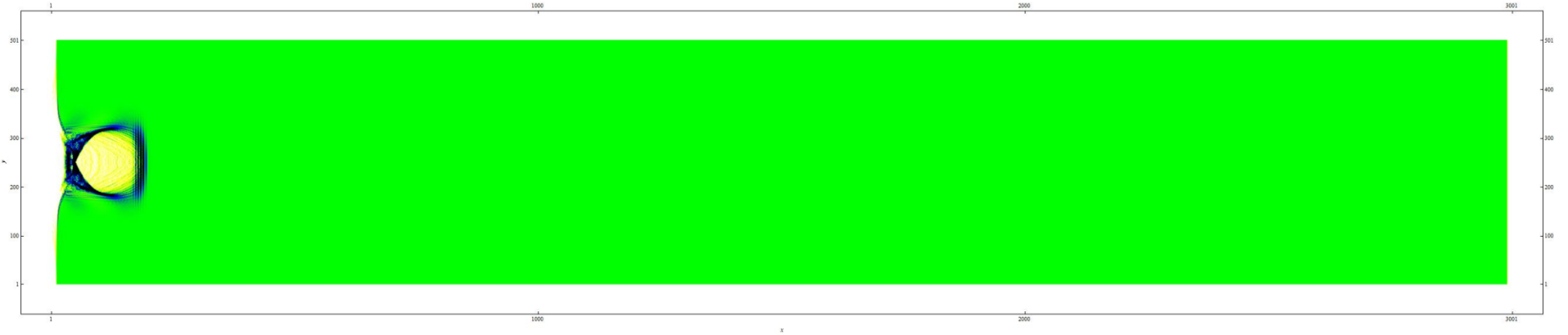


Self-cleaning due to defocusing at the injection of two laser pulses

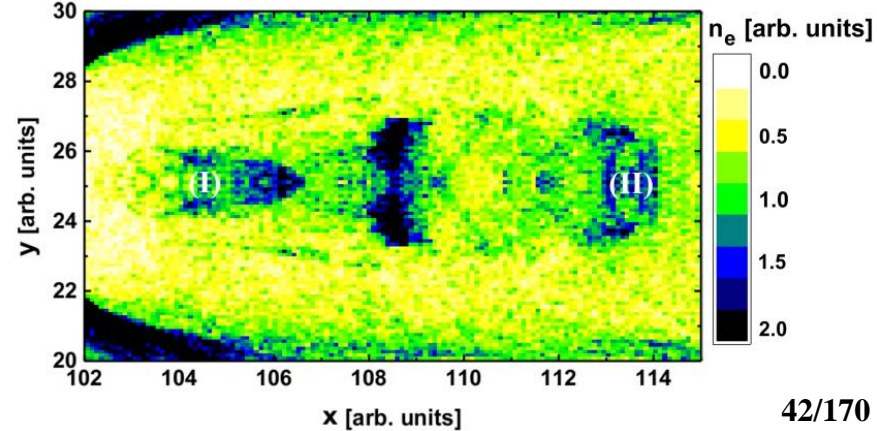
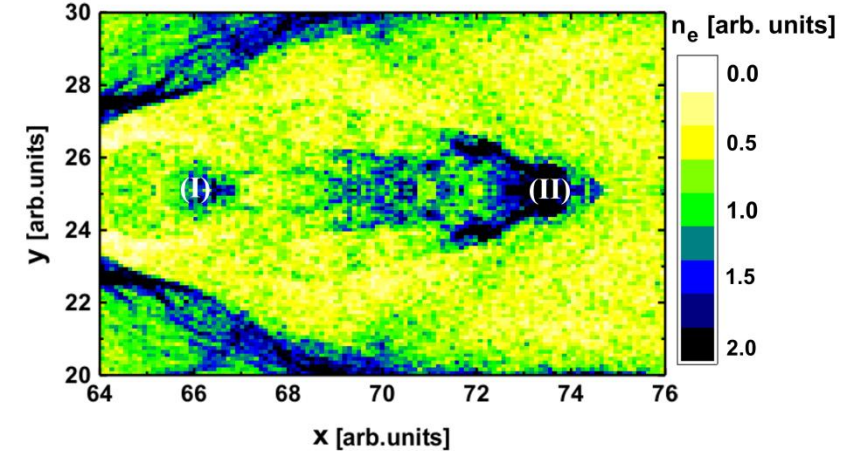
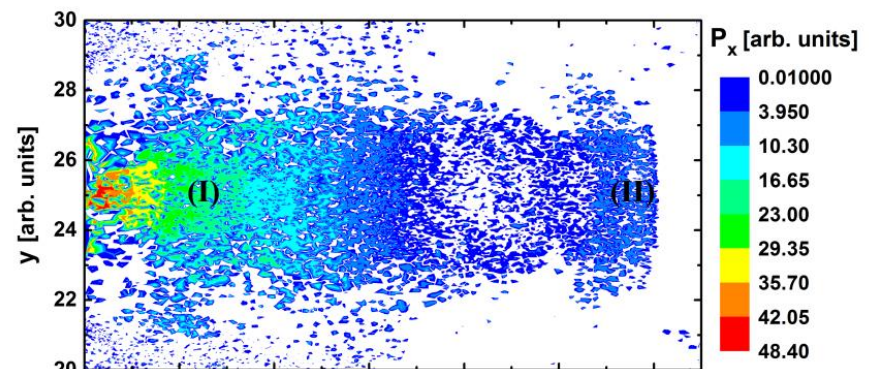
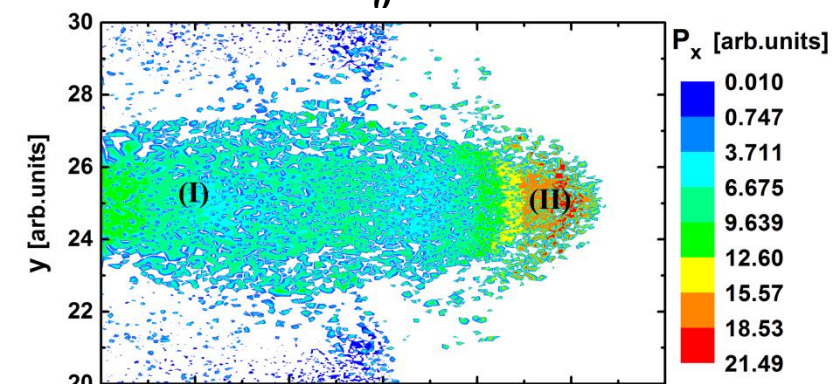
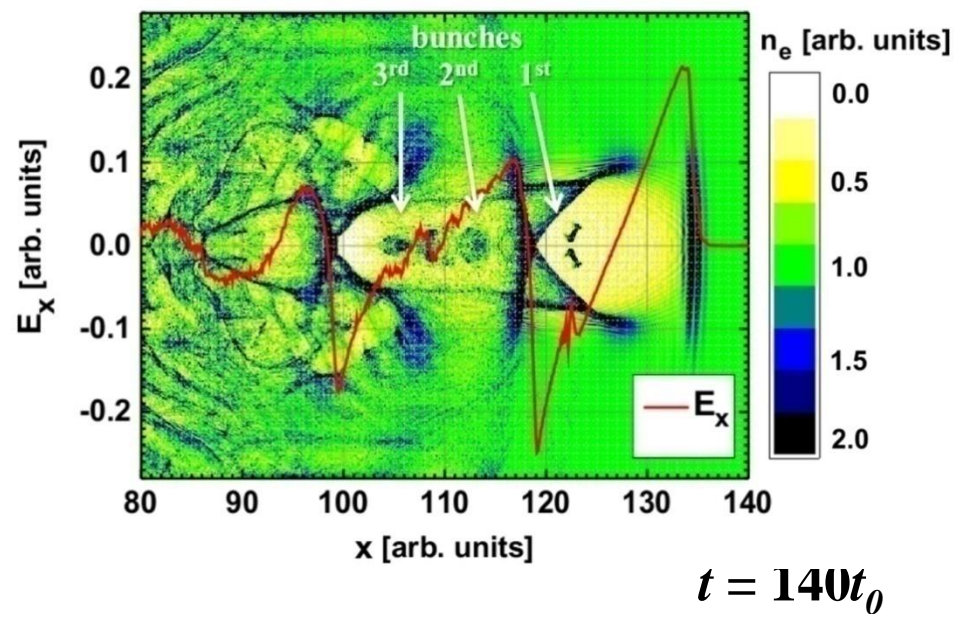
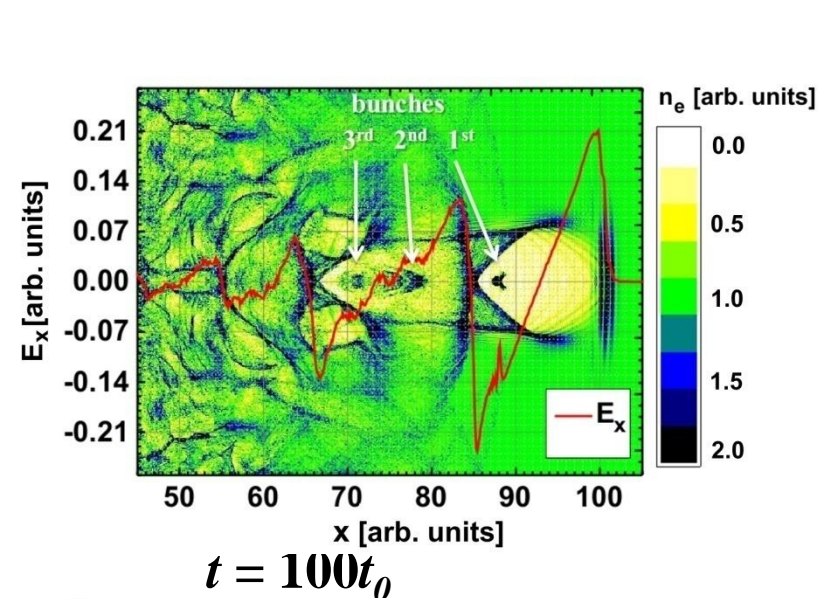


Relative shift of the region of self-injection of electron bunches and the region of strong defocusing

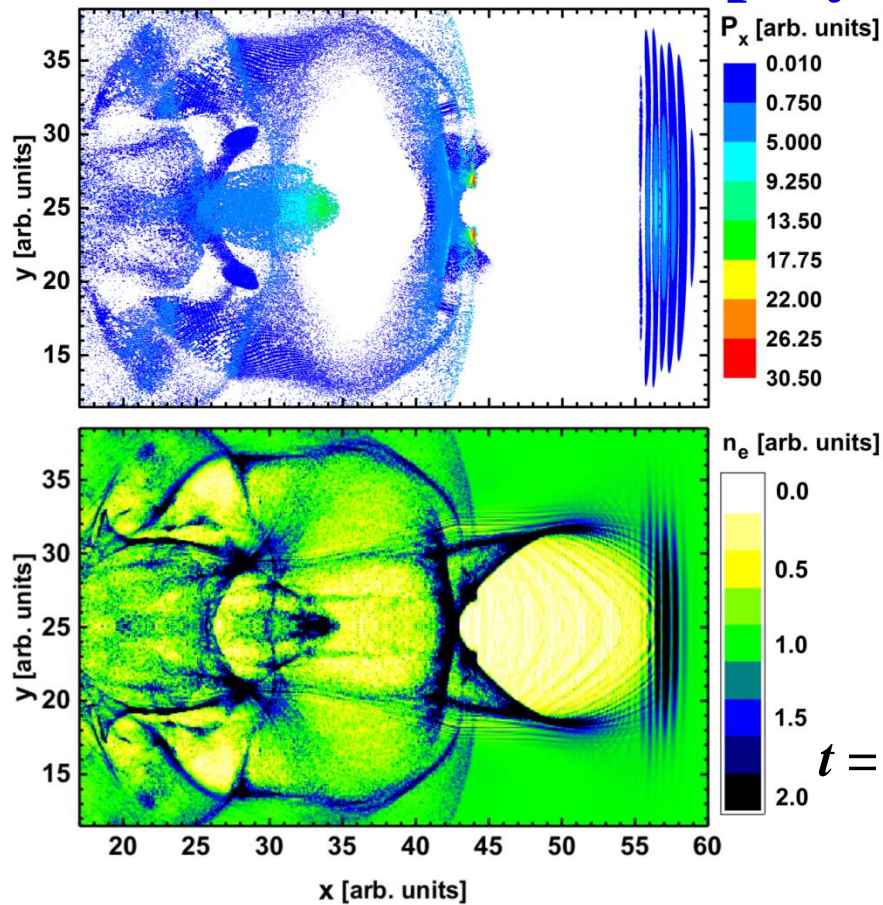




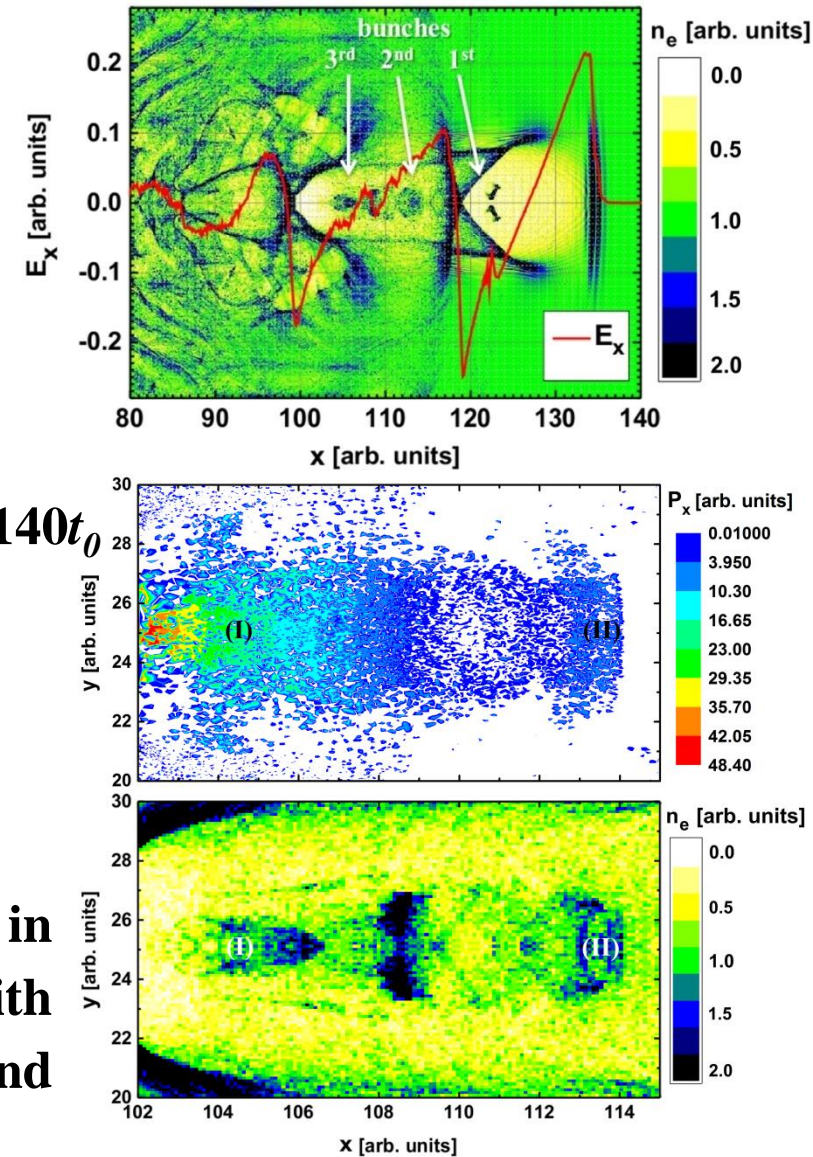
Penetration into the plasma



1-st mechanism of amplify the laser wakefield acceleration

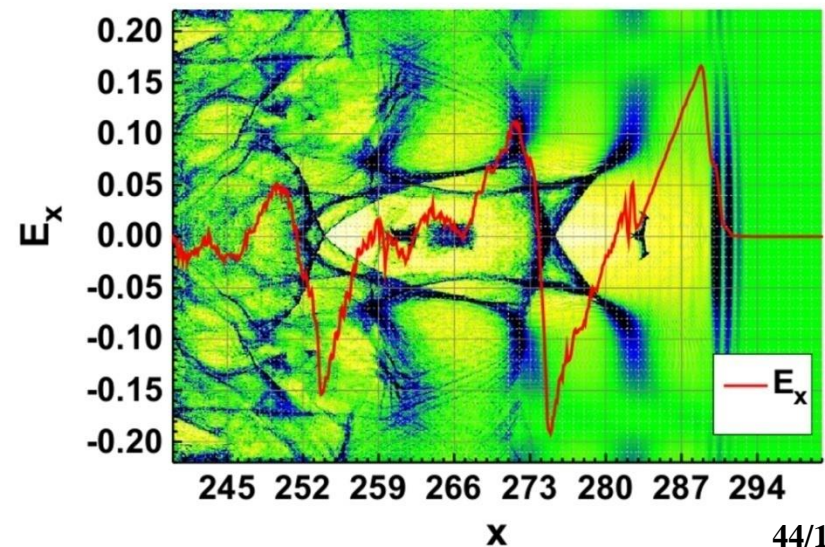
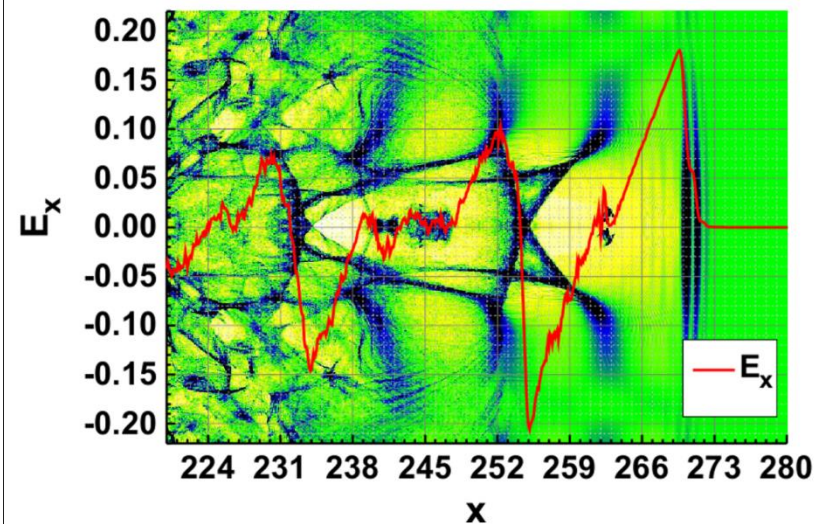
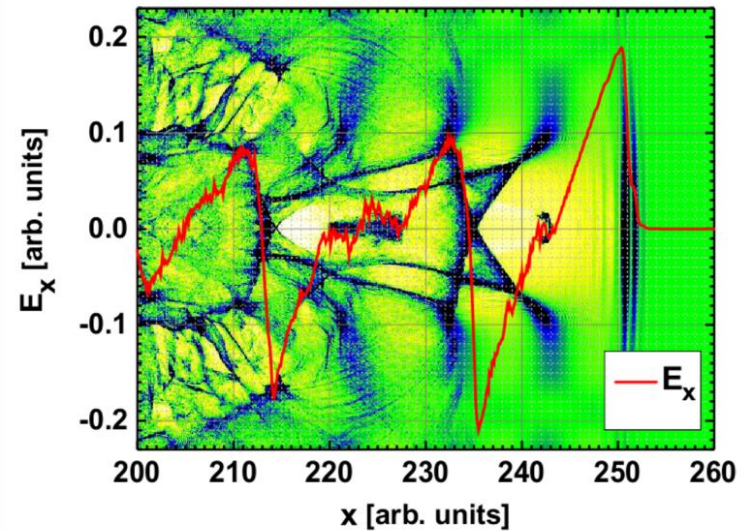
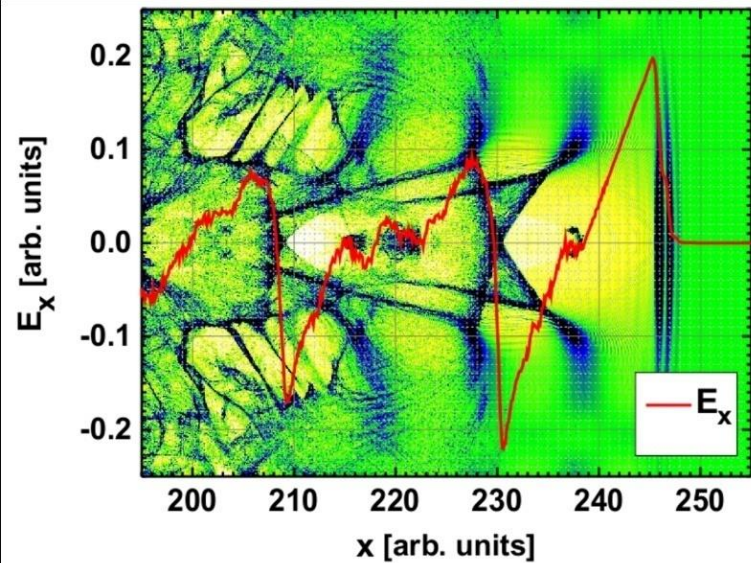


$t = 140t_0$

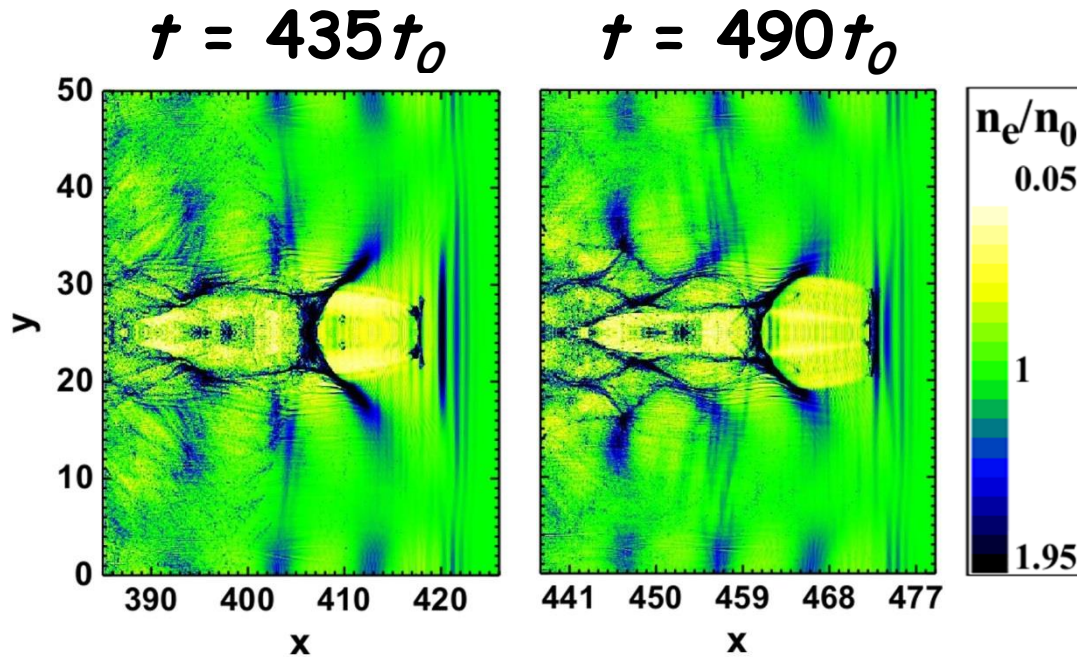


Transformation of 1-st witness-bunch in 1-st bubble into driver-bunch

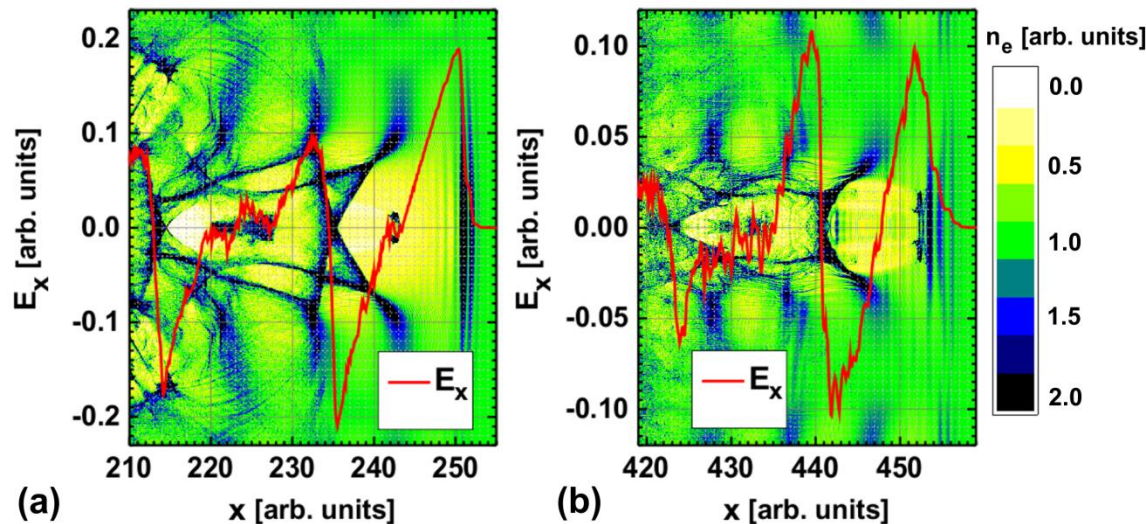
$\gamma=140,5$



Transformation of 1-st witness-bunch into driver-bunch



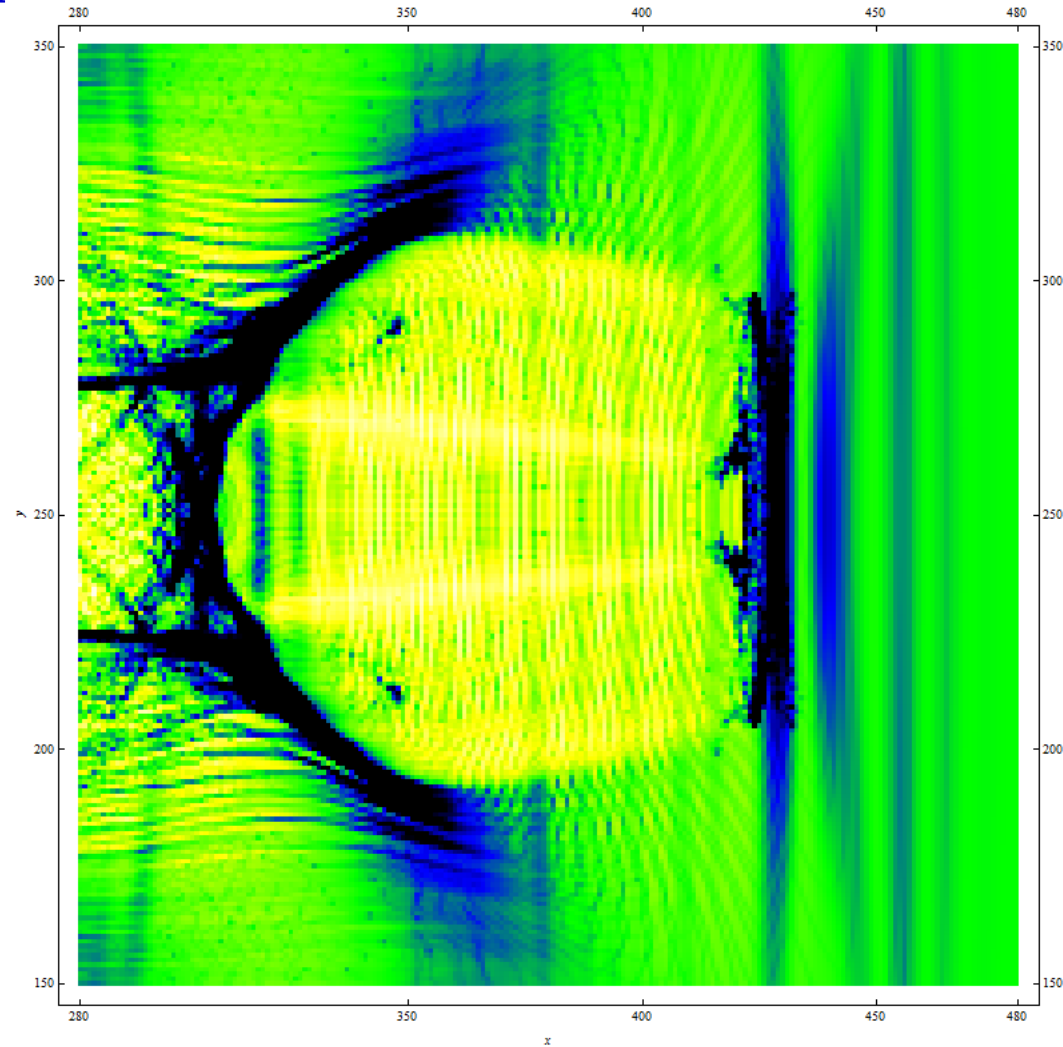
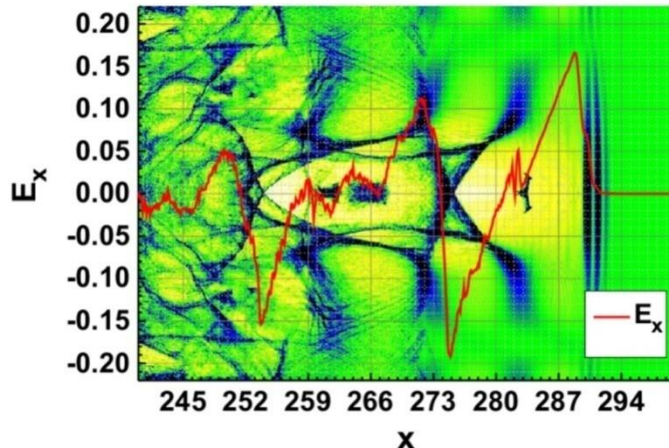
Wake perturbation of plasma electron density and longitudinal wakefield E_x (red line), excited by single laser pulse

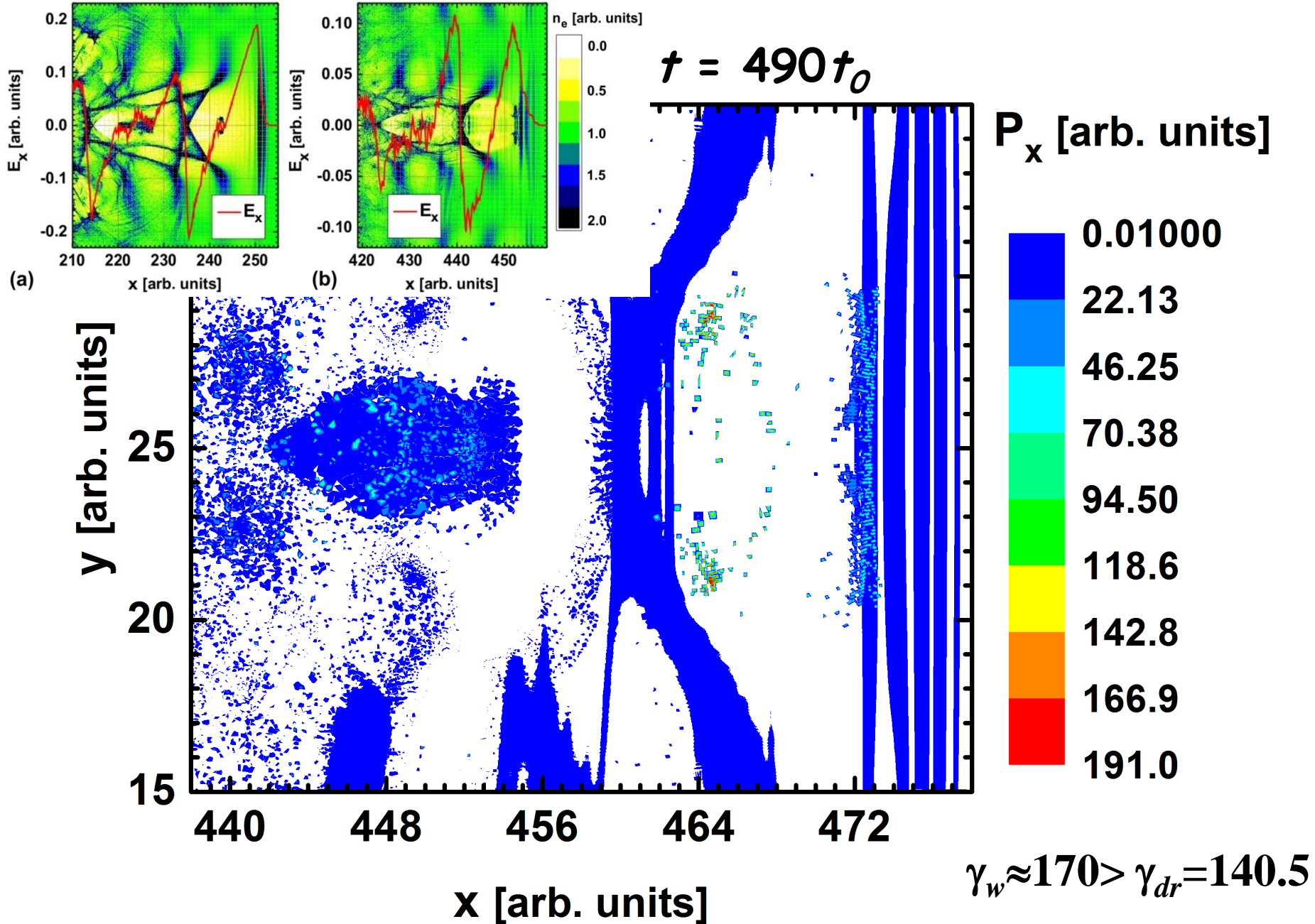


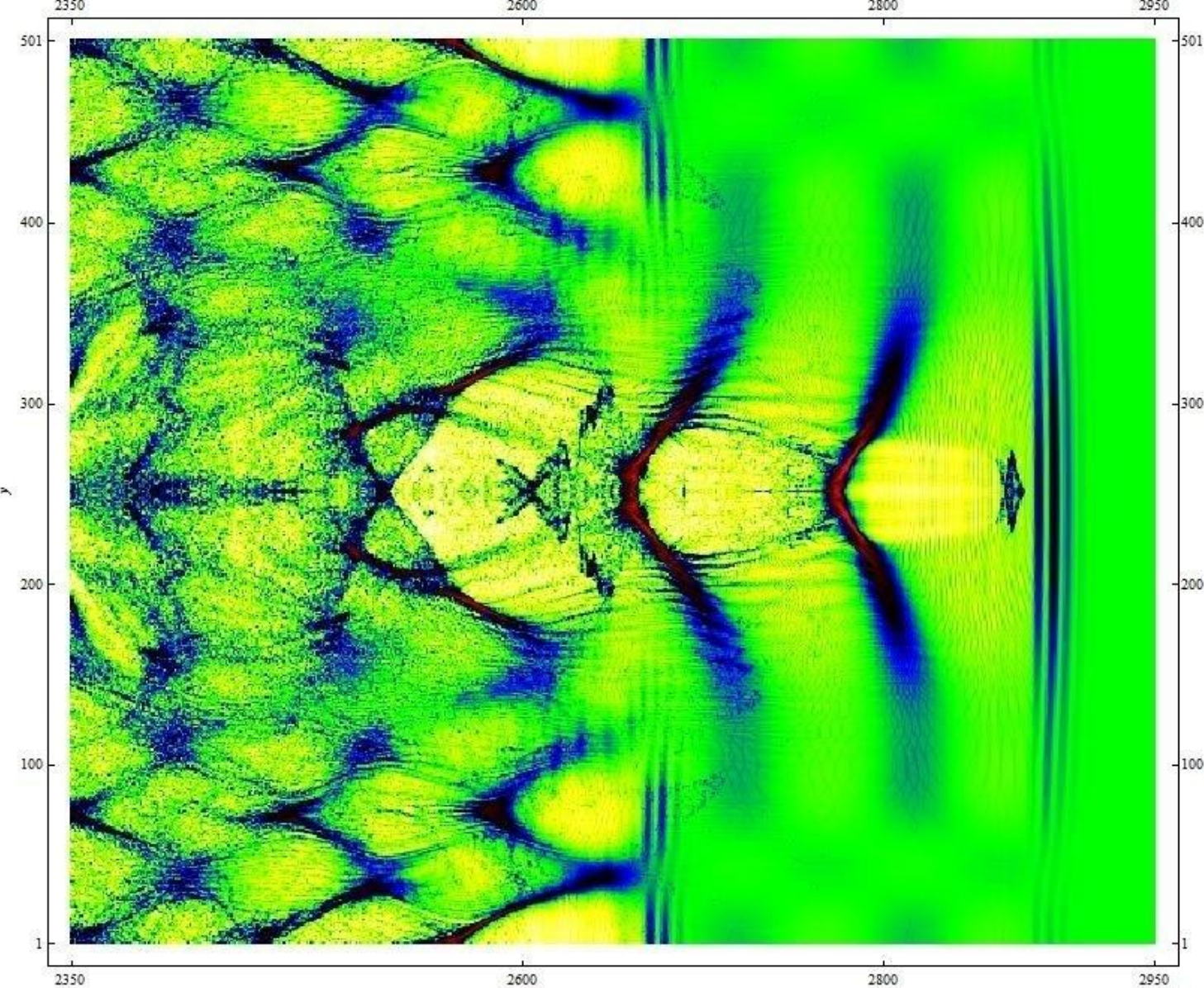
1-st witness-bunch in 1-st bubble becomes driver-bunch together with partially dissipated laser pulse. They provide a further acceleration of electrons.

2-nd mechanism of amplify the laser wakefield acceleration by bunch (plasma) wakefield acceleration

For 2-nd witness-bunch in 1-st bubble and for 2-nd witness-bunch in 3-rd bubble accelerating fields are larger than for 1-st witness-bunch in 1-st bubble and than for 1-st witness-bunch in 3-rd bubble .



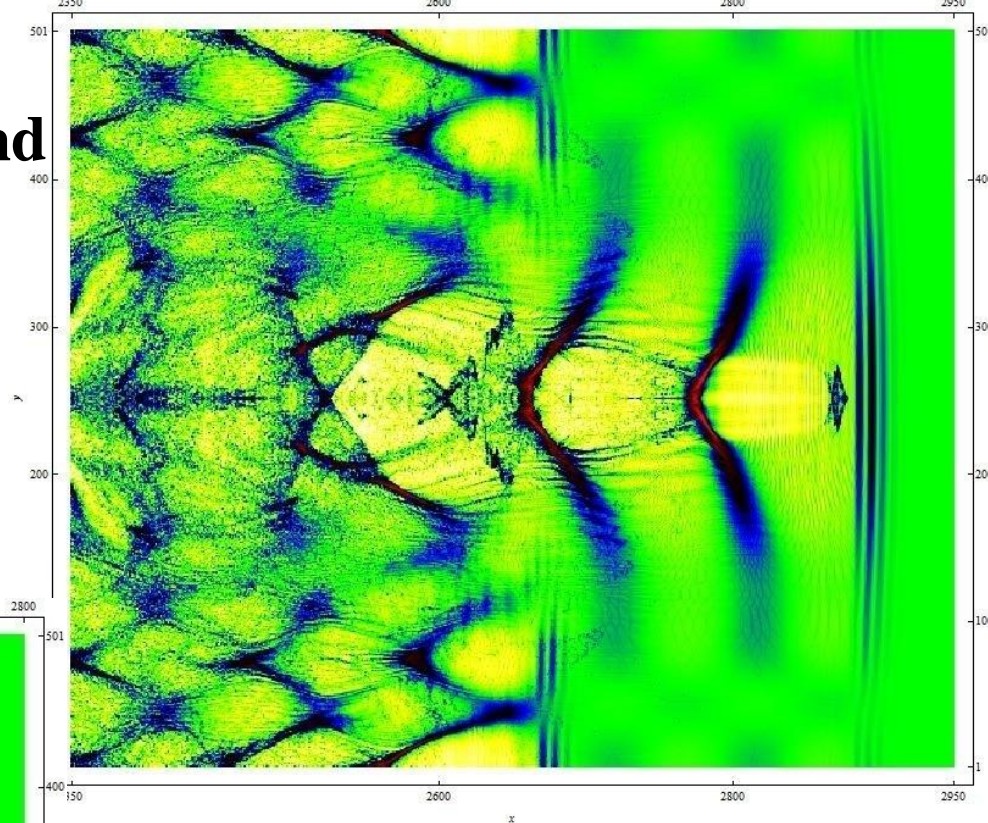
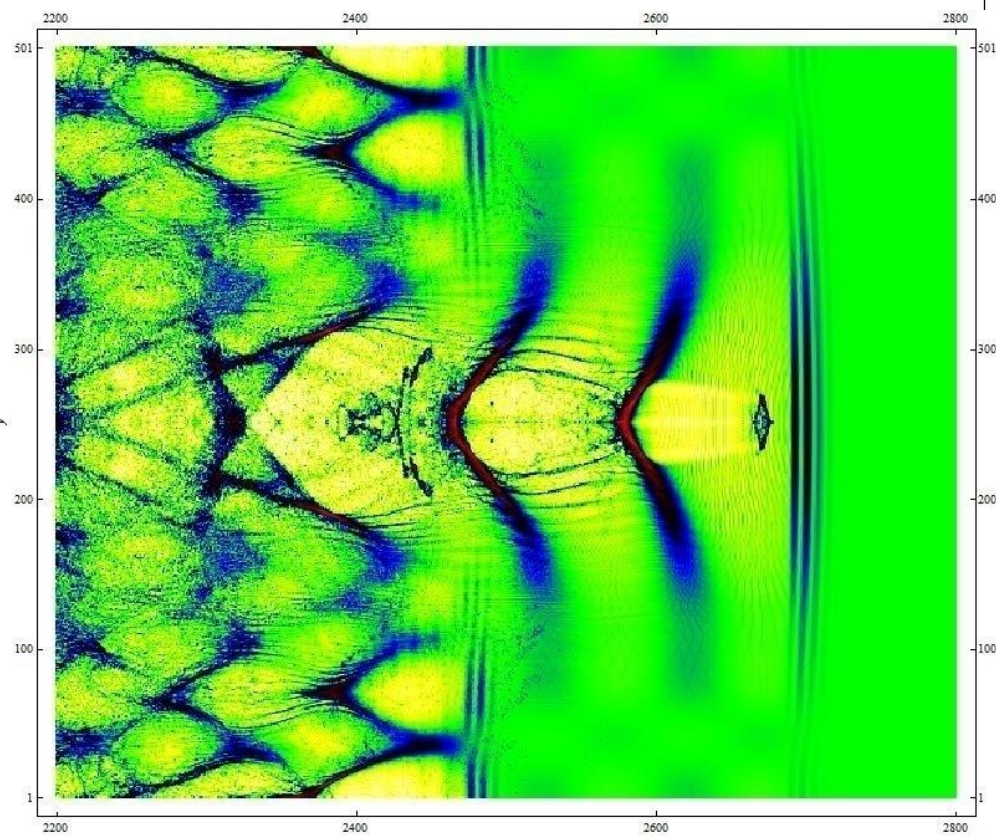


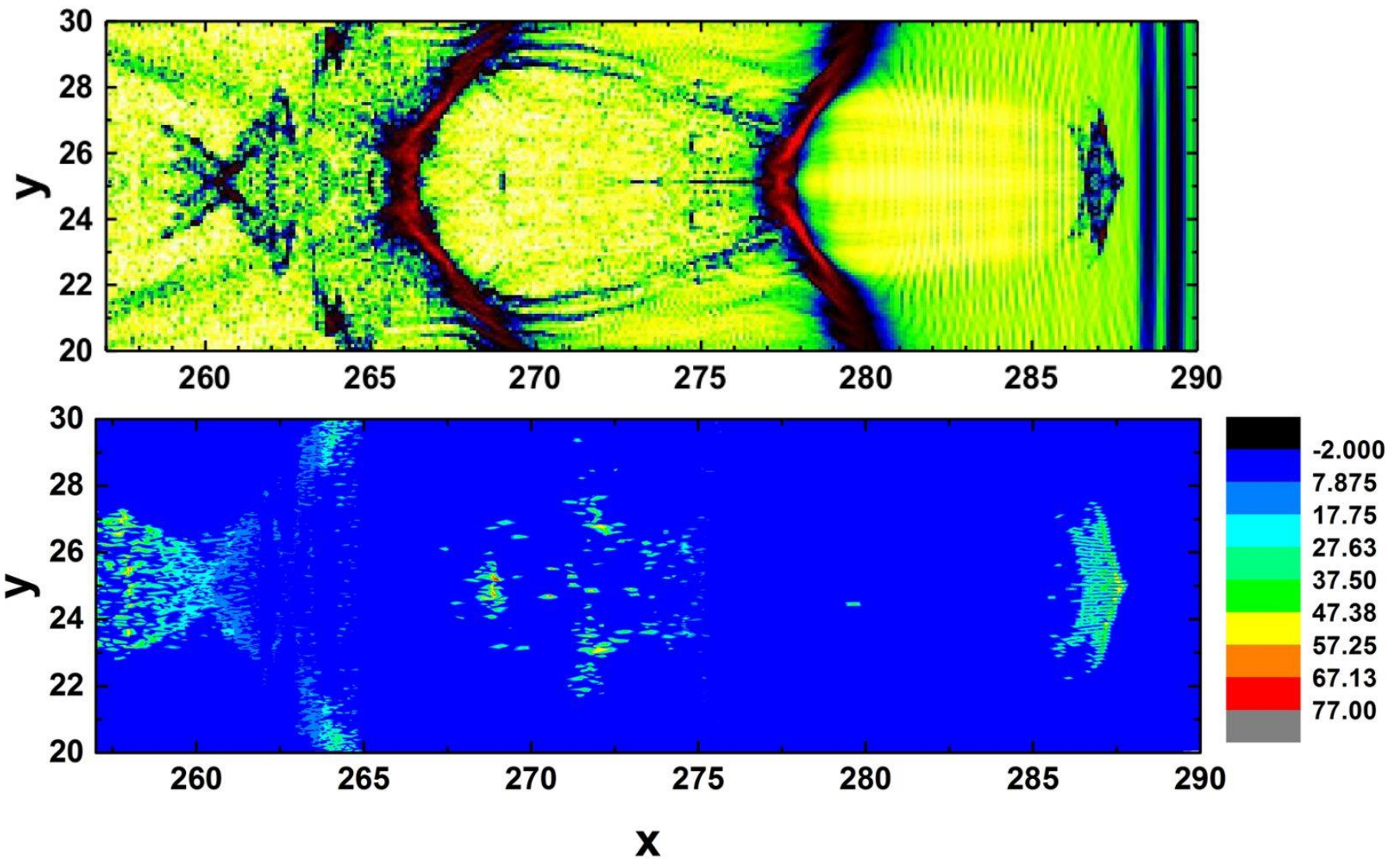


**2-nd witness-
bunch and 1-st
driver-bunch in
3-rd bubble in
the case of two
laser pulse
injection**

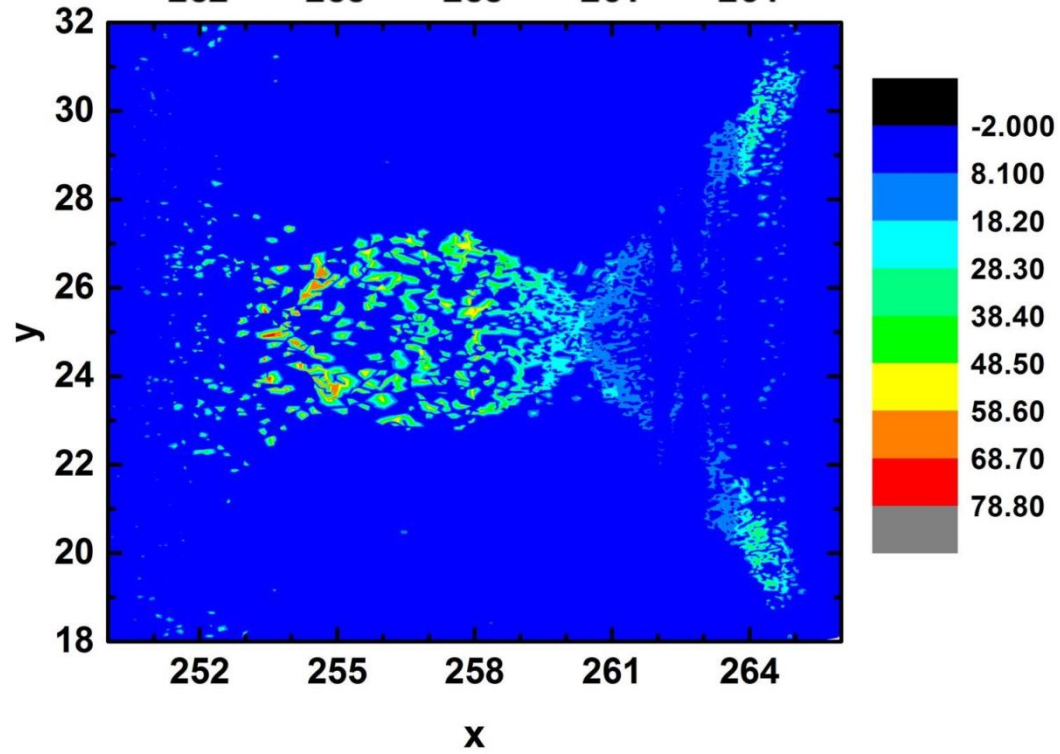
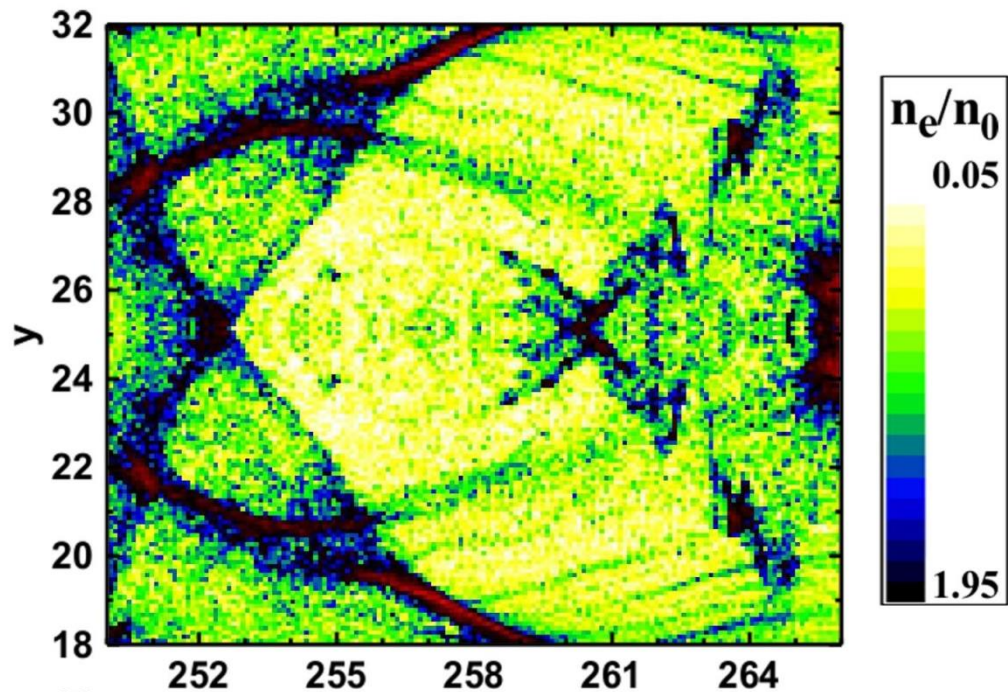
r-dynamics of electron^x bunches in the conventional metal accelerators is bad and in plasma accelerators, it turns out, can be good.

Oscillations on radius of 2-nd witness-bunch in 3-rd bubble

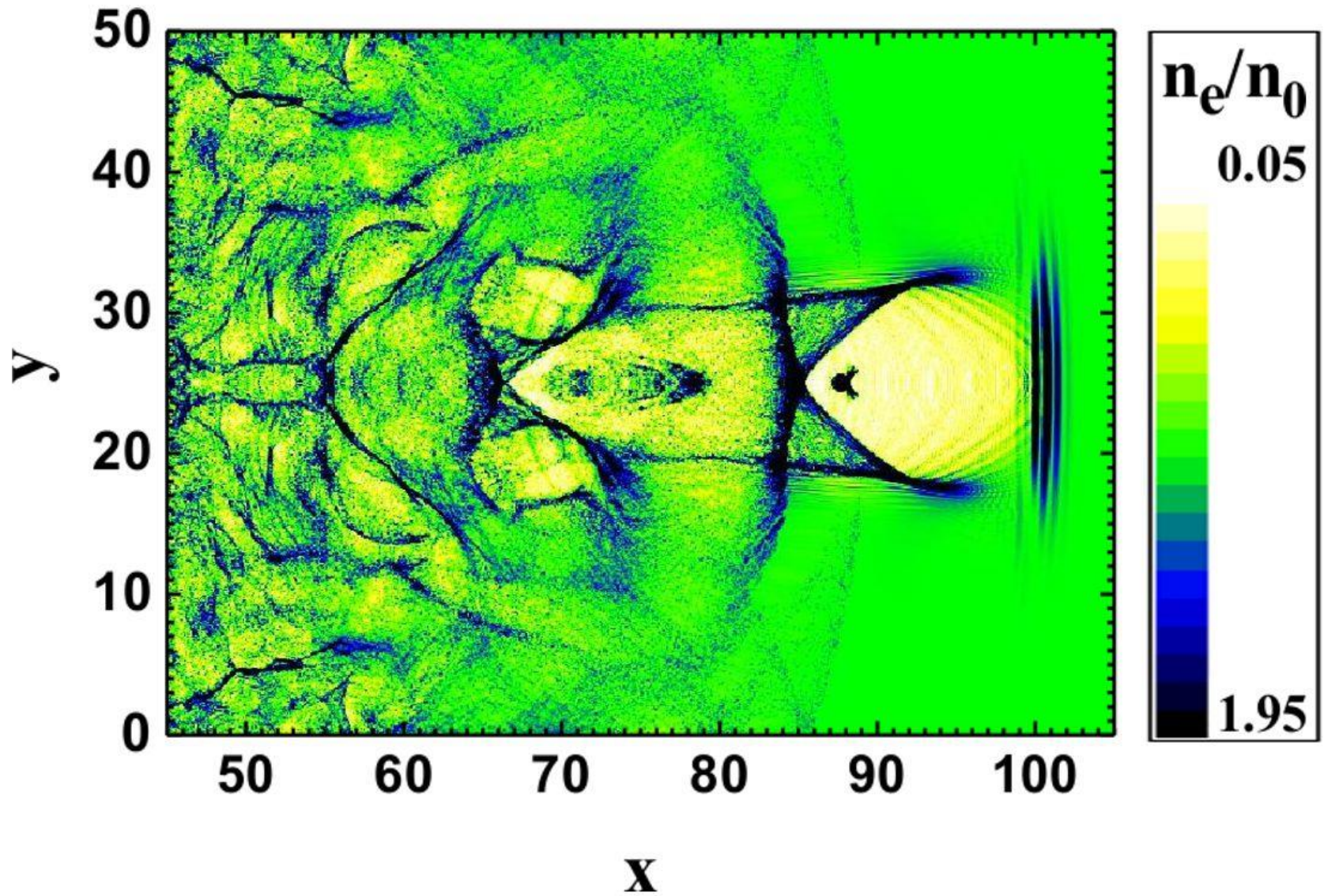




After expansion of 2-nd laser pulse the electron bunch is self-injected in 2-nd bubble in the case of two laser pulse injection.

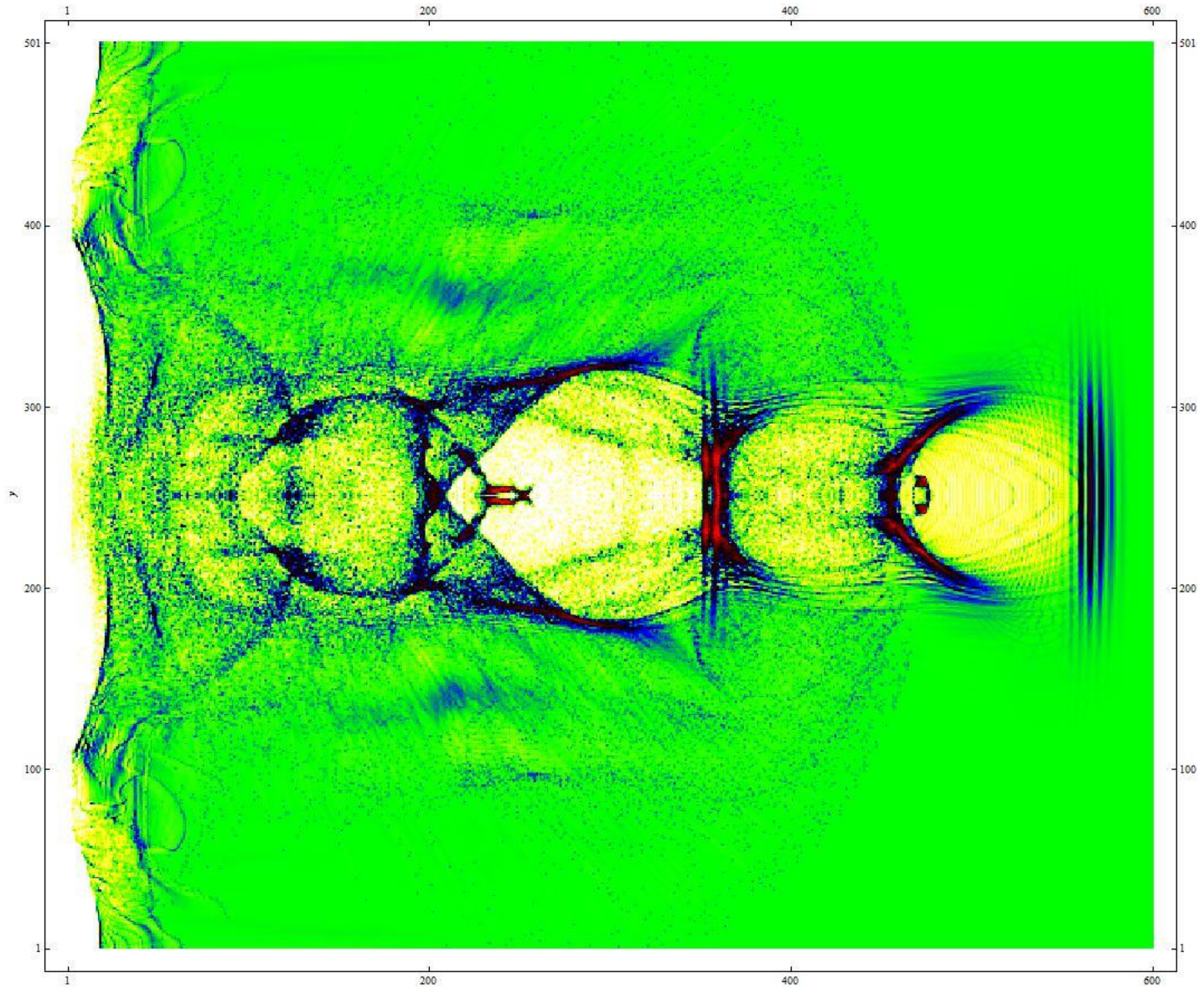


Charge of 1-st witness-bunch in 1-st bubble



$q \approx 3 \text{ pC}$

Charge of 1-st witness-bunch in 3-rd bubble



$$n_b \approx 6n_{0e}, q \approx 2.4 \text{ pC}$$

Conclusion

Dynamics of self-injected electron bunches has been demonstrated by numerical simulation in blowout regime or bubble regime (the nonlinear regime) at self-consistent change of mechanism of electron bunch acceleration by plasma wakefield, excited by a laser pulse, to additional accelerating mechanism of electron bunch by plasma wakefield, excited by self-injected electron bunch.

Two scenarios of intensification of electron bunch acceleration by wakefield, excited by laser pulses, by 1st self-injected electron bunches, which become drivers, have been observed by numerical simulation.

The radial dynamics of electron bunches leads to intensification of electron bunch acceleration by wakefield, excited by laser pulses.

The charge of bunches, self-injected and accelerated by laser pulses in plasma, equals some pC.

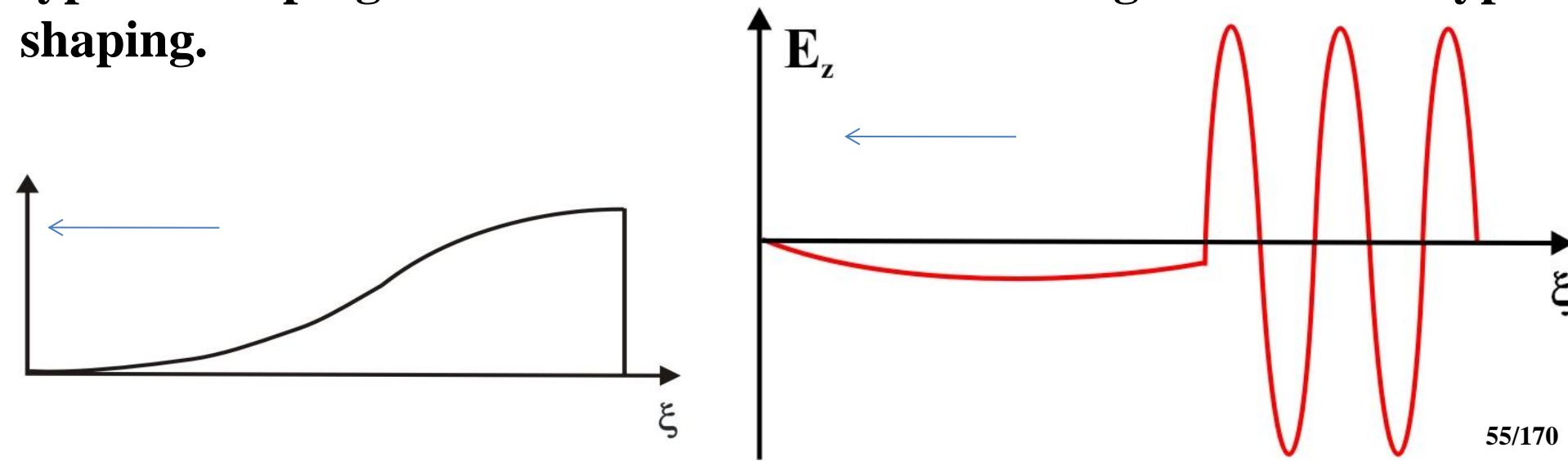
Transformation ratio at plasma wakefield excitation by laser pulse with ramping of its intensity according to cosine

At electron acceleration by wakefield the transformation ratio is important. It determines the maximal energy to which a witness can be accelerated. It is determined as

$$\text{TR}_\varepsilon = \Delta\varepsilon_w / \Delta\varepsilon_{\text{dr}}$$

ratio of the energy, received by witness bunches, to energy, lost by driver.

One can provide large TR, shaping laser profile. There are several types of shaping. We consider more natural Semigaussian- cos- types shaping.



Wakefield excitation in plasma by shaped laser pulse

We consider the wakefield excitation by shaped laser pulse with intensities $b_0 = eE_x / (m_e c \omega_0) = 3$ and $b_0 = 4$. In the case of a laser pulse with an intensity $b_0 = 3$ maximum TR equals 4.3 at the time $t = 160t_0$ (Fig. 1). $TR = 5.9$ at the time $t = 120t_0$ in the case of wakefield excitation by laser pulse with intensity $b_0 = 4$ (Fig. 2).

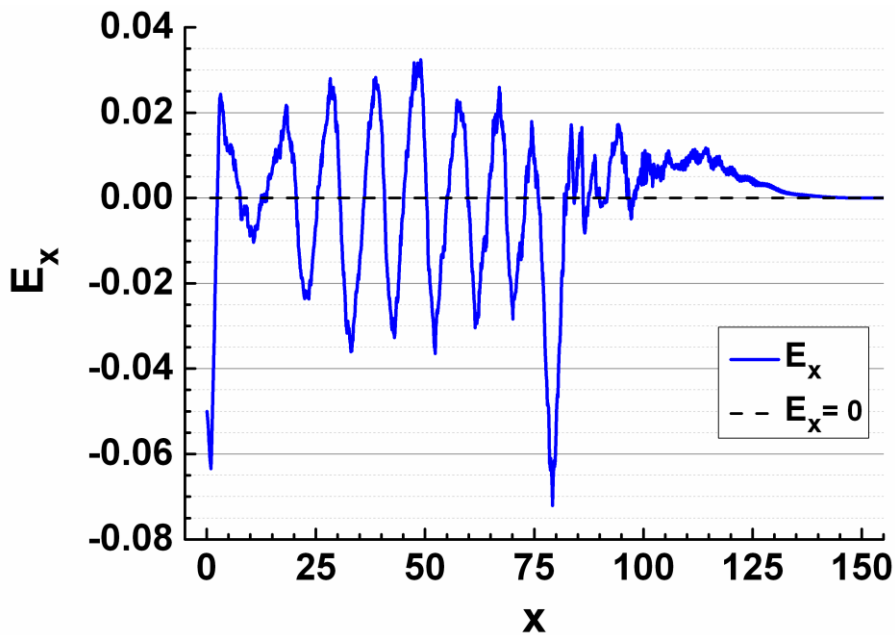


Fig.1. Longitudinal wakefield E_x excited by laser pulse with intensity $b_0 = 3$ at the time $t = 160t_0$

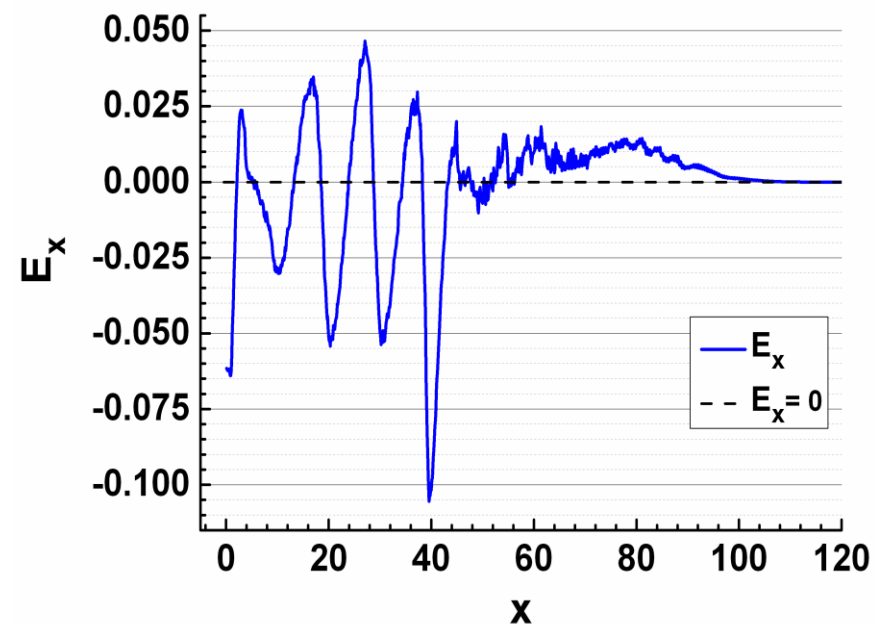


Fig.2. E_x excited by laser pulse with intensity $b_0 = 4$ at the time $t = 120t_0$

The bunch of accelerated electrons is formed after 160 laser periods (Fig.3) and destroyed after 280 laser periods. If an intensity of the laser pulse is $b_0 = 5$ TR reaches 6.26 at the time $t = 120t_0$ and the accelerated electron bunch, which was formed after 160 laser periods, is not destroyed up to $t = 300t_0$.

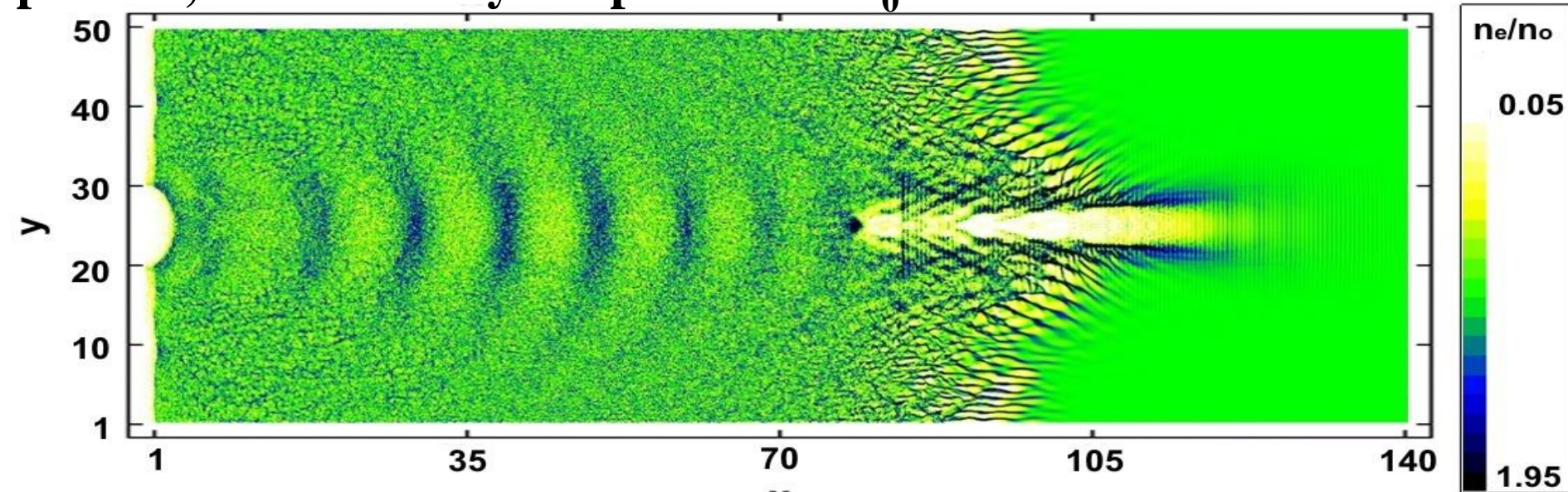


Fig.3. Wake perturbation of plasma electron density, excited by laser pulse of intensity $b_0 = 4$ at the time $t = 160t_0$

Conclusions

For an asymmetric laser pulse distribution in which the pulse intensity rises gradually according to cosine from the front to the peak and then falls off sharply behind the peak, TR can be $TR > 2$.

Shaped train of laser pulses

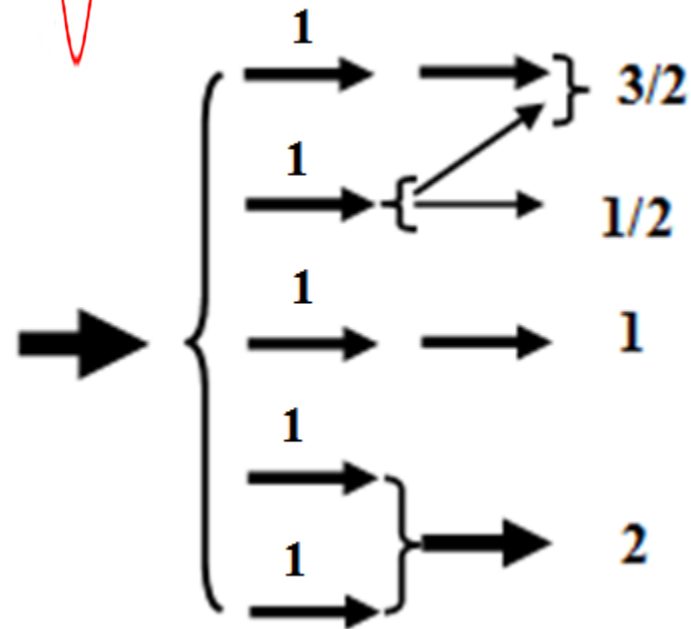
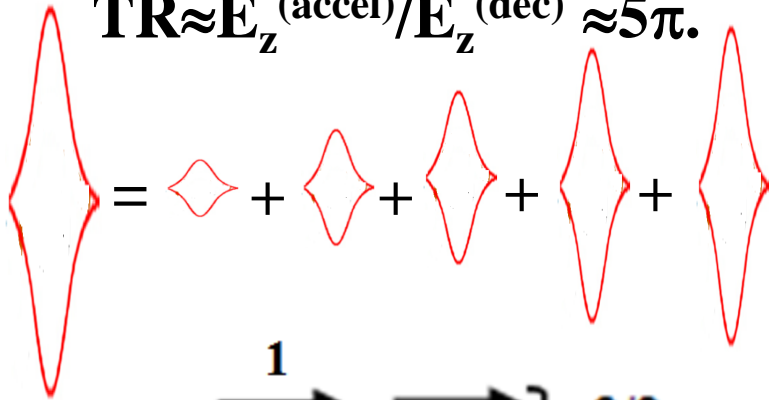
In this way it is possible to prepare shaped on intensity train of laser pulses to increase transformation ratio of laser pulse energy into energy of accelerated electron bunches.

Versions of experimental realization of shaping of train of laser pulses by splitting (branching) laser pulse

1) $I_1:I_2:I_3:I_4: \dots = 1:2:3:4: \dots$,
or the same

$$I_1:I_2:I_3:I_4: \dots = 1/2:1:3/2:2: \dots$$

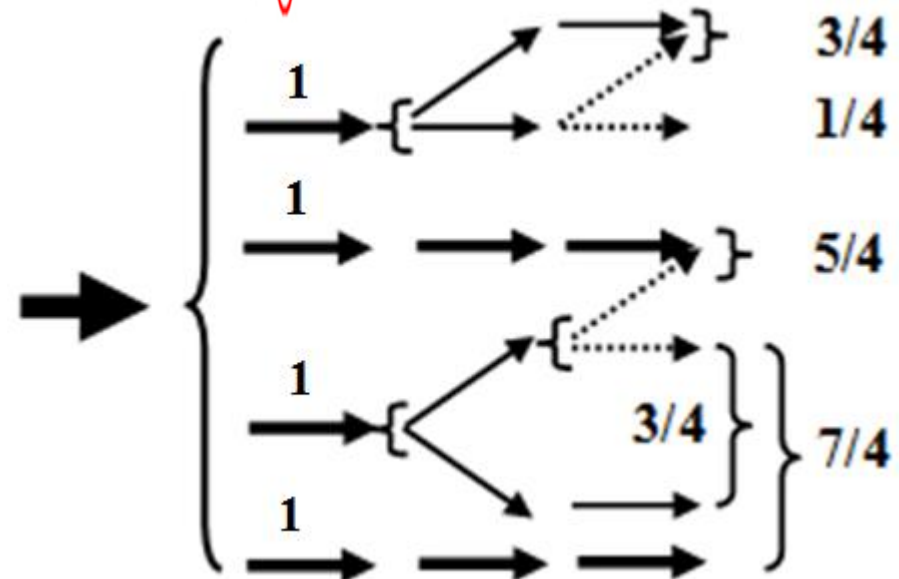
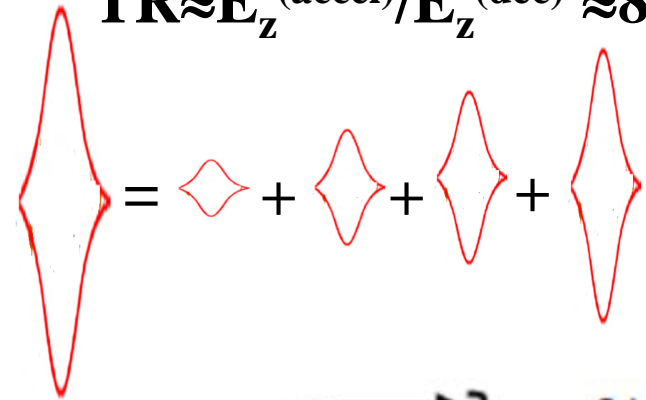
$$TR \approx E_z^{(\text{accel})}/E_z^{(\text{dec})} \approx 5\pi.$$



2) $I_1:I_2:I_3:I_4: \dots = 1:3:5:7: \dots$,
or the same

$$I_1:I_2:I_3:I_4: \dots = 1/4:3/4:5/4:7/4: \dots$$

$$TR \approx E_z^{(\text{accel})}/E_z^{(\text{dec})} \approx 8.$$

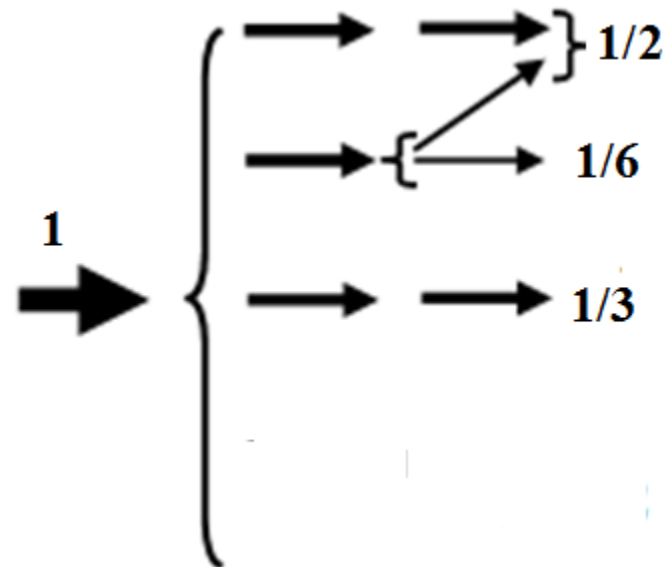
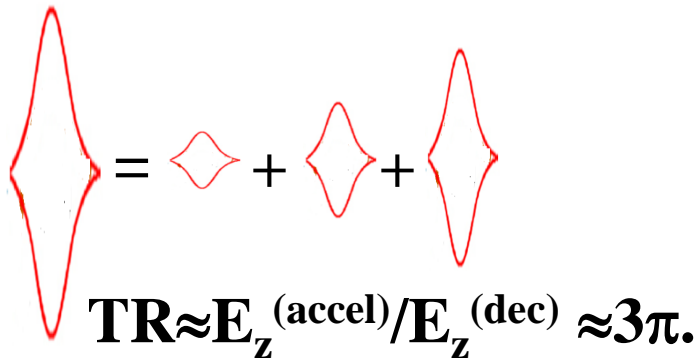


Shaped train of laser pulses

Versions of experimental realization of shaping of train of laser pulses by splitting (branching) of laser pulse

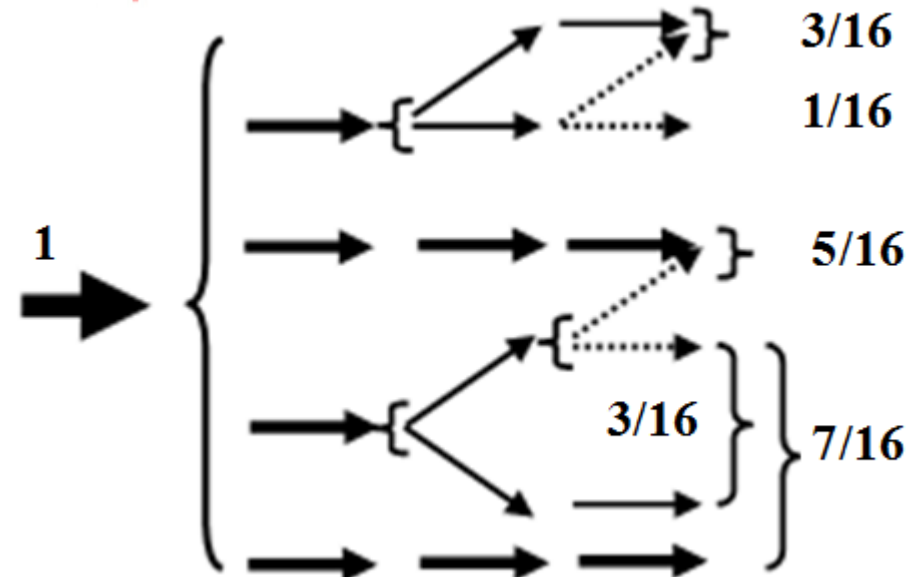
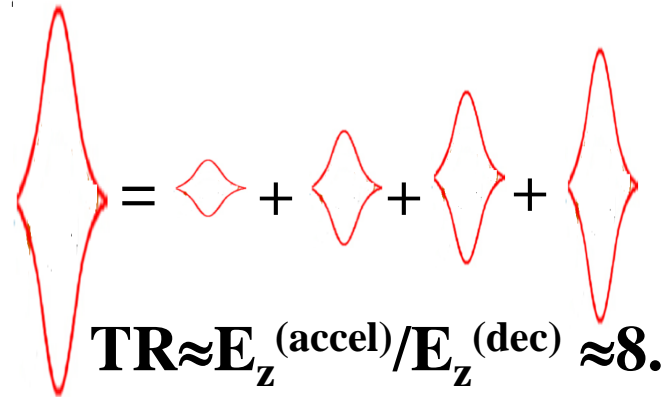
or

$$I_1:I_2:I_3:\dots = 1/6:1/3:1/2:\dots$$



or

$$I_1:I_2:I_3:I_4:\dots = 1/16:3/16:5/16:7/16:\dots$$



Some Problems of Wakefield Excitation and Electron Acceleration in Plasma and Dielectric Cavities by Drivers and by Train of Drivers

Resonant excitation of wakefield by RESONANT train of drivers - relativistic electron bunches

Introduction

The plasma wakefield excitation by a single dense bunch has allowed to achieve accelerating electrical field above 40 GeV/m and to double energy of 42 GeV-bunch by a plasma of the length less than 1m

I.Blumenfeld et al. 2007.

The question arises to what value the wakefield can grow if it is excited by a long train of drivers. To address this question, experiments

V.A.Kiselev et al. 2008

have been performed. Here the results of simulation of self-consistent dynamics of short electron bunches in plasma are presented.

Numerical simulation of plasma wakefield excitation by a train of drivers has been performed with 2.5D code LCODE.

The bunches are treated as ensembles of macro-particles.

Parameters are taken close to those of plasma wakefield experiments

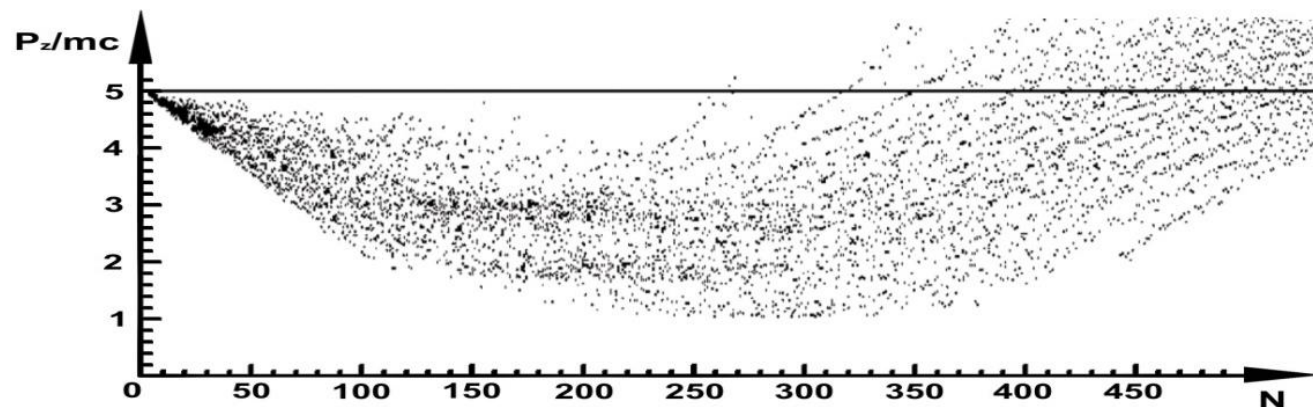
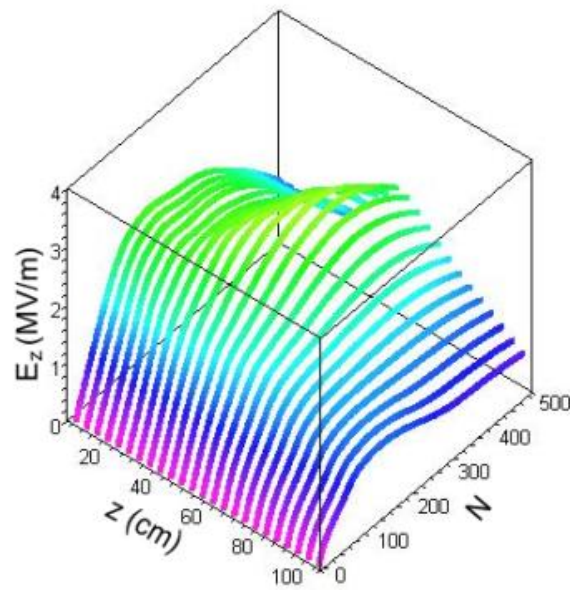
V.A.Kiselev et al. 2008.

Drivers, represented by a regular train of 6000 electron bunches, each of energy 4 MeV, charge 0.32 nC, rms length $2\sigma_z=1.7$ cm, rms radius $\sigma_r=0.5$ cm, and rms angular spread $\sigma_\theta=0.05$ mrad, excites wakefield in the plasma of density $n_p=10^{11}$ cm⁻³ and length of about 1 m, so that the repetition frequency of the bunches coincides with the plasma frequency ω_p (so called resonant train).

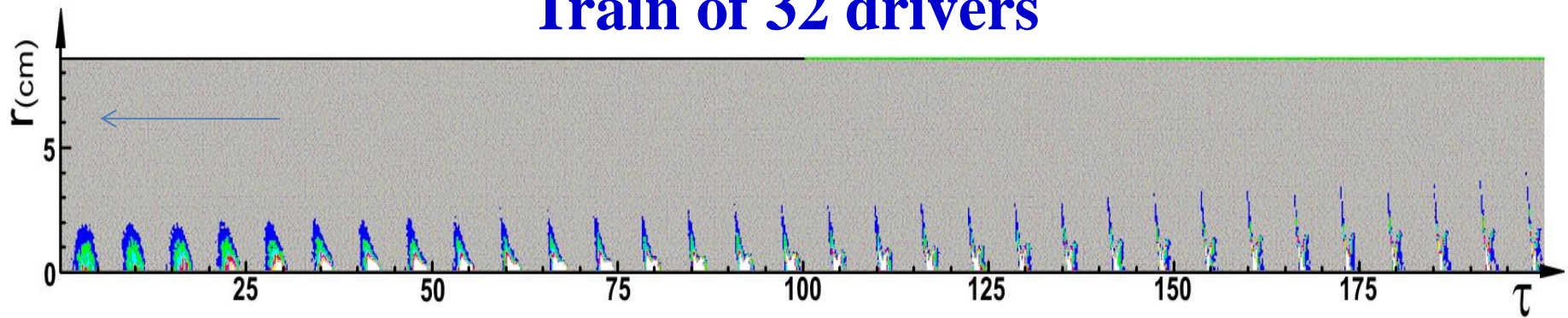
Simulations of wakefield excitation by trains of 32, 239, 318, 500 and 1300 drivers have been performed.

For observed times and bunch energy **the radial shift of bunch electrons is larger than longitudinal** in rest frame of wave. r-dynamics of electron bunches in the conventional metal accelerators is bad and in plasma accelerators it is real property.

It has been shown that the train of approximately 300 drivers excites the wakefield. Then, the resonance is broken. In resonant case $n_e^{(\text{res})}$ the wakefield is achieved 3MV/m, i.e. 10% of the wavebreaking limit. In optimal case $(n_e/n_e^{(\text{res})}-1 \approx 0.35\%)$ the wakefield is larger.

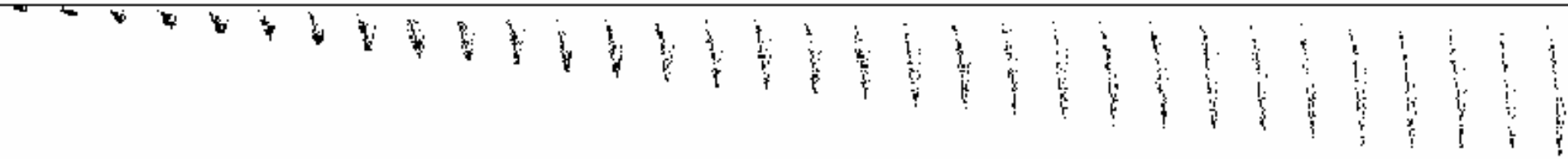


Train of 32 drivers



Time evolution of the beam density in the middle of the plasma (at $z=50$ cm from the injection boundary).

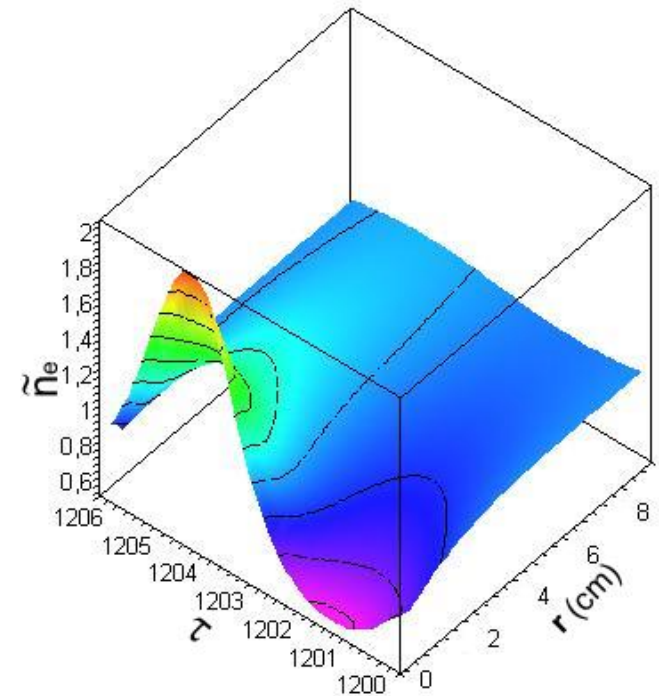
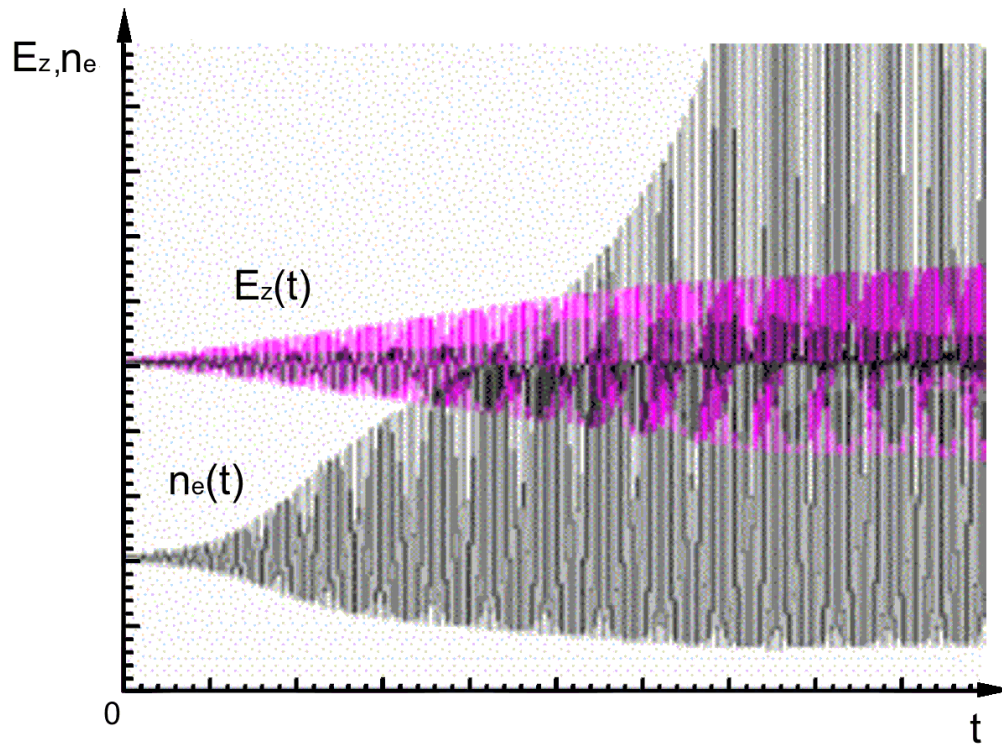
Bunches are focused by wakefield but focusing is inhomogeneous. The 1-st front of the bunch is defocused and back front of the bunch is focused.



Longitudinal momentum.

Train of 32 bunches excites wakefield. Middle of bunch gets in $E_z^{(\max)}$.

Train of 239 drivers

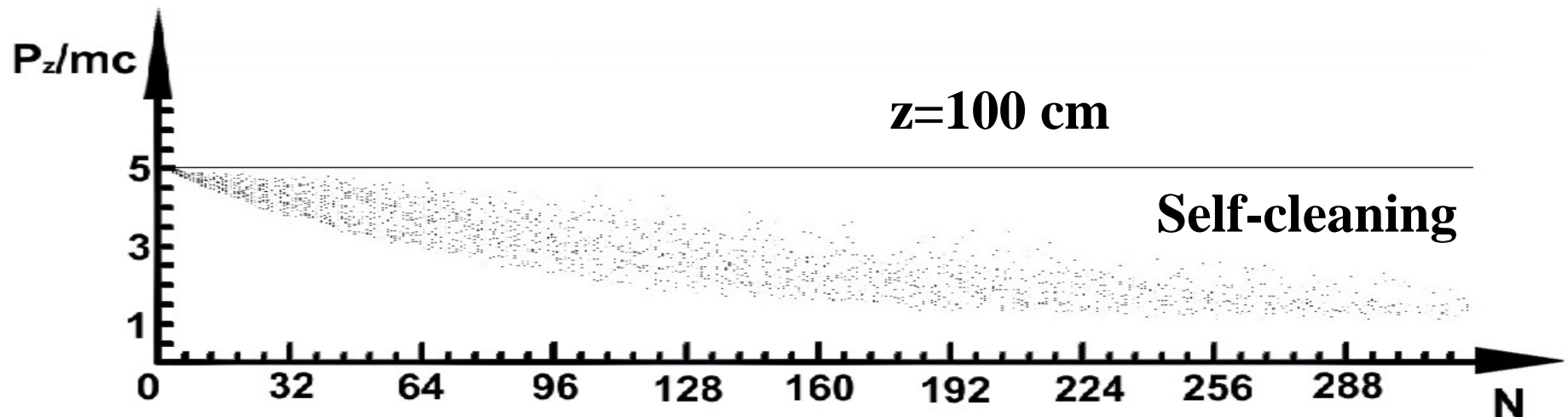
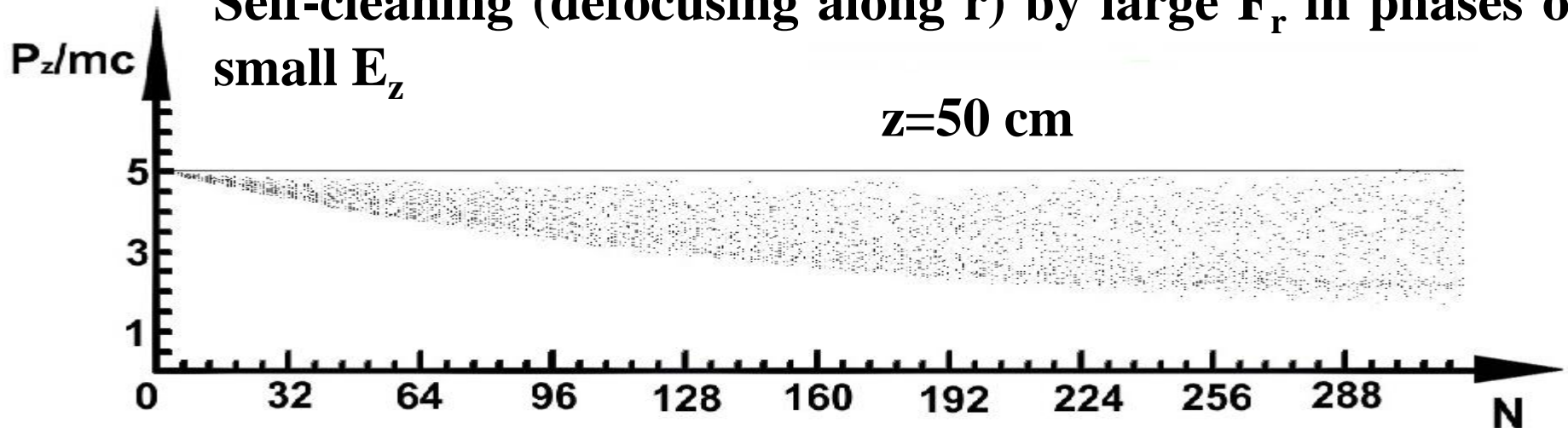


Electron density perturbation at $z=50$ cm during one period of wave after passage of 200 drivers .

E_z (red) and n_e (black) (non- sinusoidal: asymmetrical relative to n_{0e}) by the train of 239 drivers ($z=33$ cm)

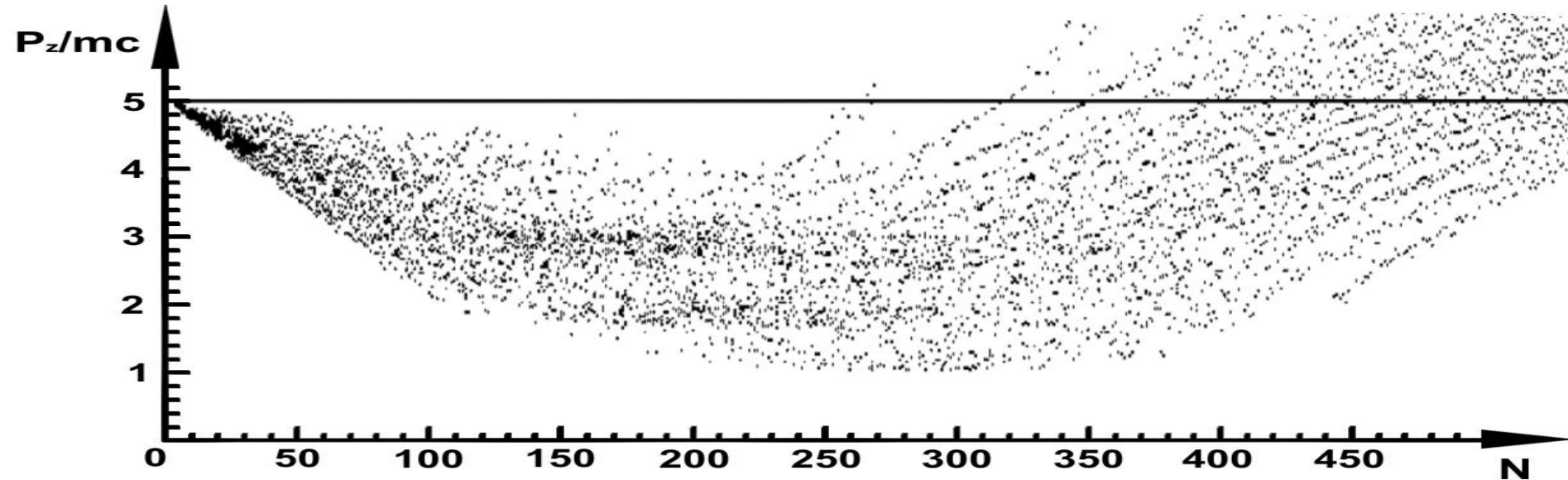
Train of 318 drivers

Self-cleaning (defocusing along r) by large F_r in phases of small E_z



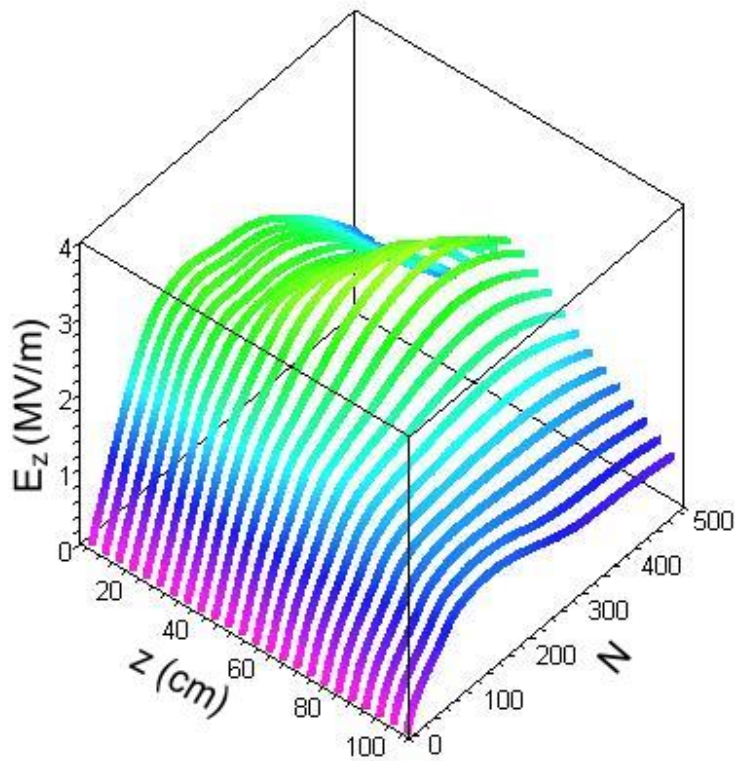
Longitudinal momentum of train of 318 drivers in the middle ($z=50$ cm) and near exit of the plasma ($z=100$ cm)

Train of 500 drivers



Longitudinal momentum of train of 500 bunches in the middle of the plasma ($z=50$ cm)

Train of approximately 300 drivers excites wakefield.



The amplitude of the on-axis electric field as a function of the coordinate along the plasma and the number of injected bunches

There is maximum on z (focusing and over focusing) and N (resonance breaking)

The wakefield of 3 MV/m, i.e., 10 % of the wavebreaking limit, is achieved.

Near injection boundary the field grows linearly until the wave becomes nonlinear and goes out of resonance with the train. At $z \sim 50$ cm, the bunches are focused, and we observe faster field growth and a higher saturation level.

Near the exit of the plasma, the bunches are mostly defocused and overfocused, and the excited wakefield is low.

Energy exchange of drivers with wakefield

Energy losses of N-th driver on wakefield excitation equals

$$\varepsilon_N = 2\pi e n_b c E_{Nc} / \omega_p, \quad E_{Nc} = E_N + (\beta - 1) \delta E_N, \quad \beta \approx 1/2.$$

Then wave energy W changes on

$$W_N - W_{N-1} = \eta \varepsilon_N.$$

η is the part of volume, occupied by drivers in comparison with volume, occupied by wakefield

$$\delta E_N = E_N - E_{N-1}, \quad E_N \approx N \delta E_N, \quad \delta E_N = e n_b c \eta (2\pi)^2 / \omega_p.$$

$$\eta \approx 1.7 \cdot 10^{-3}, \quad \ell_b / \lambda \approx 1/6, \quad \partial_t E_N = 2\pi e n_b c \eta.$$

Energy losses of one dense bunch ε_0 and train of bunches $\sum \varepsilon_i$ equal

$$\varepsilon_0 = \pi e n_0 c E_0 / \omega_p, \quad E_0 = (2\pi)^2 e n_0 c \eta_0 / \omega_p \approx E_N.$$

$$\sum \varepsilon_i = \pi \eta (e c n_b N 2\pi / \omega_p)^2 \approx \varepsilon_0. \quad (\text{identical})$$

It is the same for multi-pulse laser wakefield

Energy losses of train of N bunches are proportional to N^2

$$\sum \varepsilon_i = \pi \eta (e c n_b N 2\pi / \omega_p)^2$$

On 1st asymptotic the wakefield amplitude grows linearly with time or with N

$$\partial_t E_N \sim e n_b c = \text{const.}$$

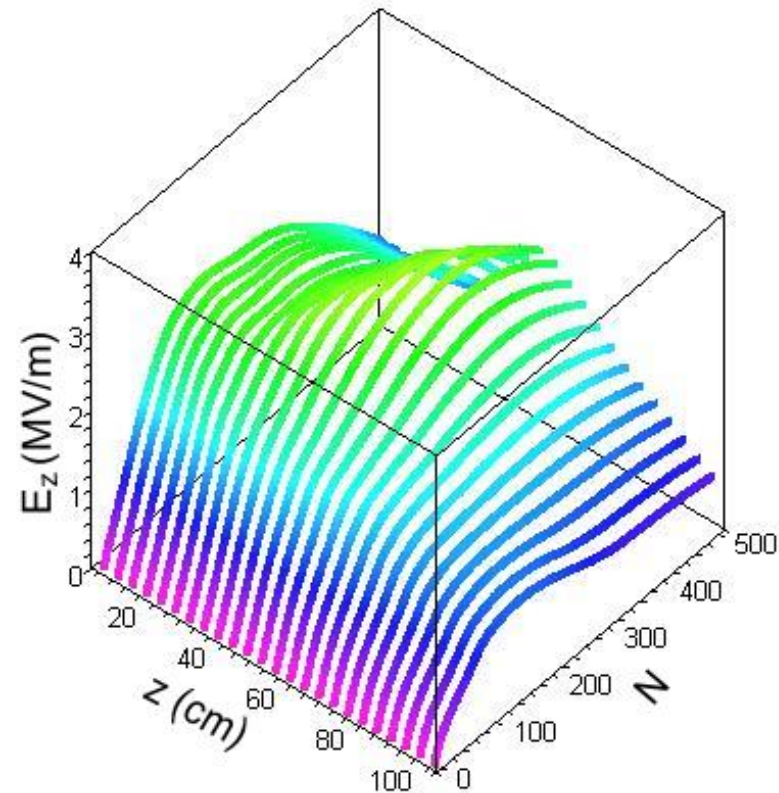
1st asymptotic: $E_N \sim t$ or $\sim N$.

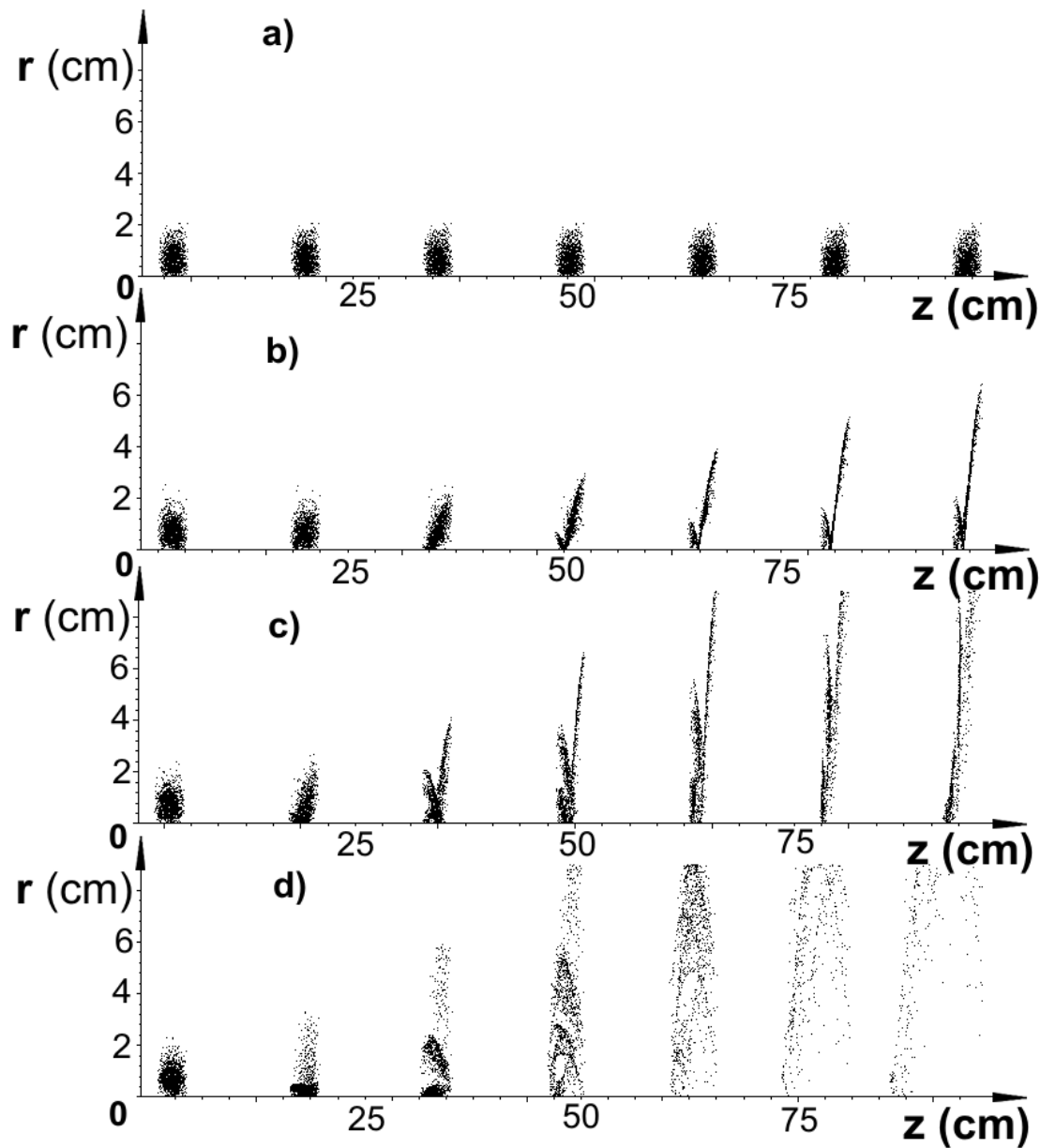
When bunches completely lose the energy

$$\varepsilon_0 \approx m c^2 (\gamma_0 - 1) n_0, \quad \sum \varepsilon_i = (N - K) m c^2 (\gamma_b - 1) n_b + \pi \eta c (e c n_b K 2\pi / \omega_p)^2.$$

When each bunch loses a significant part of the energy, the wakefield amplitude grows with time as \sqrt{t} (2nd asymptotic) or \sqrt{N} .

2nd asymptotic: $E_N \sim \sqrt{t}$ or $\sim \sqrt{N}$.

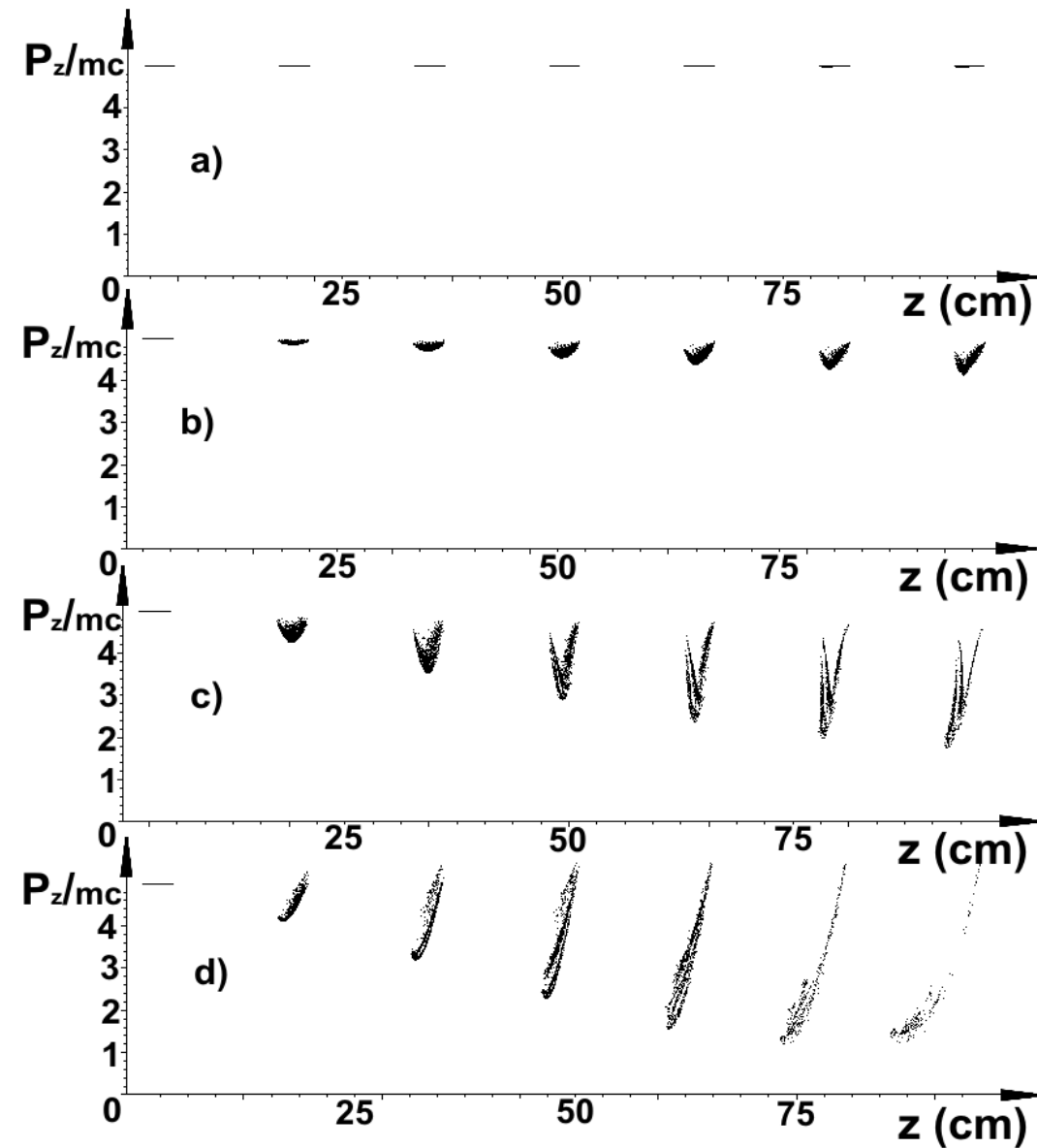




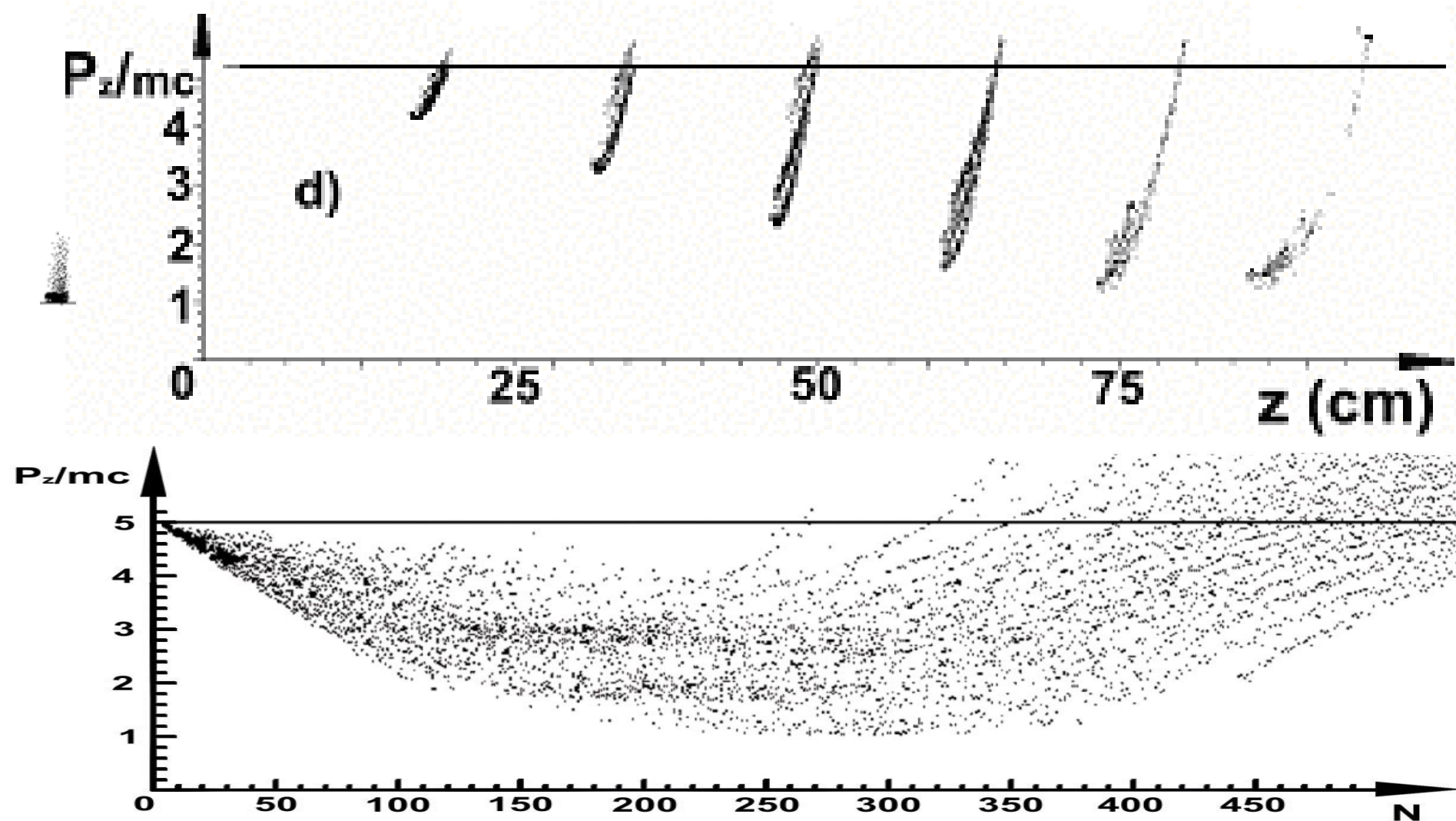
Evolution of the driver shape in plasma; 1st (a), 20th (b), 100th (c), and 300th (d) driver at seven instants as they move through the plasma.

First fronts of 20th and 100th bunches are defocused.

Longitudinal phase space portraits of the 1st (a), 20th (b), 100th (c), and 300th (d) bunch at the seven instants.



20th and 100th bunches are decelerated by excited wakefield. The deceleration rate is highest at bunch centers. 300th bunch loses much of its energy.



300th bunch from the beginning is compressed as a whole. Its main part is decelerated and small part is accelerated.

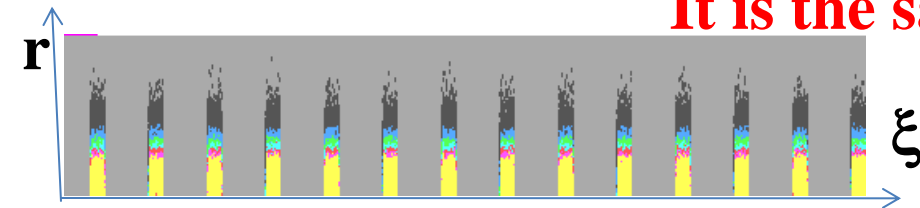
Resonant excitation of wakefield by NONRESONANT

$\omega_d < \omega_{pe}$ train of drivers - relativistic electron bunches

It is difficult to support resonance in experimental inhomogeneous nonstationary plasma. $\Delta n/n = 1/6000$.

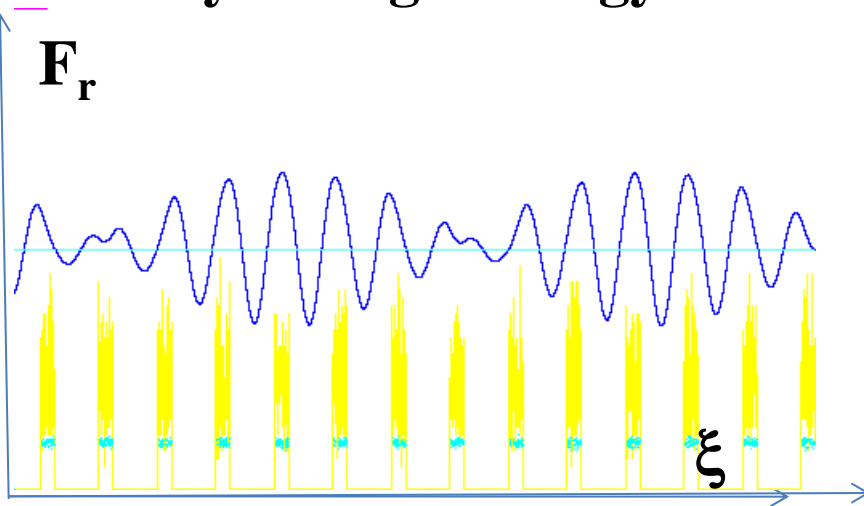
How is the wakefield excited in experiment ?

It is the same for multi-pulse laser wakefield

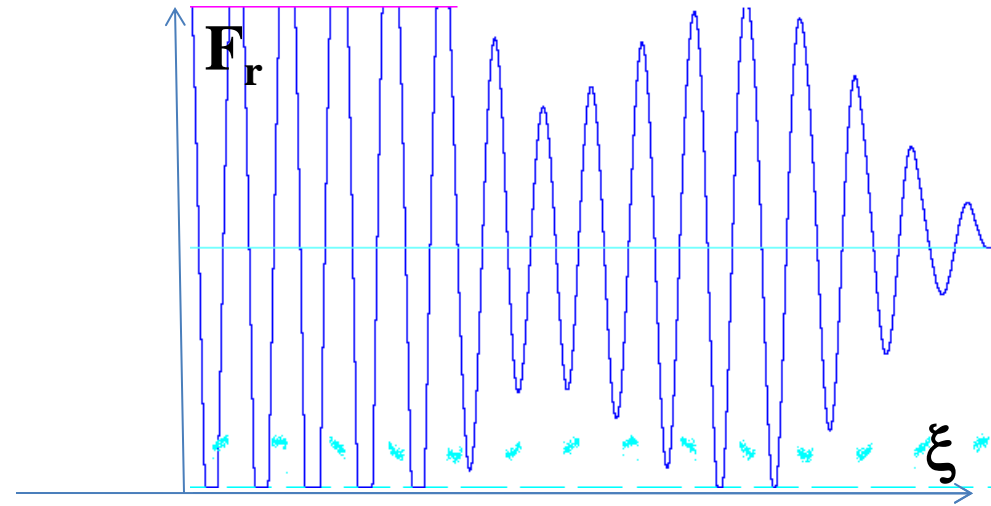


Drivers of length $\xi_b = \lambda/4$

Density of High-energy Electron Bunches



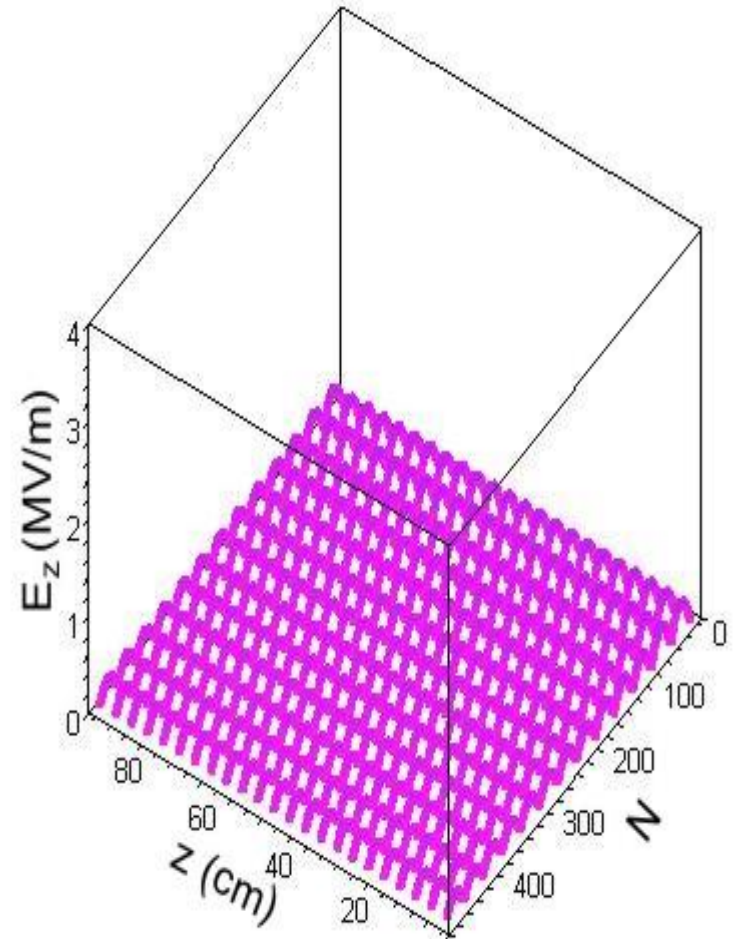
Beatings near boundary of injection



Wakefield excitation deeply

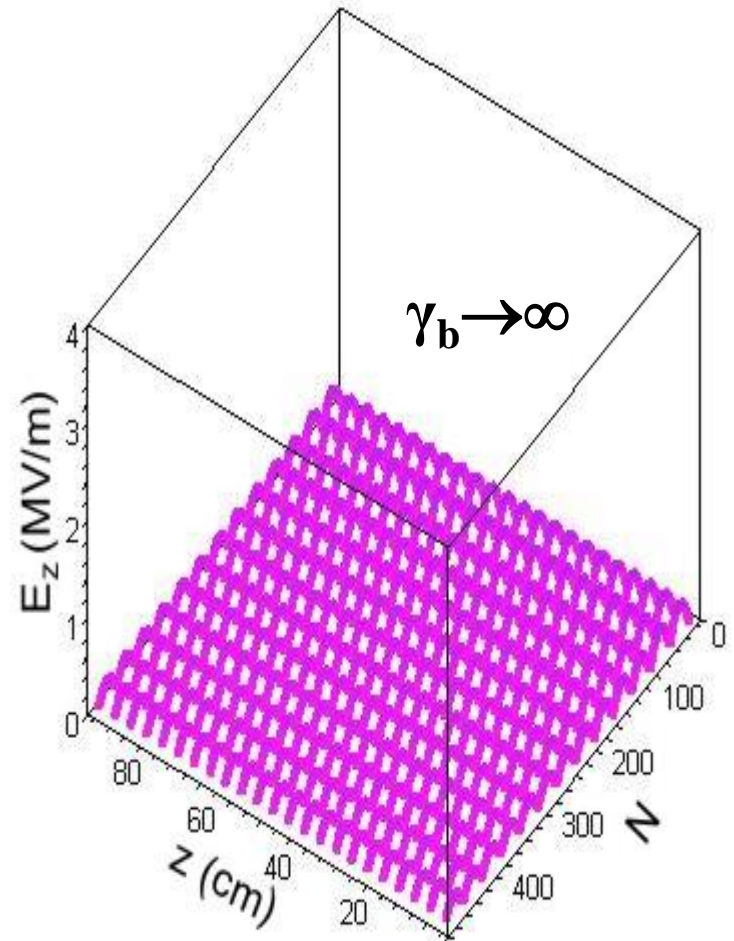
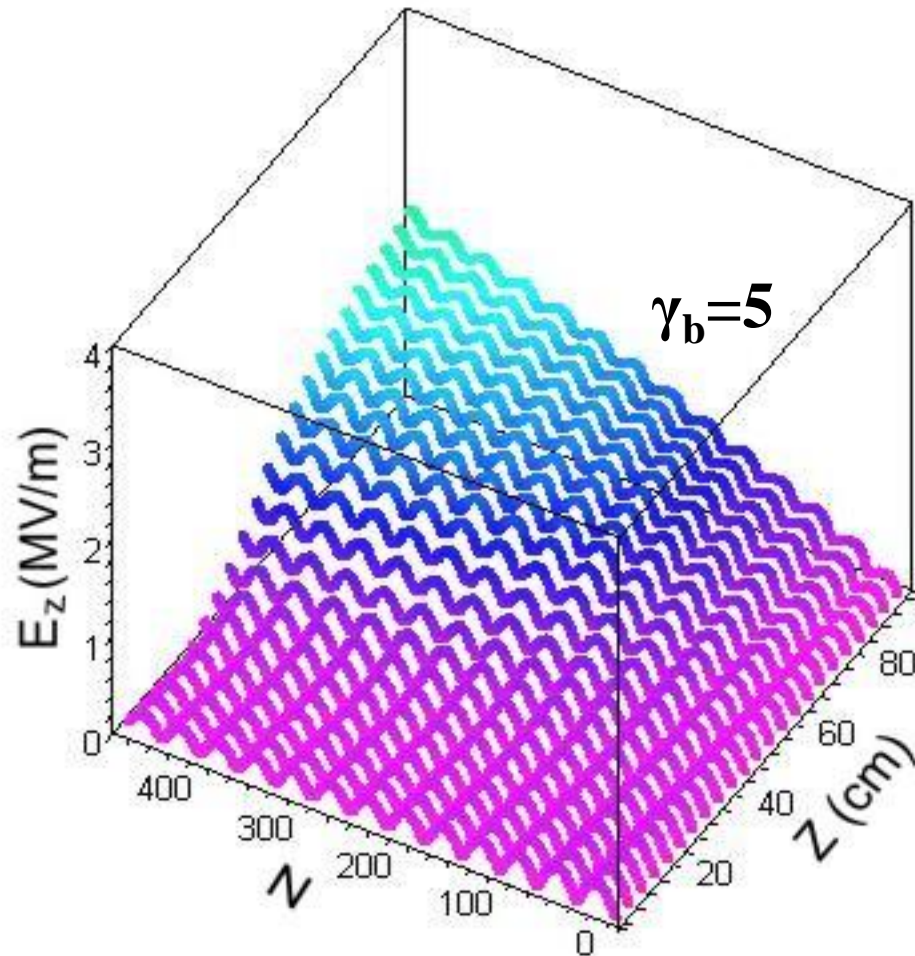
Simulation of resonant excitation of wakefield by non-resonant train of drivers - relativistic electron bunches

Only beatings (periodical excitation and damping) of small amplitude for $\gamma_b=1000$ or near boundary of injection.



Wakefield can not be excited in non-resonant case, only beatings. But in experiment wakefield is excited. Why? **Due to self-cleaning of train to resonant one.**

Train of 500 drivers at plasma density smaller than resonant one ($\omega_p < \omega_d$) on 5 %

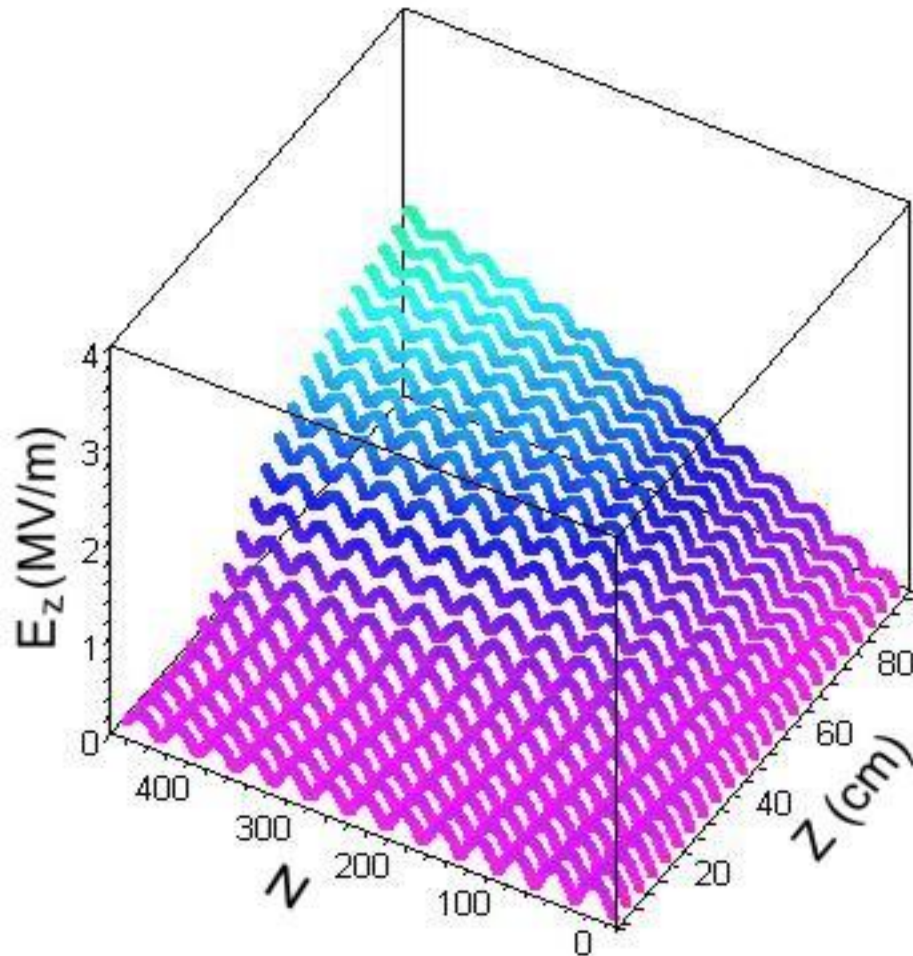


The amplitude of the on-axis electric field as a function of the coordinate along the plasma and the number of drivers for nonresonant plasma density (-5%).

It is the same for multi-pulse laser wakefield

Damping of excitation at $\gamma \rightarrow \infty$ and near boundary of injection

Wakefield amplitude is oscillated and grows with number of injected drivers N in the case of nonresonant plasma density at not large γ_b in contrary to very large γ_b . In the last case the wakefield amplitude is oscillated with N (beatings).



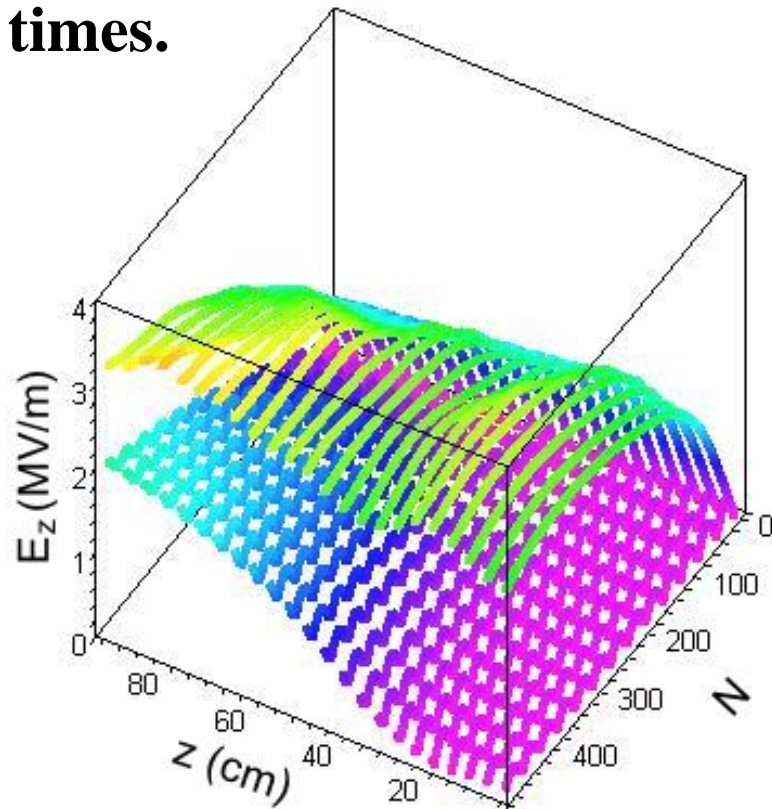
Number of drivers in beating equals $N=1/(1-\omega_p/\omega_d)\approx 39$. Number of beatings along train equals $N_0=500(1-\omega_p/\omega_d)\approx 13$.

On first 40 cm drivers are partly defocused on radius. «Self-cleaned» train has repetition rate, equal to plasma wave frequency, and excites wakefield by resonant way.

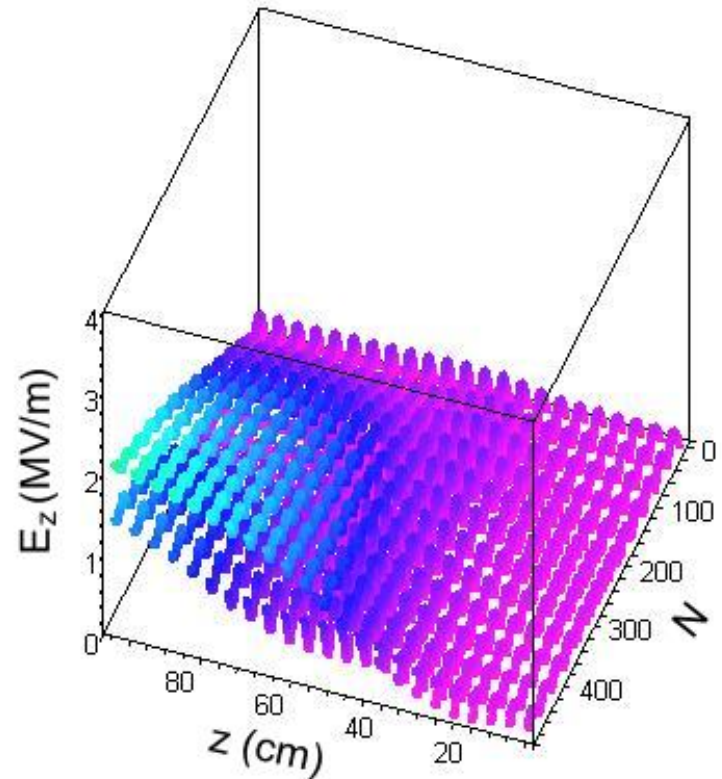
Comparison of nonresonant cases

**Cases +0.5% (close to optimal)
and - 5%.**

**The difference is only in 1.5
times.**

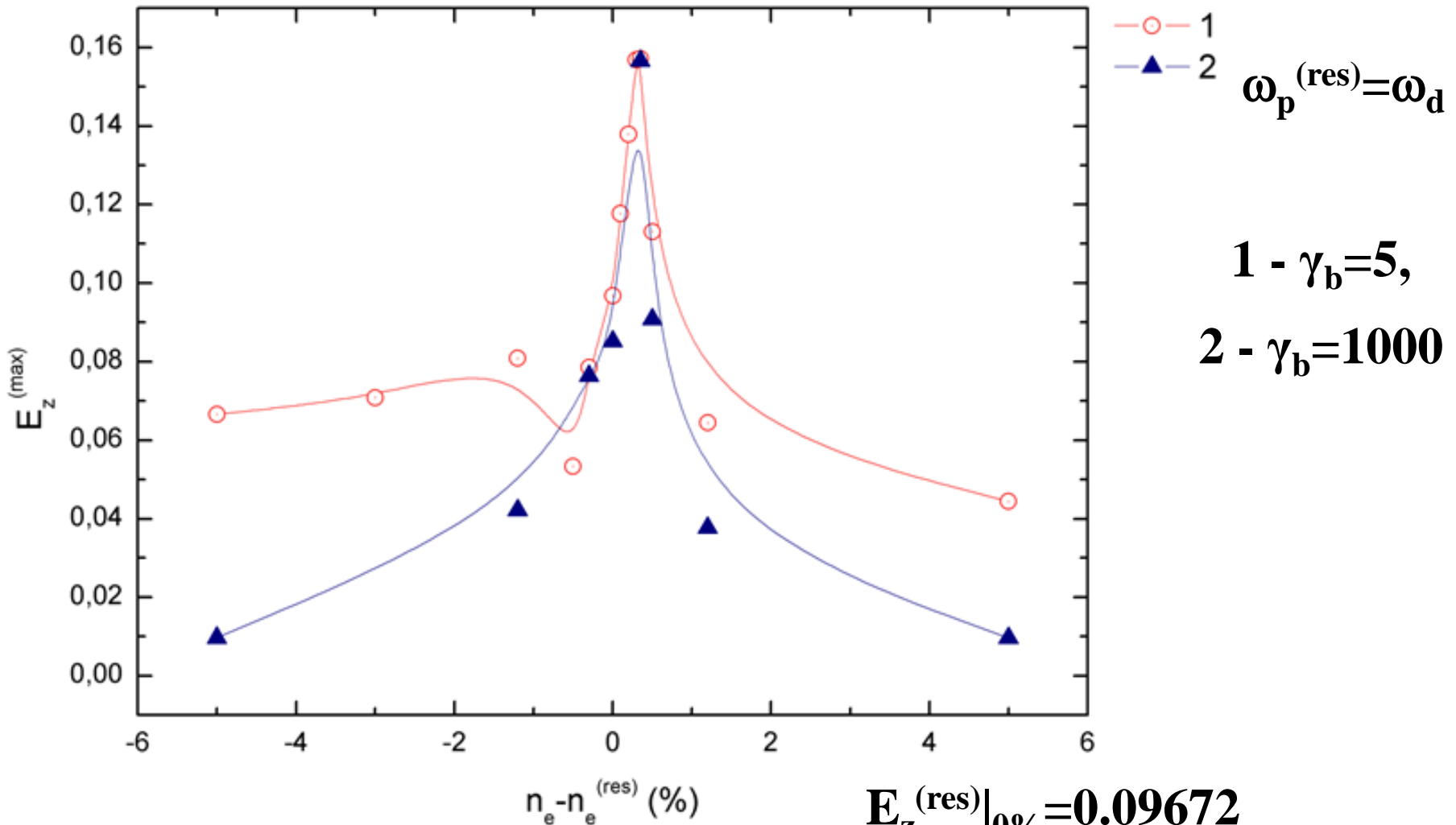


**Cases +5% ($\omega_p > \omega_d$) and -5%
($\omega_p < \omega_d$) differ small.
Achieved amplitude in the
case -5% is larger.**



**The amplitude of the on-axis electric field as a function of the
coordinate z along the plasma and the number of injected drivers N
for nonresonant plasma densities.**

Dependence of wakefield amplitude E_z on difference of plasma density and resonant one $n_e - n_e^{(res)}$, determined by repetition frequency of drivers ω_d

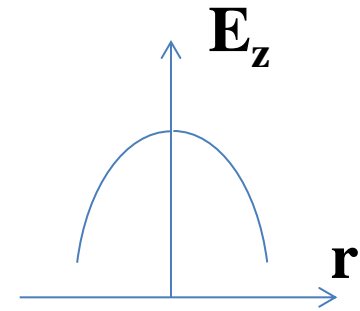
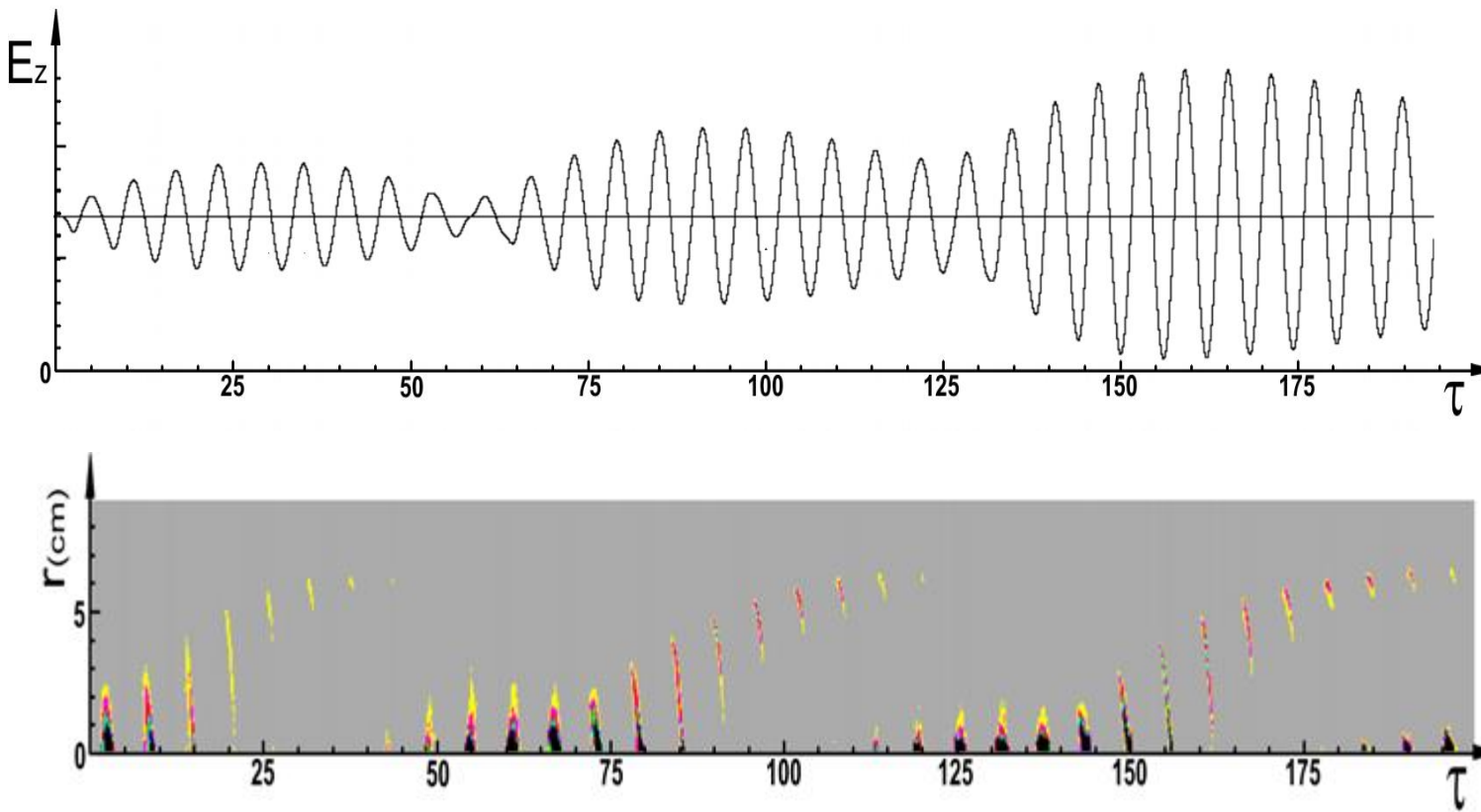


$$E_z^{(res)}|_{0\%} = 0.09672$$

$$E_z^{(opt)}|_{+0.35\%} = 0.1571$$

Maximum is at $n_e > n_e^{(res)}$

Mechanism of excitation in nonresonant case

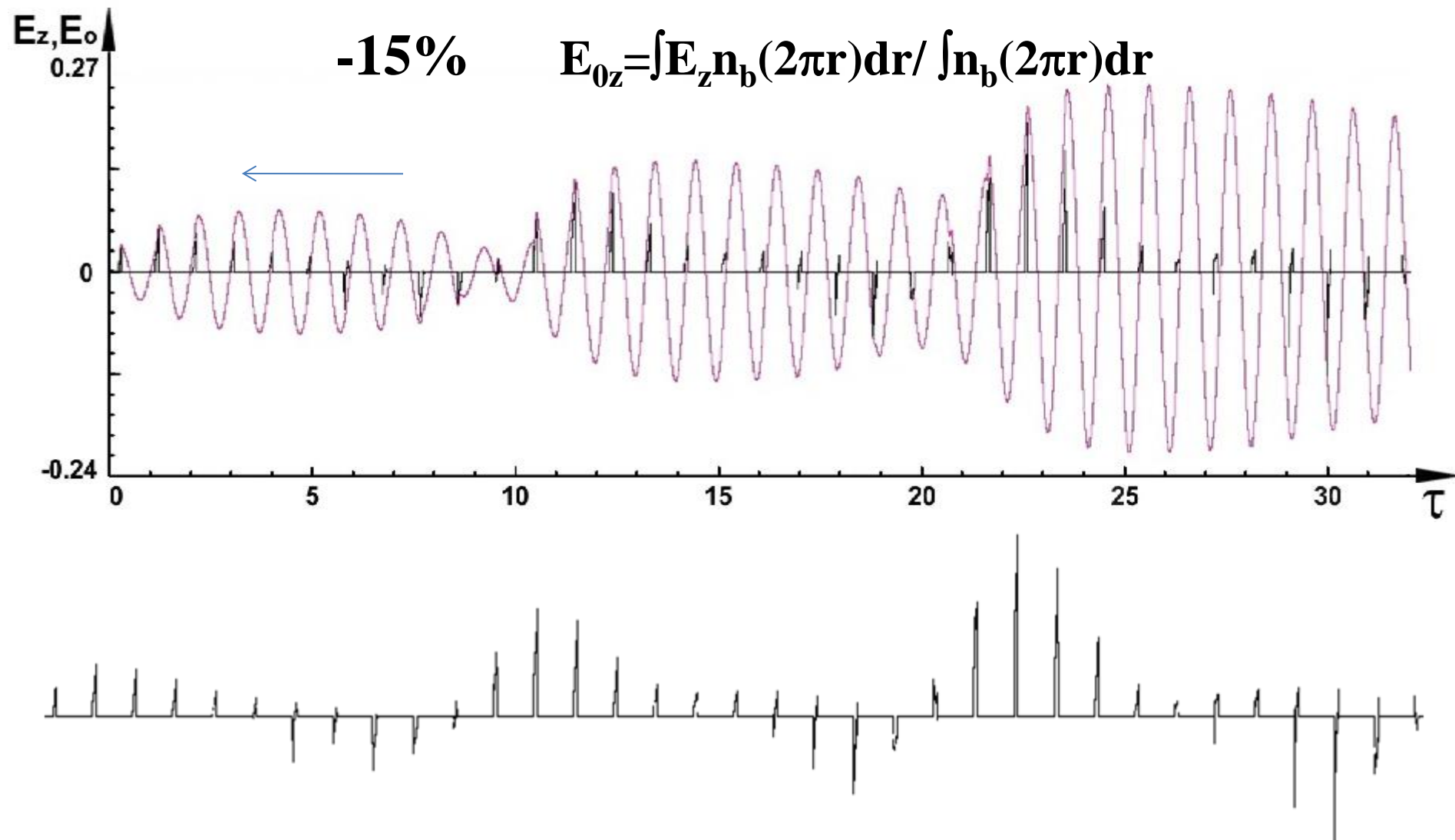


Reason of excitation is asymmetry of electron dynamics (Self-cleaning).

Temporal evolution of bunch density and on-axis wakefield E_z , excited by train of 32 drivers ($z=100$ cm) in the case of nonresonant plasma density (-15%)

Wakefield amplitude is oscillated and grows with number of injected drivers N in the case of nonresonant plasma density at not large γ_b in contrary to very large γ_b . In the last case the wakefield amplitude is oscillated with N .

Mechanism of excitation in nonresonant case using coupling drivers and wakefield



Coupling of drivers with wakefield

Mechanisms of defocusing and synchronization of drivers at wakefield excitation in plasma

Aim

Numerical simulation, using 2d3v code LCODE, of mechanisms of defocusing of train of drivers - relativistic electron bunches at wakefield excitation in plasma

We consider defocusing mechanisms of drivers at wakefield excitation by them in plasma.

1) In nonresonant case due to $\omega_d \neq \omega_{pe}$ the drivers are shifted relatively to a wave and some of them get in large radial force $F_r \neq 0$.

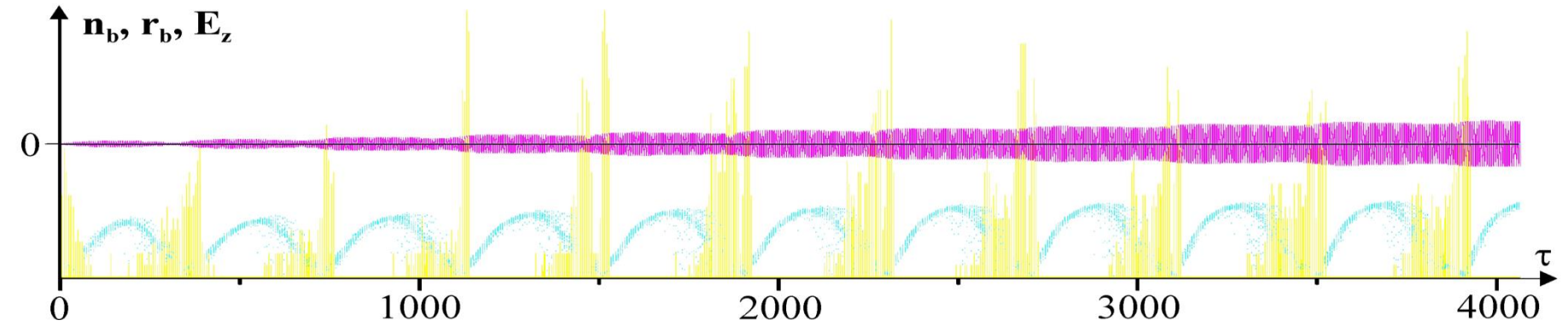


Fig. 1. Longitudinal wakefield E_z (red), density n_b (yellow) and radius of 675 short bunches r_b (blue) in nonresonant case (electron plasma density n_e is smaller on 3% than resonant one)

In non-resonant case electrons of bunches in the midpoints of beatings get in large $F_r \neq 0$, and their radius is greatly increased there, and their density on the axis (yellow) is strongly reduced.

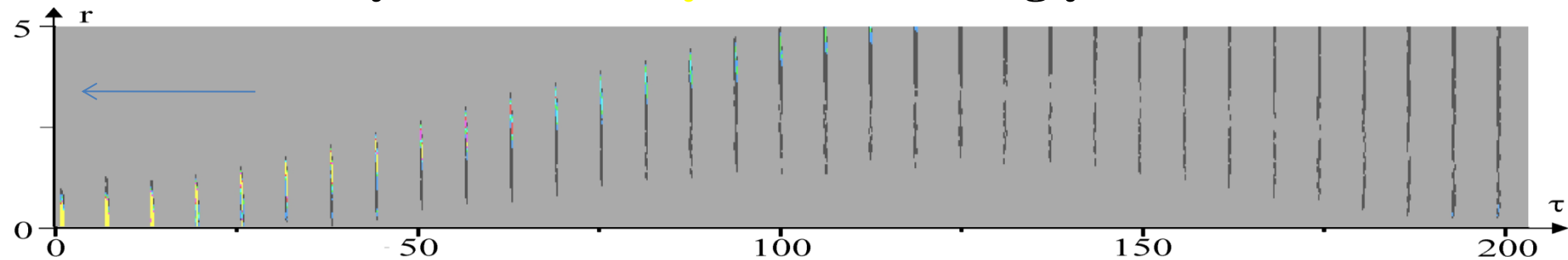


Fig. 3. The density of 33 “point” drivers far from the boundary of injection

Drivers in the midpoints of beatings get in large $F_r \neq 0$, and their radius is greatly increased.

2) At inhomogeneous focusing/defocusing warping of bunches (bunches-bells) happens simultaneously (see Fig. 6), which can effect on the wave warping (wave is a periodical chain of bells).

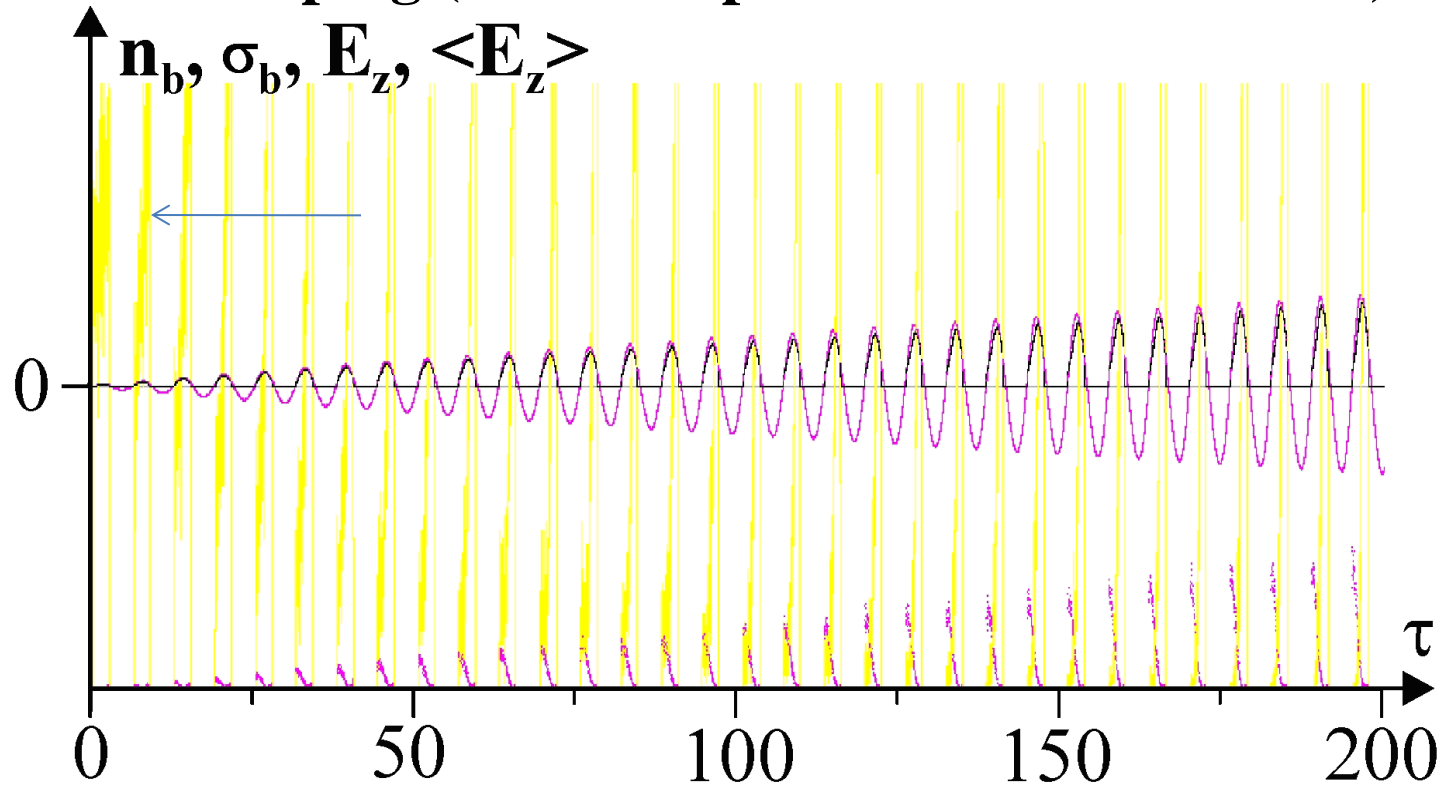


Fig. 6. Warping of bunches at E_z excitation in resonant case. Density (yellow) and radius r_b (red) of 33 “point” bunches, E_z (red) and $\langle E_z \rangle$ (black) coupling rate of bunches with E_z

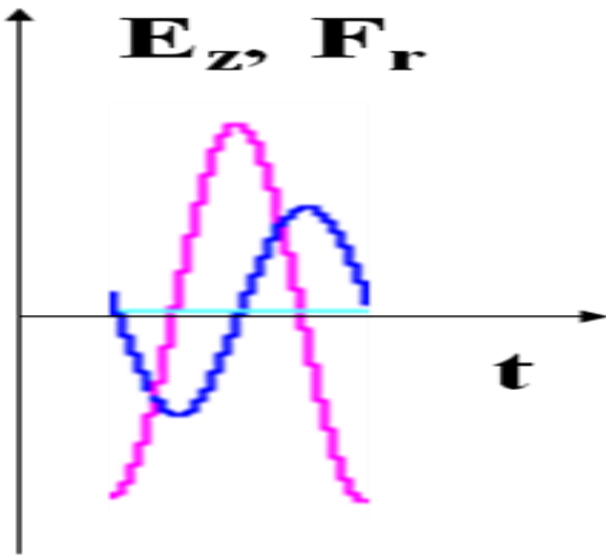
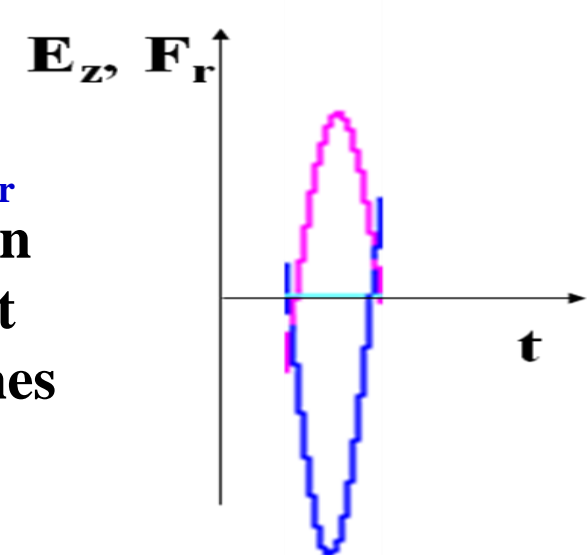


Fig. 7. Shift of \mathbf{E}_z and \mathbf{F}_r relatively to each other in longitudinal direction at radial dynamics of bunches



It changes relation \mathbf{E}_z and \mathbf{F}_r and changes conditions of focusing /defocusing. Bunches, which are decelerated, are focused.

3) finite length of bunches $\Delta\xi_b \neq 0$ leads to that their 1-st fronts are defocused and back fronts are focused. Thus also the bunches are simultaneously warped. This can effect on the wave warping.

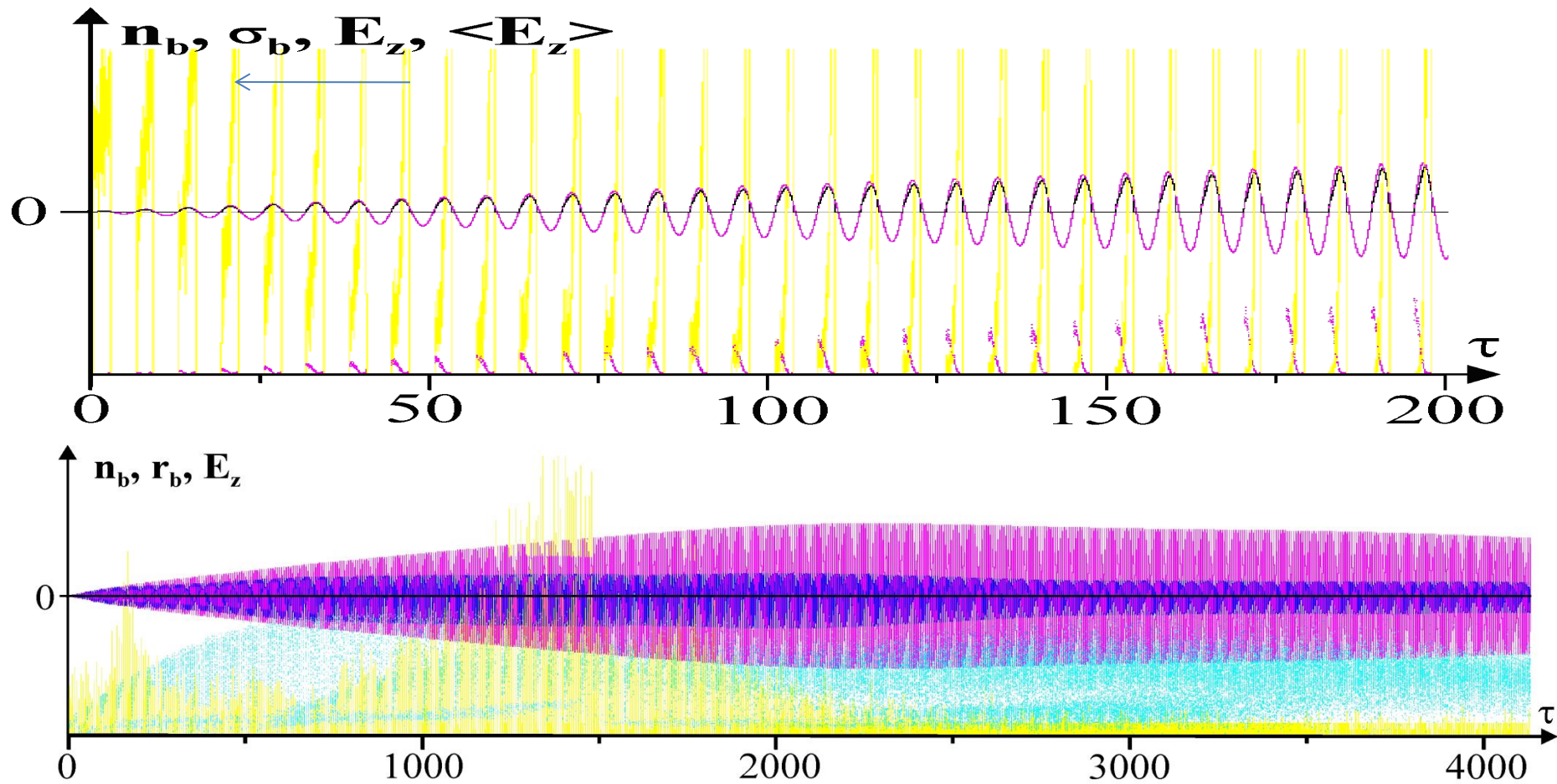
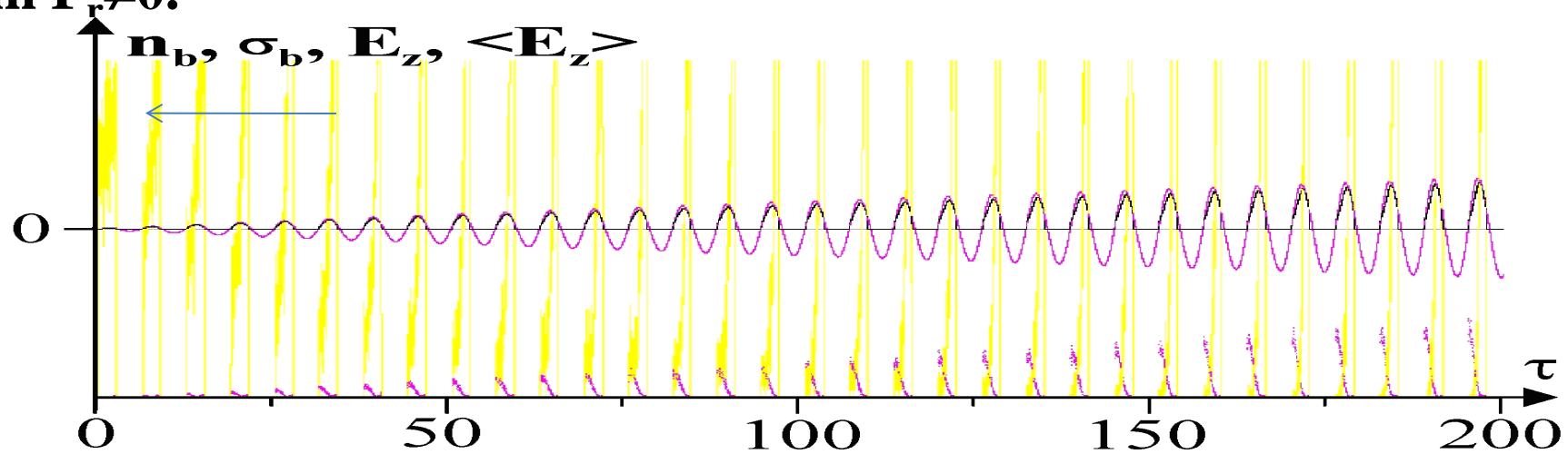


Fig. 8. E_z (red), n_b (yellow) and radiuses r_b (blue) 675 of bunches in resonant case

4) Dependence on $r_b \neq 0$ at $F_r|_{r=0}=0$. Due to finite radius of bunches $r_b \neq 0$ some their electrons get in the finite radial field F_r . Even if bunches are short on z (i.e. they are pancakes), at wave warping (due to leaving of compensative plasma electrons) fields E_z and F_r are shifted relatively to each other in longitudinal direction (see Fig. 7). Thus, if before warping bunches-pancakes were in $E_z^{(\max)}$ and $F_r \approx 0$, then after wave warping the periphery (on r) of bunches-pancakes gets in $F_r \neq 0$.



I.e. explanation of "rapid" radial evolution of bunches, "point" on z of finite radius, is following. Wave warping results in relative shift of E_z and F_r . Consequently, bunches get in large F_r .

5) bunches, getting in focusing phases of wakefield, are expanded due to broadening of betatron oscillations, because wakefield amplitude decreases along the axis to the front of train of bunches.

Broadening of betatron oscillations (see the betatron oscillations in Fig. 9-11)

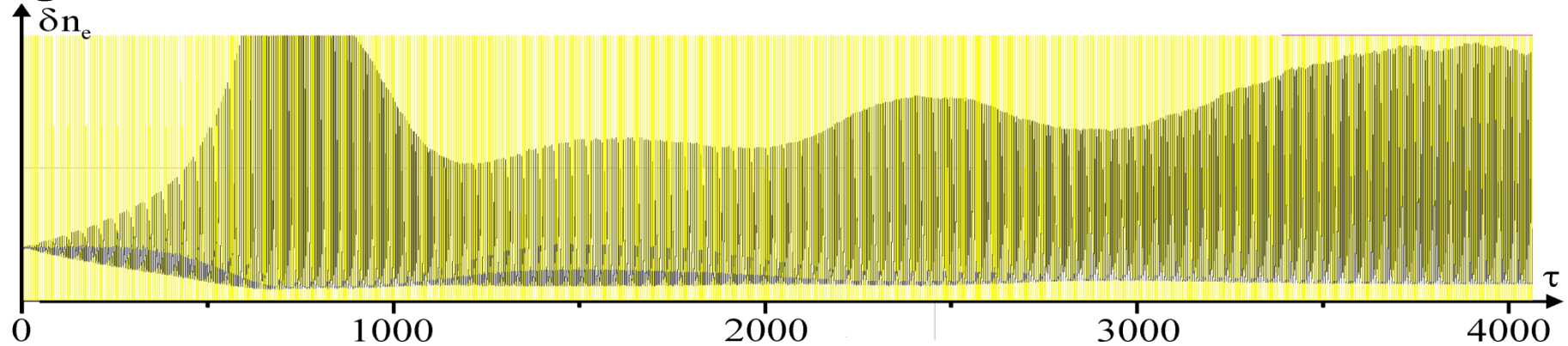


Fig. 9. δn_e (black) in wakefield, excited by 675 resonant electron bunches far from the boundary of injection. $\gamma_b=5$

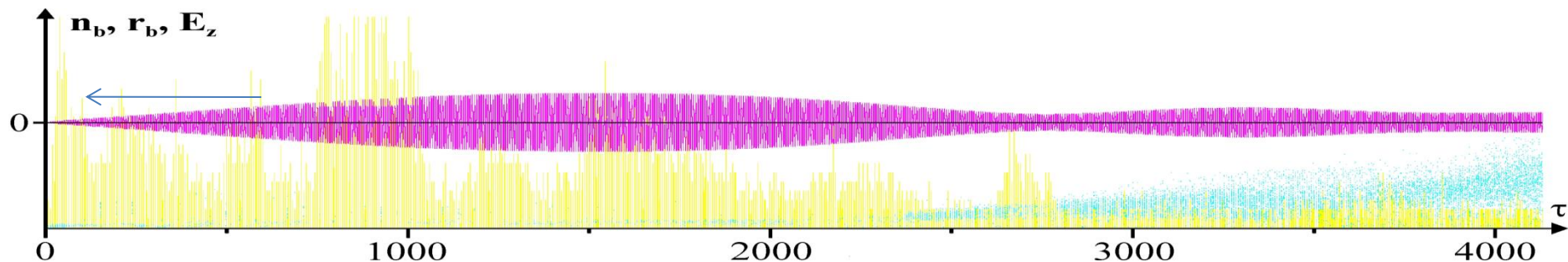


Рис. 10. E_z (red), density n_b (yellow) and radiuses r_b (blue) of bunches

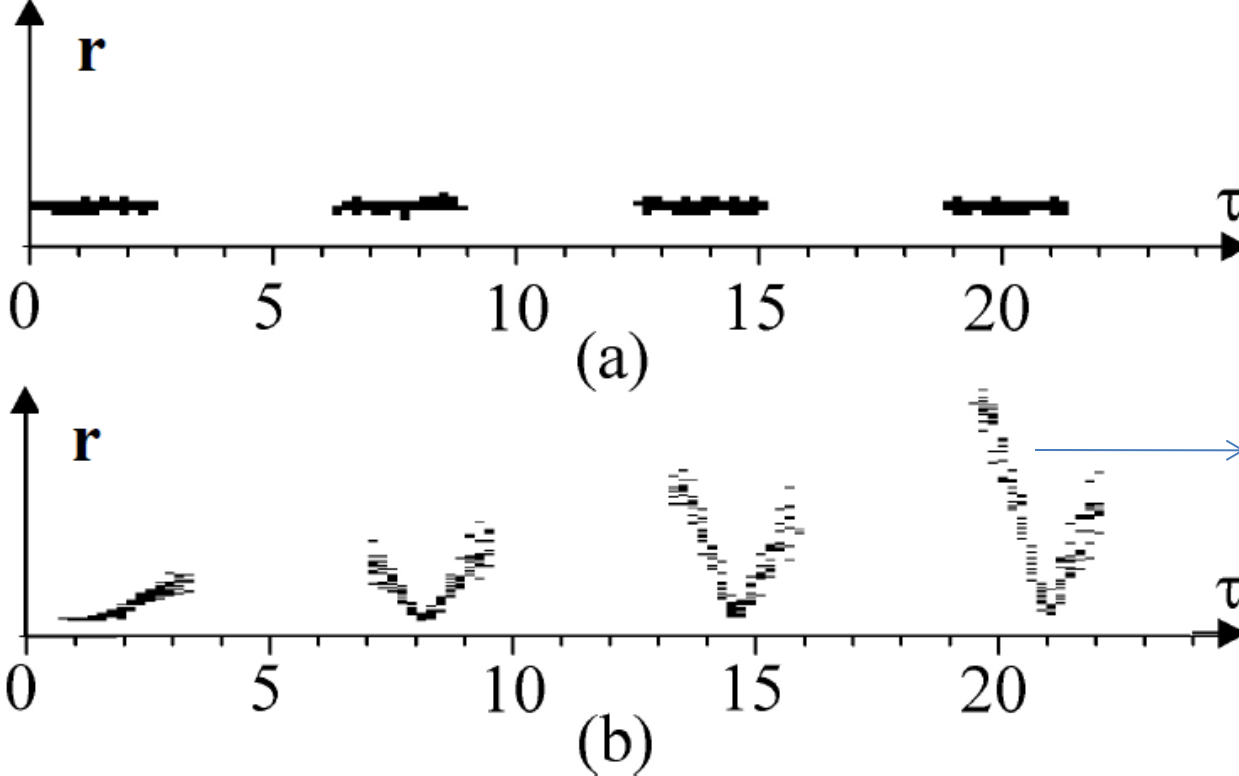


Fig. 11. Electron beam defocusing and over-focusing

The betatron oscillations are expanded as follows. At first the bunches are focused. Then the bunches are defocused because the wakefield

amplitude is spatially inhomogeneous. Namely, the wakefield decreases to the 1-st front of train of bunches.

At $\gamma_b \rightarrow \infty$ or in the case of strong external magnetic-field focusing of electron bunches is damped, and $E_z^{(\max)}$ becomes smaller, but amplitude is homogeneous along the system.

As we already mentioned, bunches are also defocused due to waves warping which can occur due to leaving of plasma electrons, compensating charge of bunches, from the axis. It leads to the fact that the plasma frequency on axis $\omega_{pe}(r=0)$ is less than the plasma frequency on the periphery $\omega_{pe}(r=0) \neq \omega_{pe}(r \neq 0)$.

The wave warping is a charge-dependent phenomenon. Really, in the case of electron bunches the wave is warped in one side (see Fig. 12), and in the case of positron bunches the wave is warped in other side (see Fig. 13).

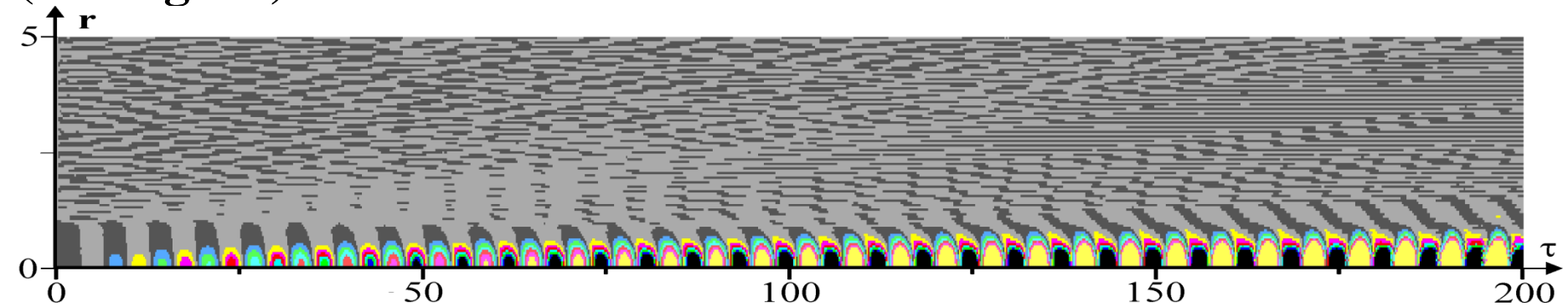


Fig. 12. Wave of n_e in the case of electron bunches

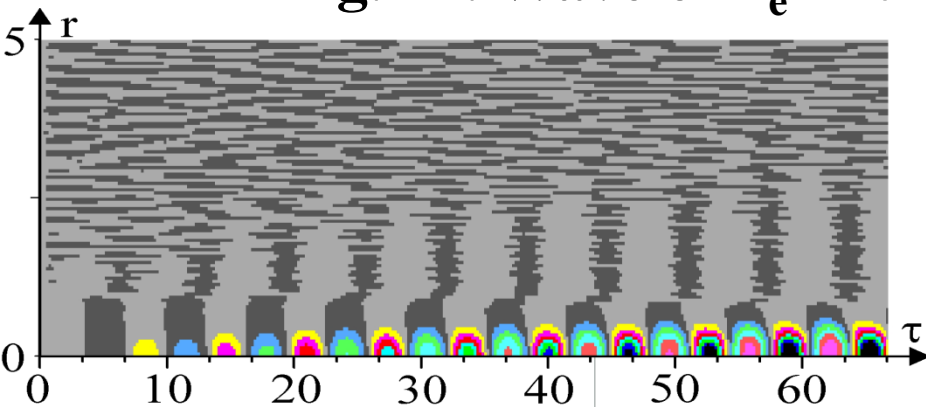


Fig. 13. Wave of n_e in the case of positron bunches

Conclusions

It has been shown that the following mechanisms can lead to defocusing of electron bunches:

- 1) shift of bunches relatively to the wave in nonresonant case;
- 2) finite length of bunches $\Delta\xi_b \neq 0$ results in that their 1-st fronts are defocused and back fronts are focused;
- 3) at the finite radius of bunches even if they are short, at a wave warping due to leaving of compensative plasma electrons from the axis the fields E_z and $\propto F_r$ are shifted relatively to each other in longitudinal direction and bunches gets in $F_r \neq 0$;
- 4) the bunch warping at focusing/defocusing can effect on the wave warping;
- 5) if bunches are in focusing phases, they can be defocused at the certain conditions due to expansion of the betatron oscillations.

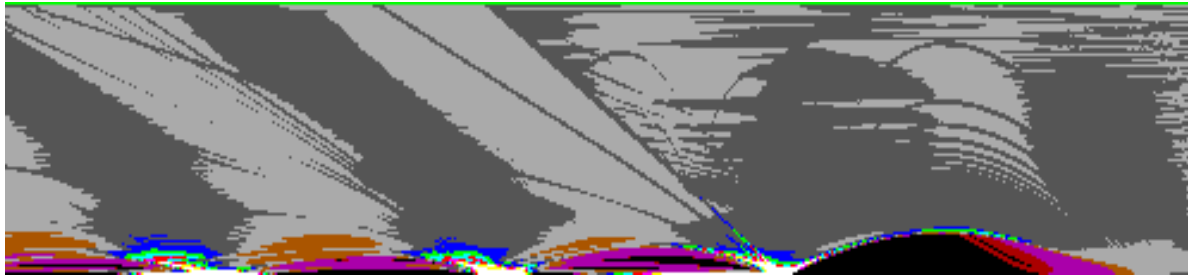
Simulation of Plasma Wakefield Bubble Excitation by Relativistic Electron Bunches

Train of plasma bubble excitation by train of bunches

Wake perturbation after one bunch



Initial bunch



Bubble and wake perturbation after one bunch.

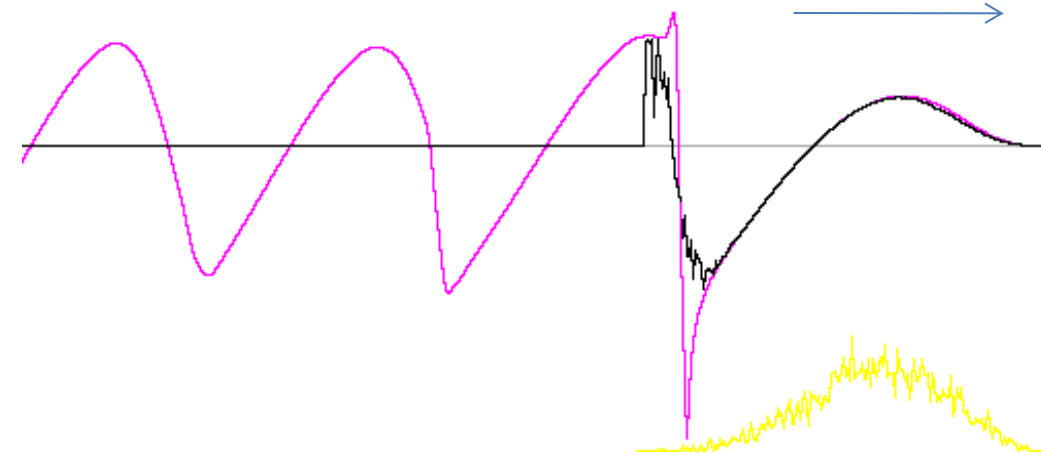


Bunch after focusing



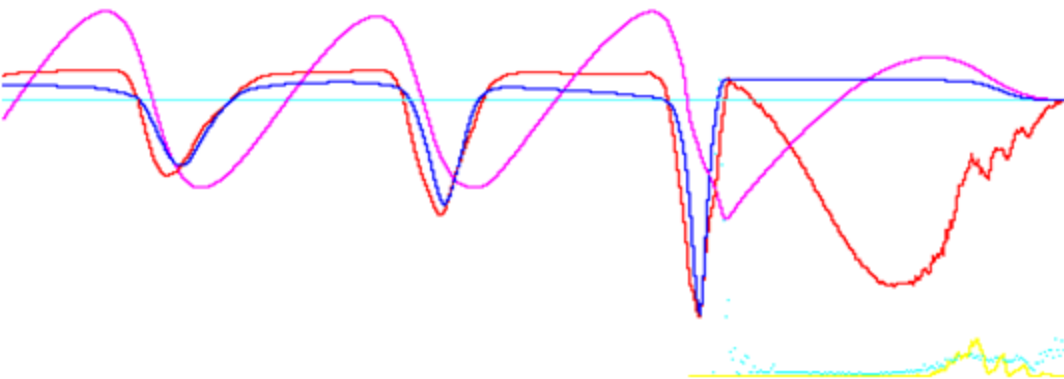
Front of bunch is decelerated and tail is accelerated

**Longitudinal wakefield E_z ,
bunch density on axis and
coupling coefficient on small
times**



**$E_r, \mathbf{F}_r, E_z \big|_{r=rb}$
on long times.**

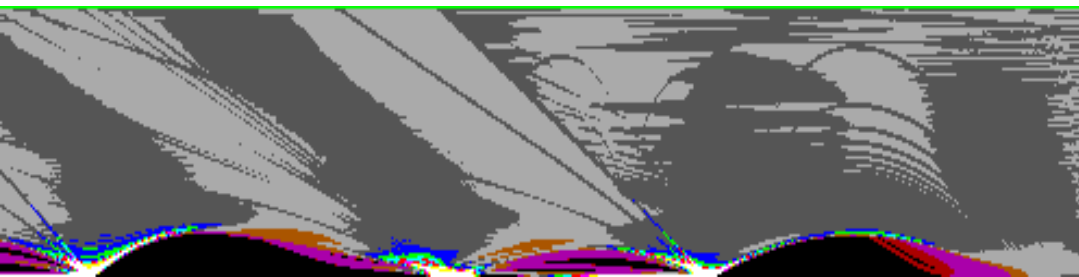
Bubble is ideal plasma lens.



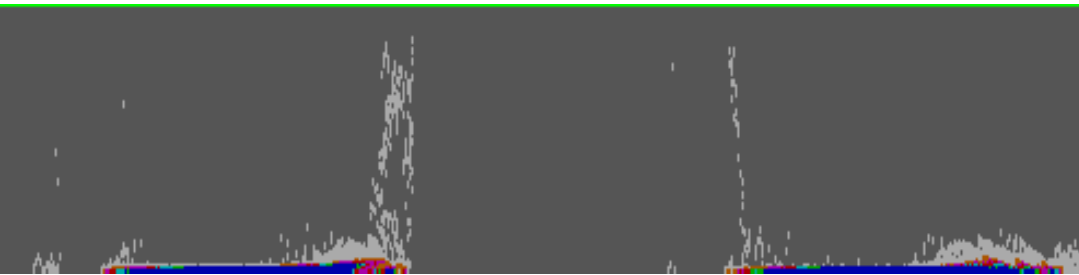
Enhancement of train of bubbles by train of drivers



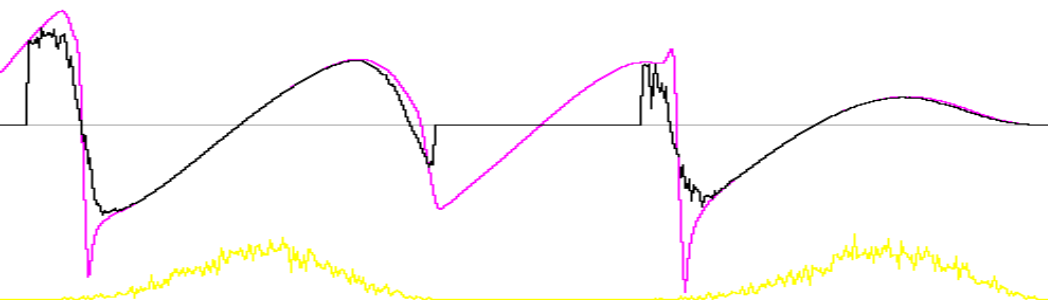
Two drivers



Wake perturbation by two drivers



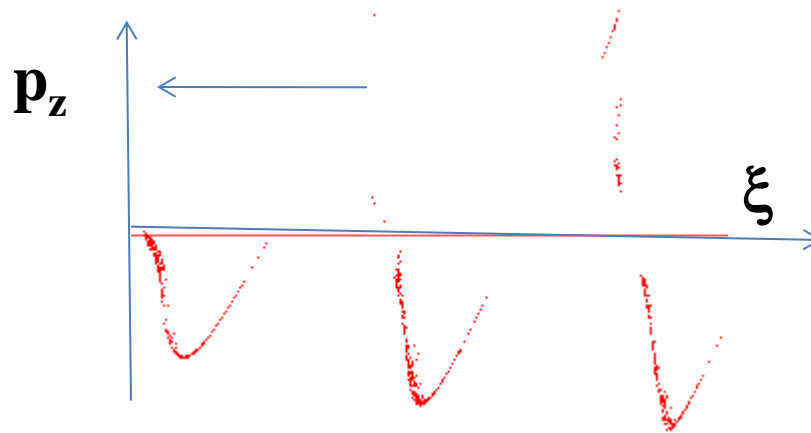
Two bunches after focusing



**Longitudinal wakefield E_z ,
bunch density on axis and
coupling coefficient on small
times**

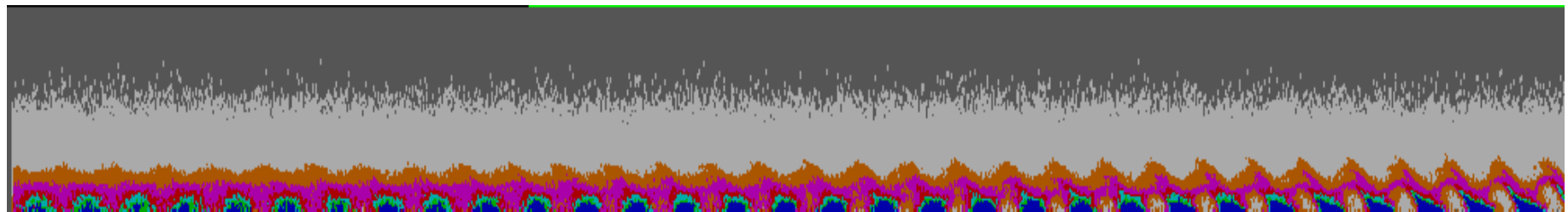
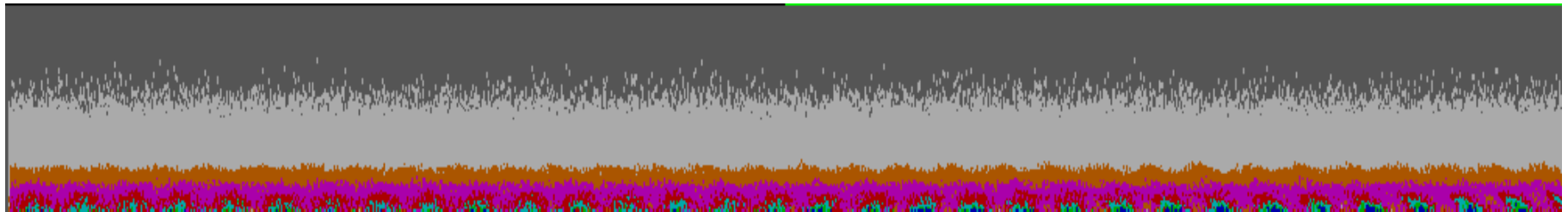
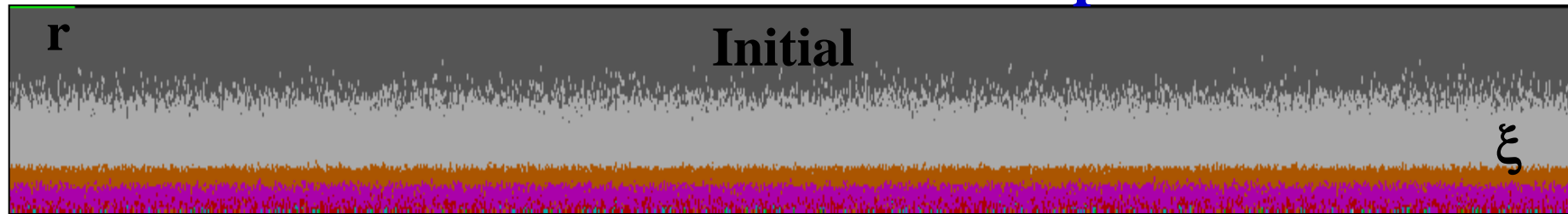
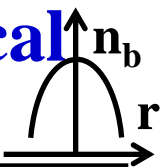
Optimal short non-resonant train of dense bunches

3 nonresonant bunches amplify wakefield bubbles and 1-st front of 3-rd bunch is accelerated.

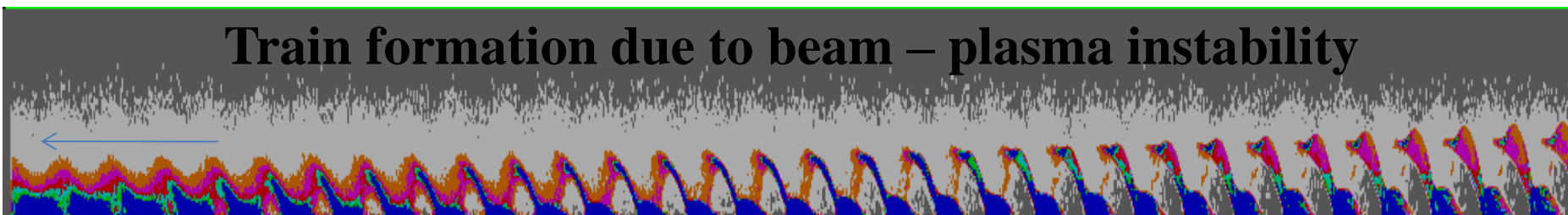


**Longitudinal phase
space portraits of 3
bunches at their
interaction with train
of bubbles**

2d3v Numerical simulation of instability of cylindrical relativistic electron beam in plasma

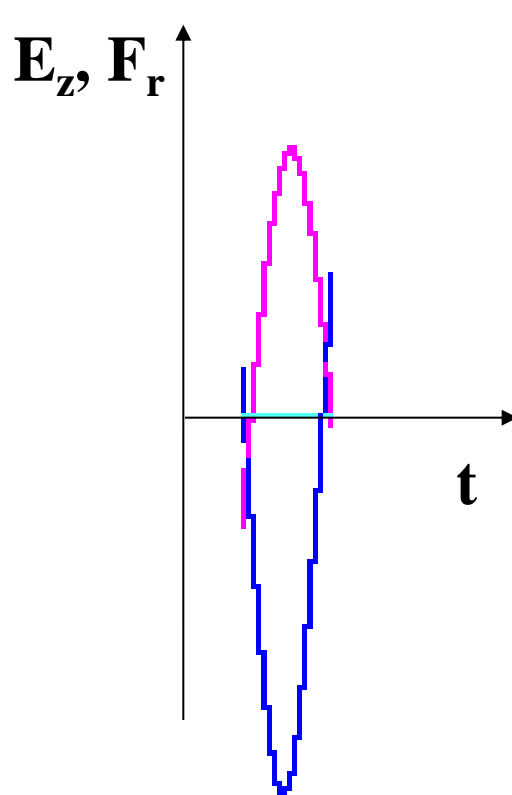
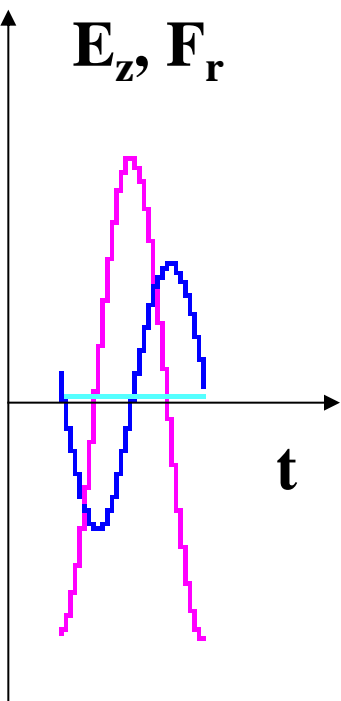


Train formation due to beam – plasma instability



Spatial (r, z) distribution of density of beam electrons

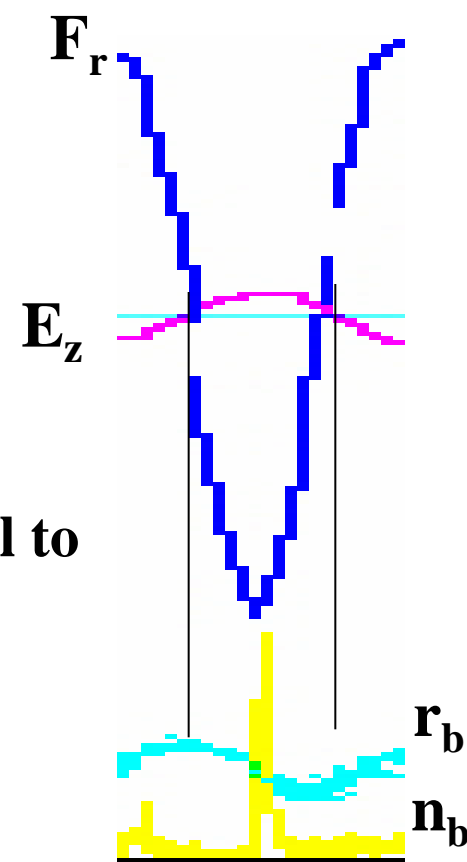
Density of formed train of bunches $n_b > n_{b0}$ due to focusing.



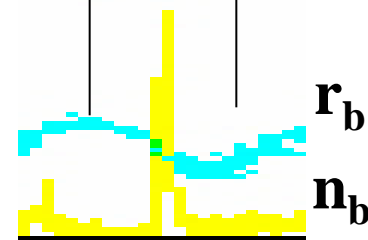
E_z (red) on radius, equal to r_b , F_r (dark blue)

At first, the relative position of E_z and F_r equals $\pi/2$, then it is changed due to focusing to π (decelerated bunches are focused), and then it returns to $\pi/2$ at the expansion.

Decelerated bunches are focused

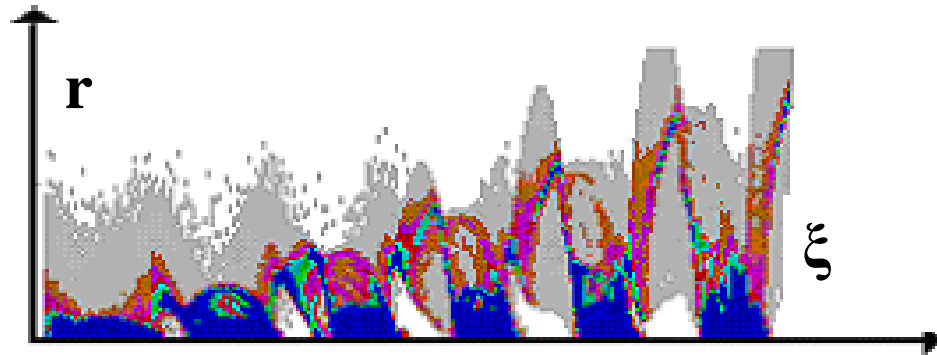
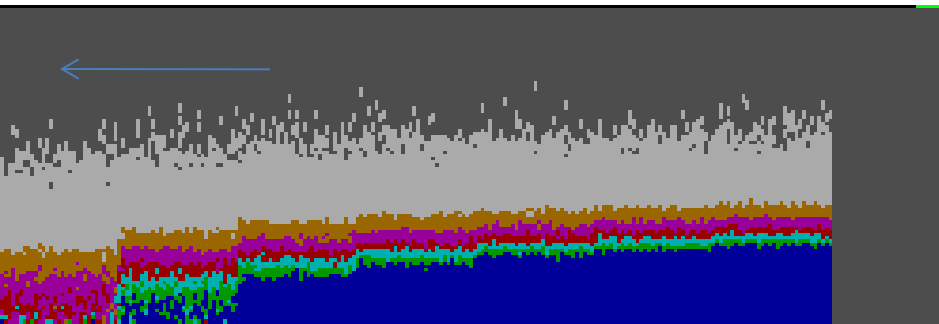


n_b (yellow) at a radius equal to r_b , E_z (red) at radius equal to r_b , F_r (blue), r_b (blue)



Beam-plasma instability for long shaped electron bunch

Transformation of long electron bunch with density, growing along z , in shaped train of bunches due to focusing and defocusing.



$\gamma_b=5$, $\sigma_r=0.3c/\omega_p$, $I_b=0.3\times 10^{-3}mc^3/4e$.

Spatial (r, ξ) distribution of density of long electron bunch
After some time $(80\omega_p^{-1})$ the train of bunches is formed.

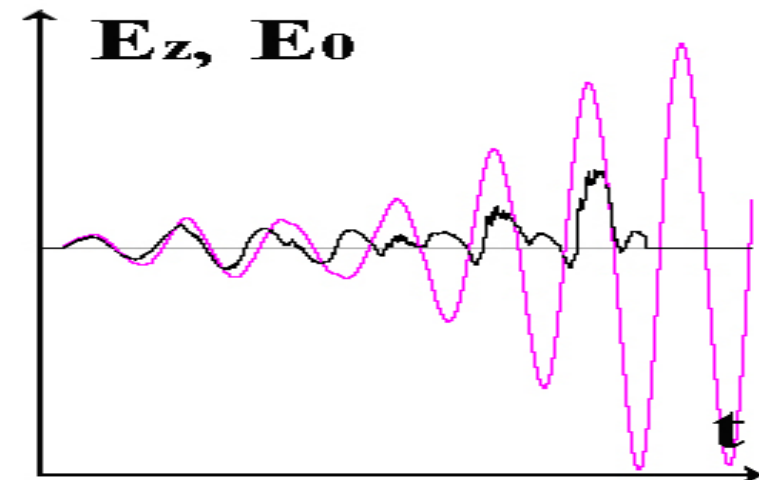
Spatial (r, ξ) distribution of density of formed train of electron bunches

$$n_{b1}:n_{b2}:n_{b3}:\dots \neq 1:3:5:\dots,$$

$$n_{b1}:n_{b2}:n_{b3}:\dots = 1:5:9:\dots$$

It also leads to $TR \approx E_z^{(\text{accel})}/E_z^{(\text{dec})} \gg 1$.

E_z (red), coupling rate of beam electrons with E_z (black)

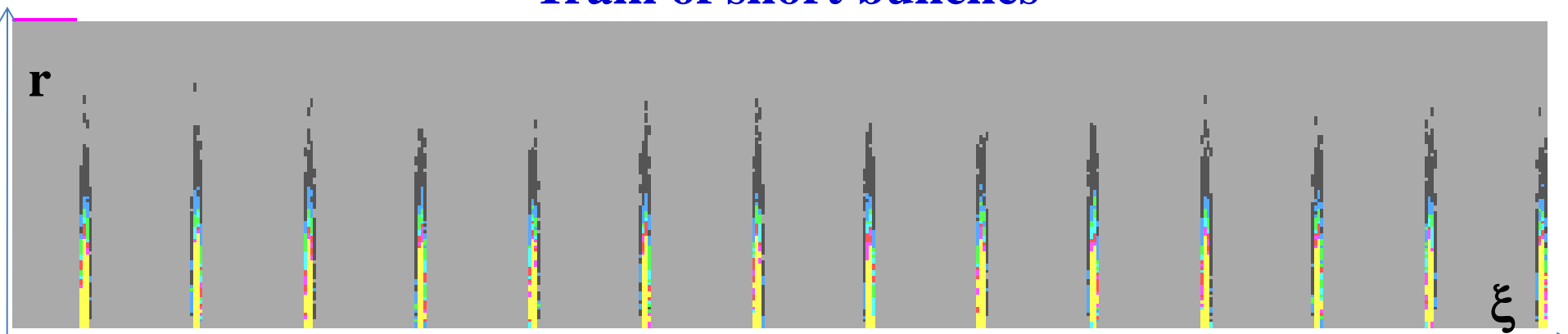


Coupling of beam electrons with E_z is larger (smaller) in decelerating (accelerating) phases.

Homogeneous Focusing of Train of Relativistic Electron Bunches in Plasma or Ideal Plasma Lens

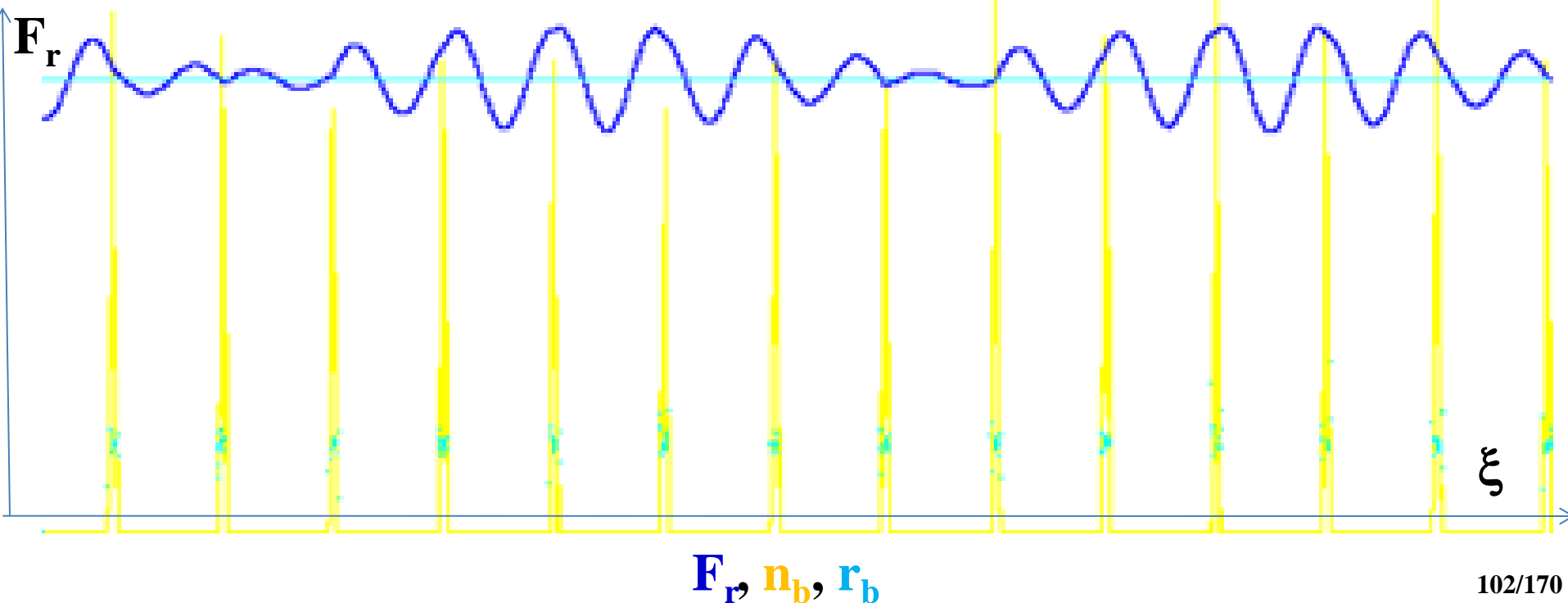
The focusing of relativistic electron bunches by wakefield, excited in plasma, is very interesting and important, **similar to laser pulses**.

Train of short bunches

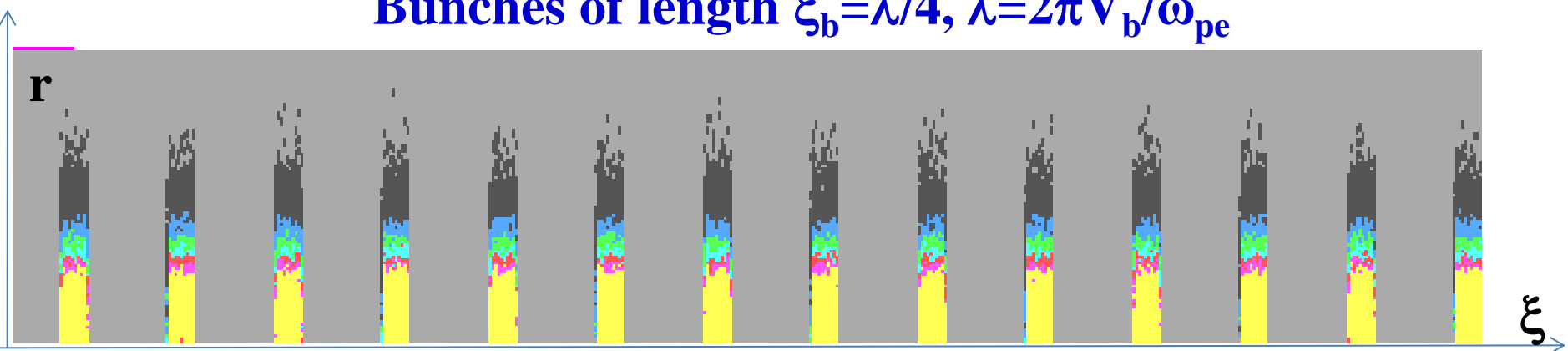


Density of High-energy Electron Bunches

At $\omega_d < \omega_{pe}$ beatings are excited. All bunches are in focusing fields of beatings except at fronts of beatings, where they are not focusing.

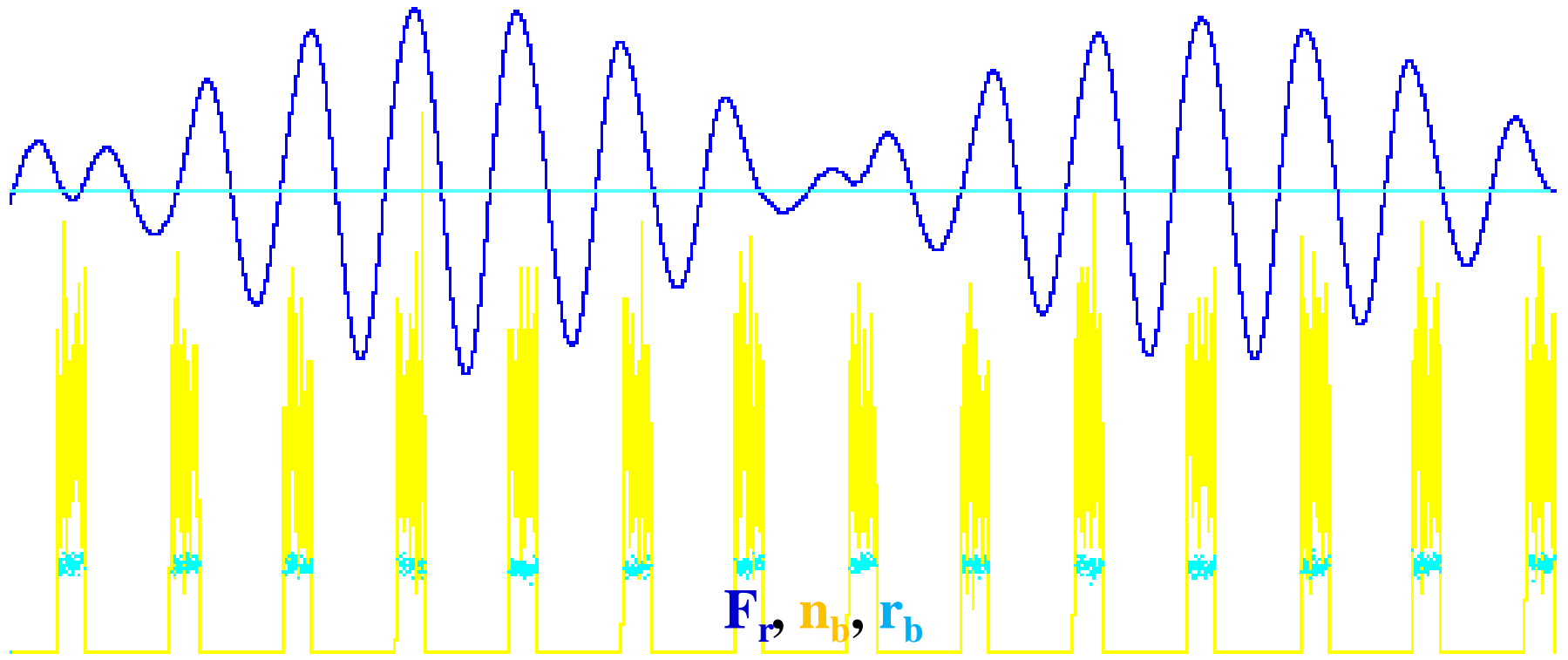


Bunches of length $\xi_b = \lambda/4$, $\lambda = 2\pi V_b / \omega_{pe}$



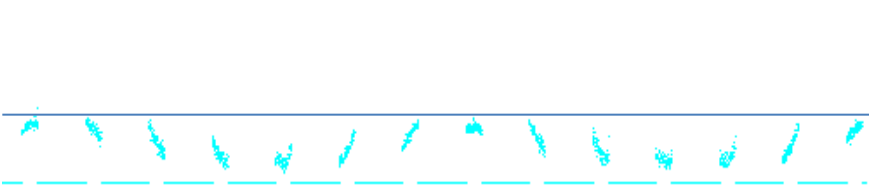
Density of High-energy Electron Bunches

All bunches are in focusing fields except at fronts of beatings, where they are not focusing.



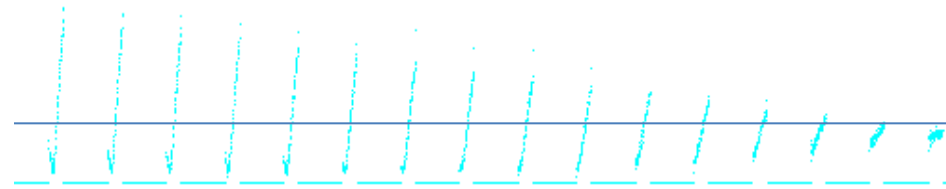
F_r , n_b , r_b

Comparison of focusing in non-resonant and in resonant cases



r_b

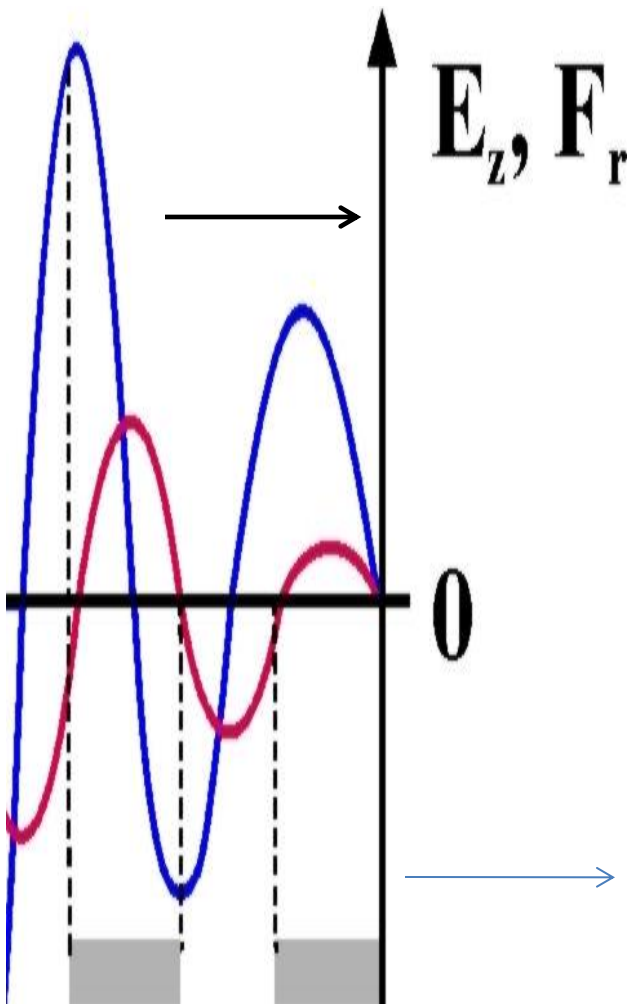
**Inhomogeneous focusing in
non-resonant case ($\omega_b < \omega_{pe}$)**



r_b

**In resonant case ($\omega_b = \omega_{pe}$) tail
of bunch is focused but it's first
front is defocused**

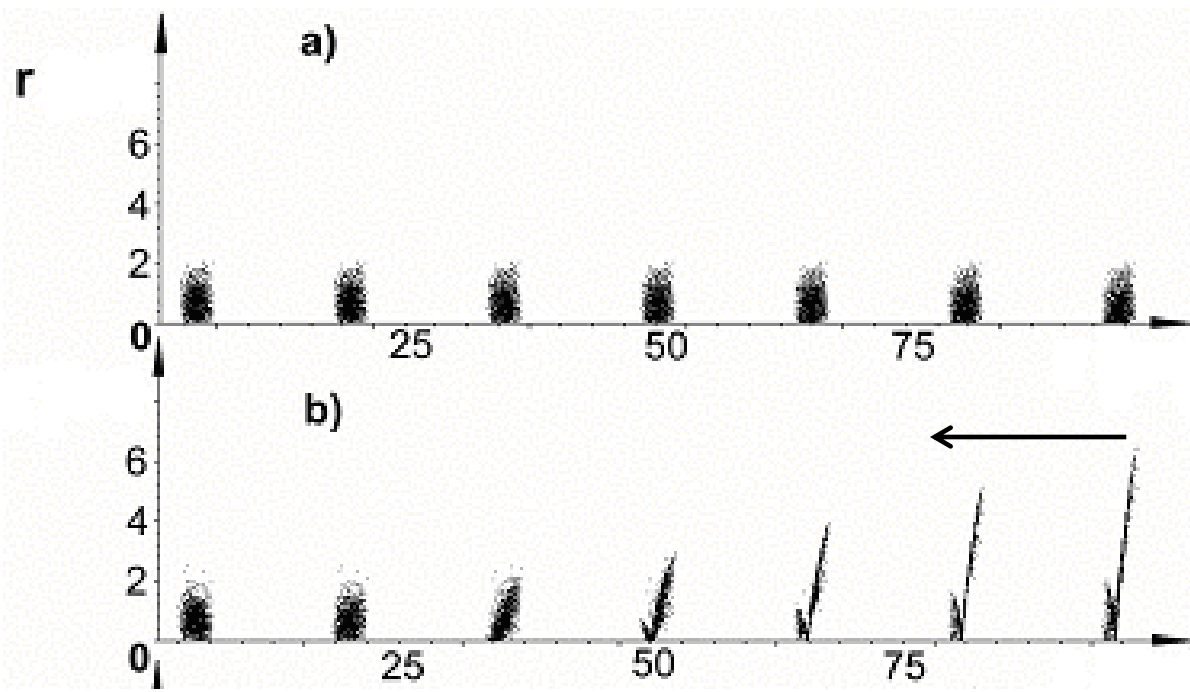
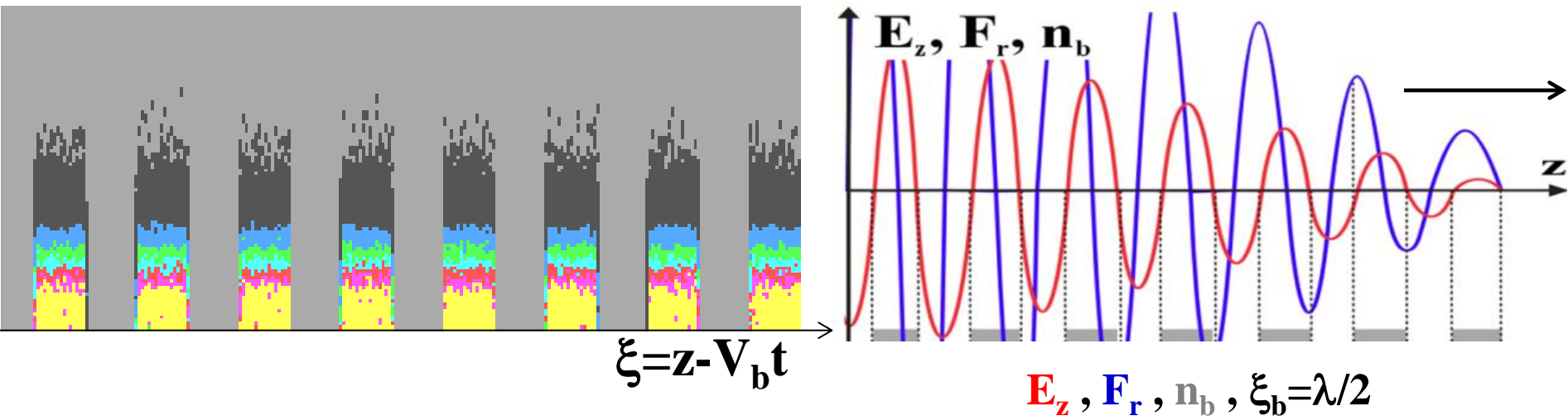
Focusing by resonant wakefield



Bunch repetition frequency ω_d coincides with the plasma frequency $\omega_d = \omega_p$. In resonant case 1-st bunch is only focused, and the back fronts of the next bunches are focused and their 1-st fronts are defocused.

Wakefield E_z , wake radial force F_r at resonant excitation of wakefield by bunches

Focusing wakefield, excited in plasma by resonant train of relativistic electron bunches



This lens is inhomogeneous. Longer part (back front) of bunch focuses by larger field F_r than shorter 1st front of bunch defocuses by smaller defocusing F_r .

Resonant wakefield lens is inhomogeneous

After 1st bunch of permanent density with $\xi_b = \lambda/2$

$$E_z \sim Z_{II}^{(\lambda/2)}(\xi) = \int_0^{\lambda/2} d\xi_0 \cos[k(\xi - \xi_0)] \approx (2/k) \sin(k\xi). \quad (1)$$

$$E_r \sim Z_r^{(\lambda/2)}(\xi) = \int_0^{\lambda/2} d\xi_0 \sin[k(\xi - \xi_0)] \approx -(2/k) \cos(k\xi). \quad (2)$$

E_z in the middle of 1st bunch equals

$$E_z \sim Z_{II.1}^{(\lambda/2)} = (1/k) \int_0^{\pi/2} dx_0 \cos(k\xi - x_0) \Big|_{k\xi = \pi/2} = (1/k). \quad (3)$$

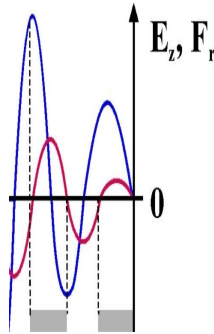
It is, as well as observed, in 2 times less than amplitude of the wakefield after 1st bunch.

Fields into 2nd resonant bunch

$$Z_{II.2}^{(\xi)}(\xi) = (2/k) \sin(k\xi) + \int_0^{\xi} d\xi_0 \cos[k(\xi - \xi_0) + 2\pi] = (3/k) \sin(k\xi),$$

$$Z_{r.2}(\xi)(\xi) = (2/k) [1 - 2\cos(k\xi)].$$

As $2\pi < k\xi < 3\pi$, $Z_{II.2}^{(\xi)}(\xi)$ changes from $Z_{II.2}^{(\xi)}(x=2\pi) = 0$ to $Z_{II.2}^{(\xi)}(k\xi=2.5\pi) = Z_{II.2}^{(\max)} = (3/k)$ and then again $Z_{II.2}^{(\xi)}(k\xi=3\pi) = 0$. Thus $Z_{r.2}^{(\xi)}(k\xi)$ changes from $Z_{r.2}(\xi)(k\xi=2\pi) = -(2/k)$ to $Z_{r.2}(\xi)(k\xi=3\pi) = (6/k)$, reaching zero in 1st half of bunch, where $\cos(k\xi_a) = 1/2$, $k\xi_a = 2\pi + \pi/3 < 2\pi + \pi/2$. I.e. **longer (in $(\pi - k\xi_a)/k\xi_a = 2$ times) part (back front) of bunch focuses in larger field E_r than 1st front (more short) of bunch defocuses (in 3 times less field E_r)**.



Large focusing wakefield, but inhomogeneous.

For identical and homogeneous focusing it is necessary that charge of 1-st bunch to be in 2 times smaller than charge of each next bunch

$$Q_1 = Q/2, Q_i = Q, i=2, 3, 4 \text{ etc.}$$

and distance between bunches should be equal $3\lambda/2$

$$\xi_{i+1} - \xi_i = 1.5\lambda.$$

All bunches with the exception of 1-st one do not change by energy with wakefield, $E_z=0$. Then wakefield after i-th bunch is the same as before it. But the bunches are focused because $E_r \neq 0$.

Focusing force is constant along the bunch

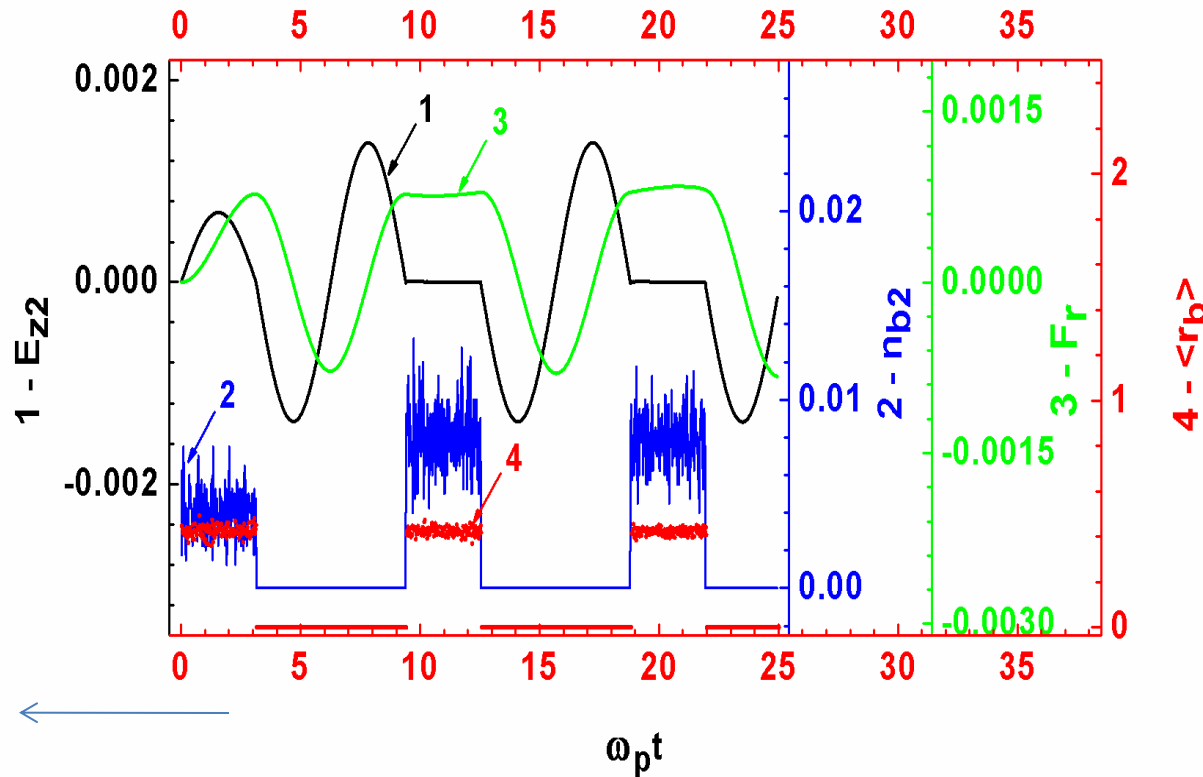
$$F_r = \text{const.}$$

We use the charge of 1-st bunch in 2 times smaller than the charge of each next bunch

$$Q_1 = Q/2, Q_i = Q, i=2, 3, 4 \text{ etc.}$$

$\Delta\xi_b = \lambda/2$ and distance between bunches equal $3\lambda/2$

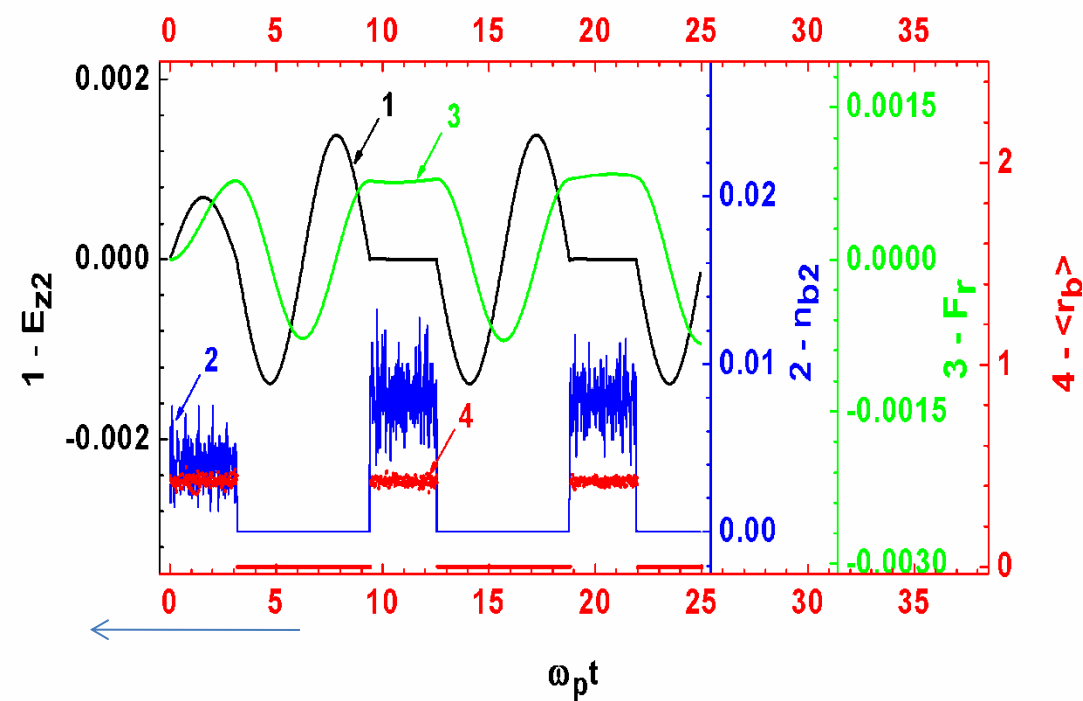
$$\xi_{i+1} - \xi_i = 3\lambda/2.$$



F_r is the plateau
in regions of
bunches and $E_z=0$.

n_b (2-dark blue), $\langle r_b \rangle$ (4-red), E_z (1-black) and F_r (3-green)

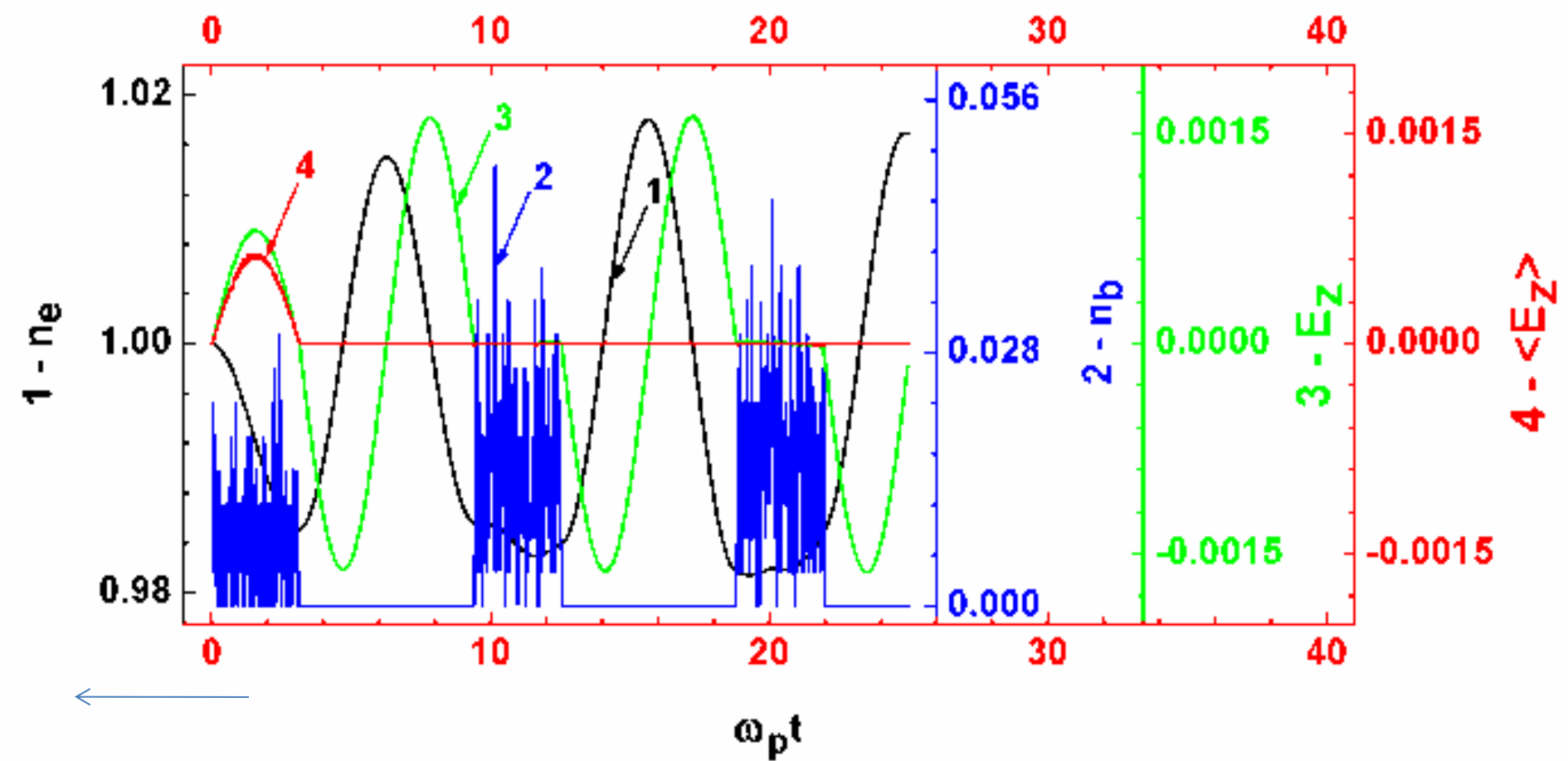
It must be the same for multi-pulse laser wakefield



All bunches with the exception of 1-st one do not change by energy with wakefield, $E_z=0$. Then wakefield after i -th bunch is the same as before it. But the bunches are focused because $F_r \neq 0$.

This wakefield “ideal” plasma lens has following qualities:

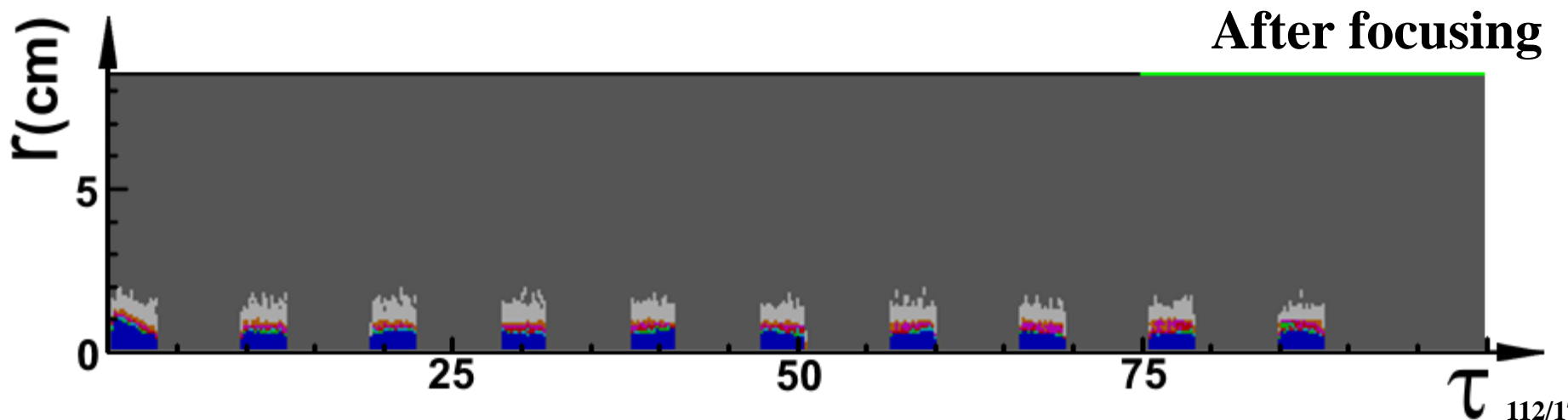
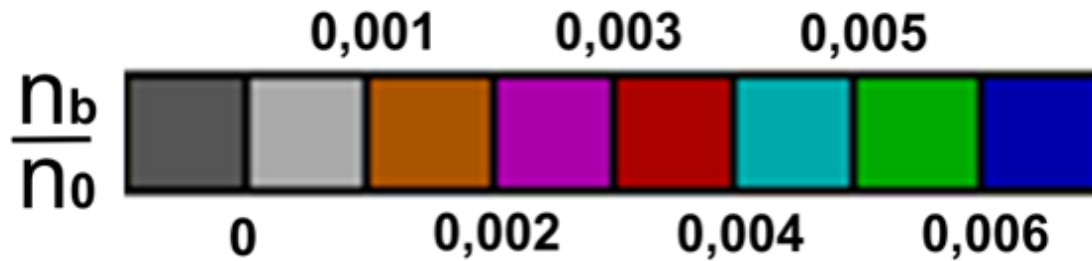
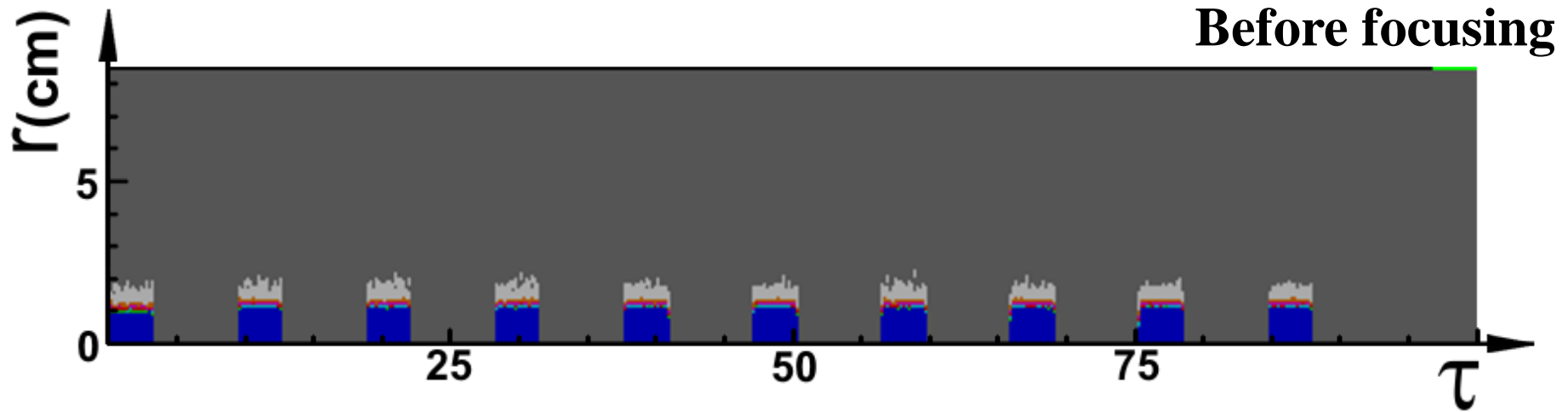
- 1) F_r does not approximately depend on longitudinal coordinate in regions, occupied by bunches, $F_r \approx \text{const}$, i.e. lengthy bunches are focused identically;
- 2) only first bunch is decelerated;
- 3) all bunches of train are under effect of identical focusing force;
- 4) $E_z=0$ in regions, occupied by bunches.



The distribution of n_e (1-black) in wakefield, E_z (3-green), n_b (2-dark blue) and coupling rate $\langle E_z \rangle$ (4-red)

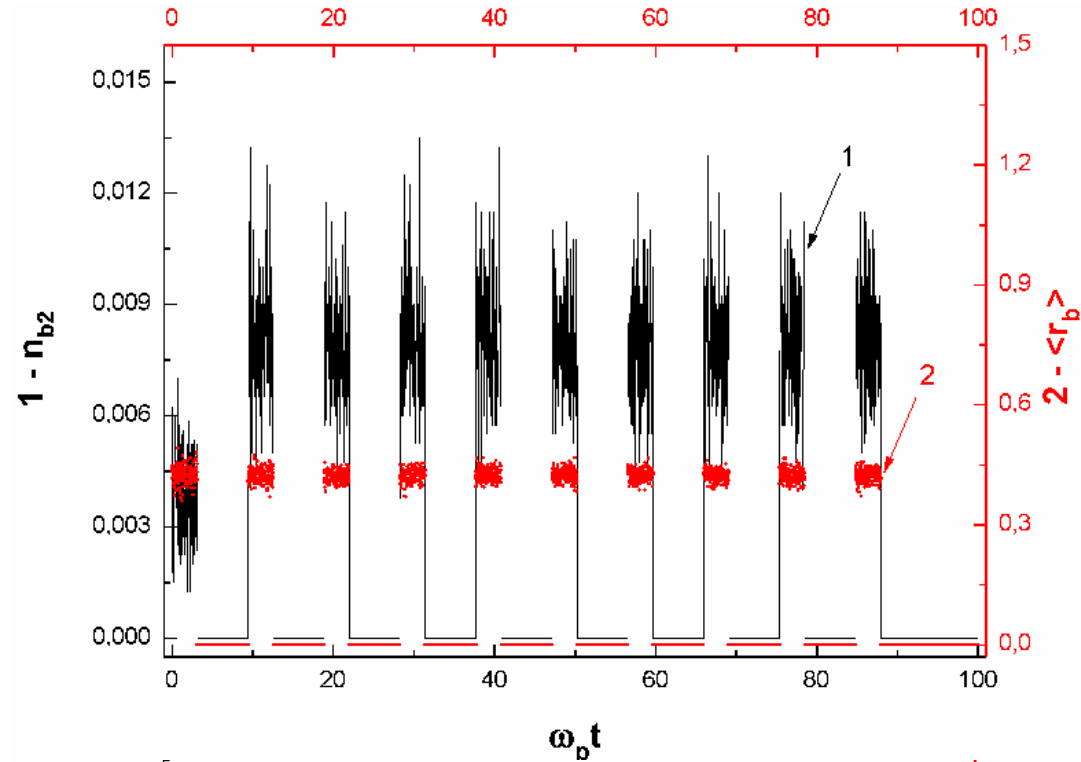
Such ideal distribution of focusing forces is realized due to formation of flat holes of plasma electron density in regions, occupied by bunches, which neutralize the charges of bunches and focus them.

Focusing of bunches

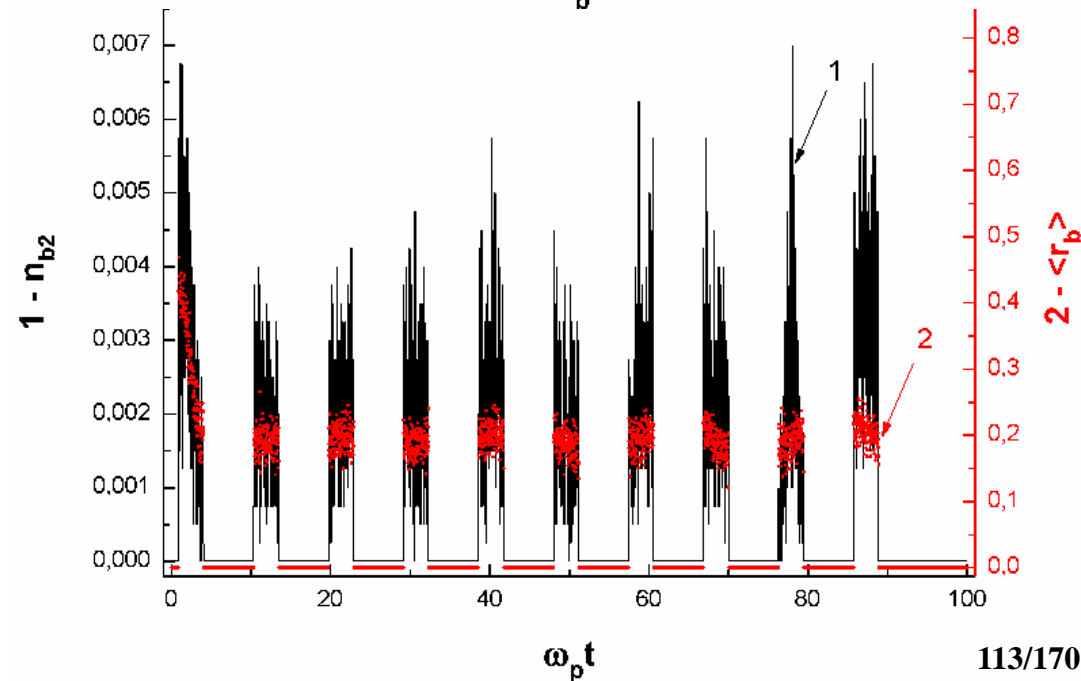


Focusing of bunches

Before focusing



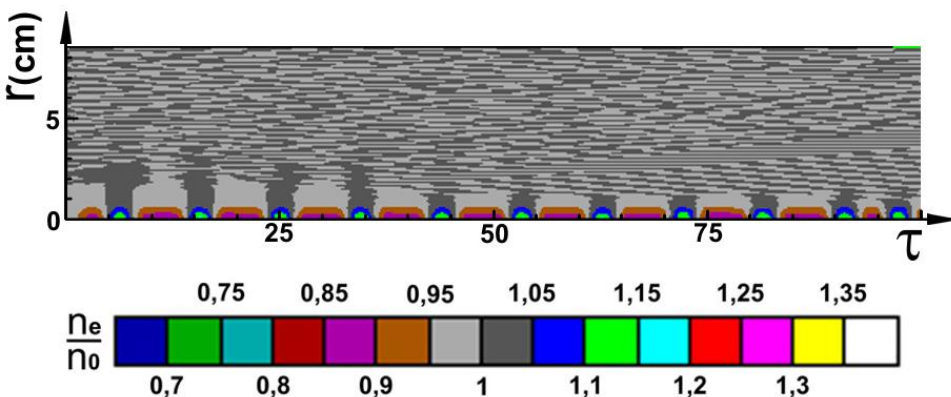
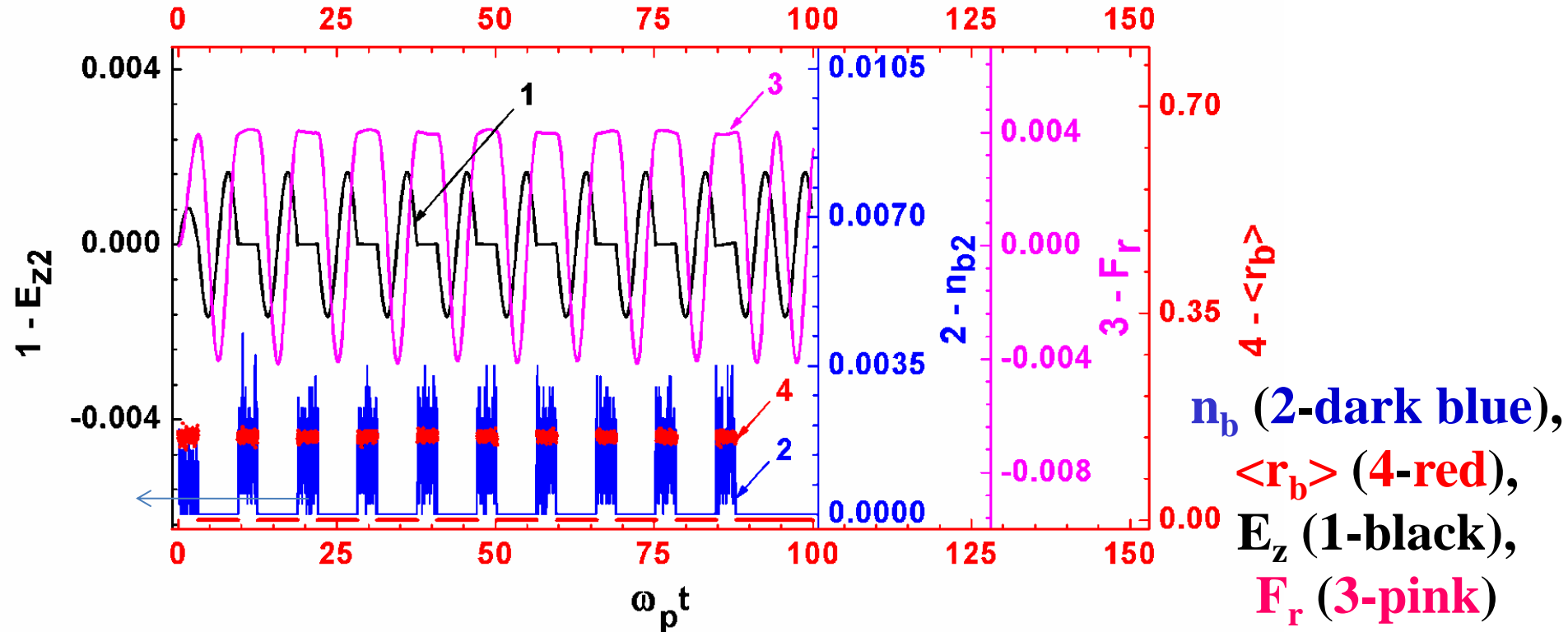
After focusing the bunch radius decreases in $\langle r_{b0} \rangle / \langle r_b \rangle \approx 2.25$ and bunch density at $r = \langle r_{b0} \rangle$ decreases in approximately 3 times.



Needle bunches

We considered bunches with close radius and length.

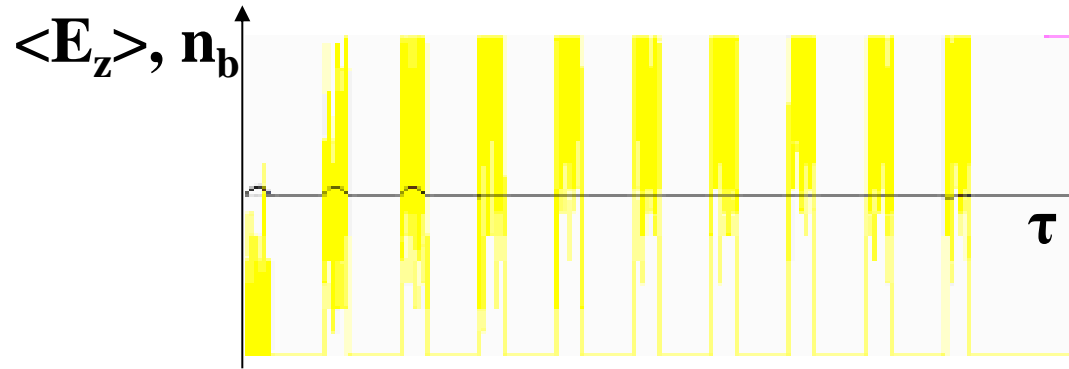
For “needle” the plateau on F_r is more ideal.



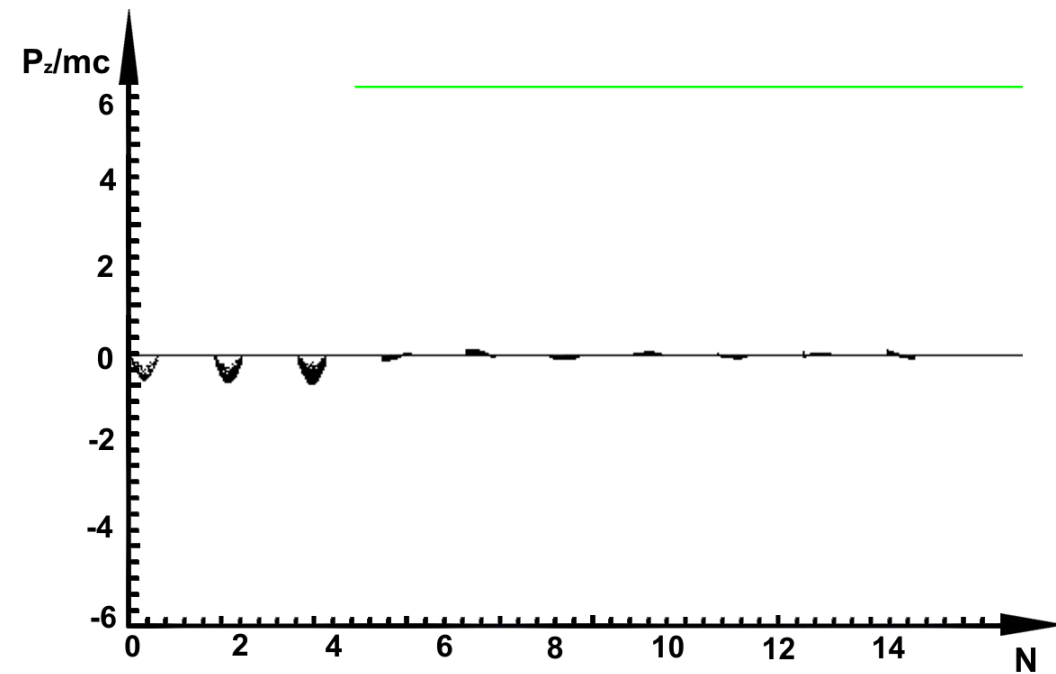
Distribution of plasma electron density in wakefield, excited by needle bunches.

$\delta n_e < 0$ is long, $\delta n_e < 0$ is short.

For focusing force enhancement let us consider the shaped train of bunches: the charges of 1-st N bunches increase according to following dependence: $2k-1$, $k \leq N$. The charges of next all bunches equal $2N$, $k > N$.

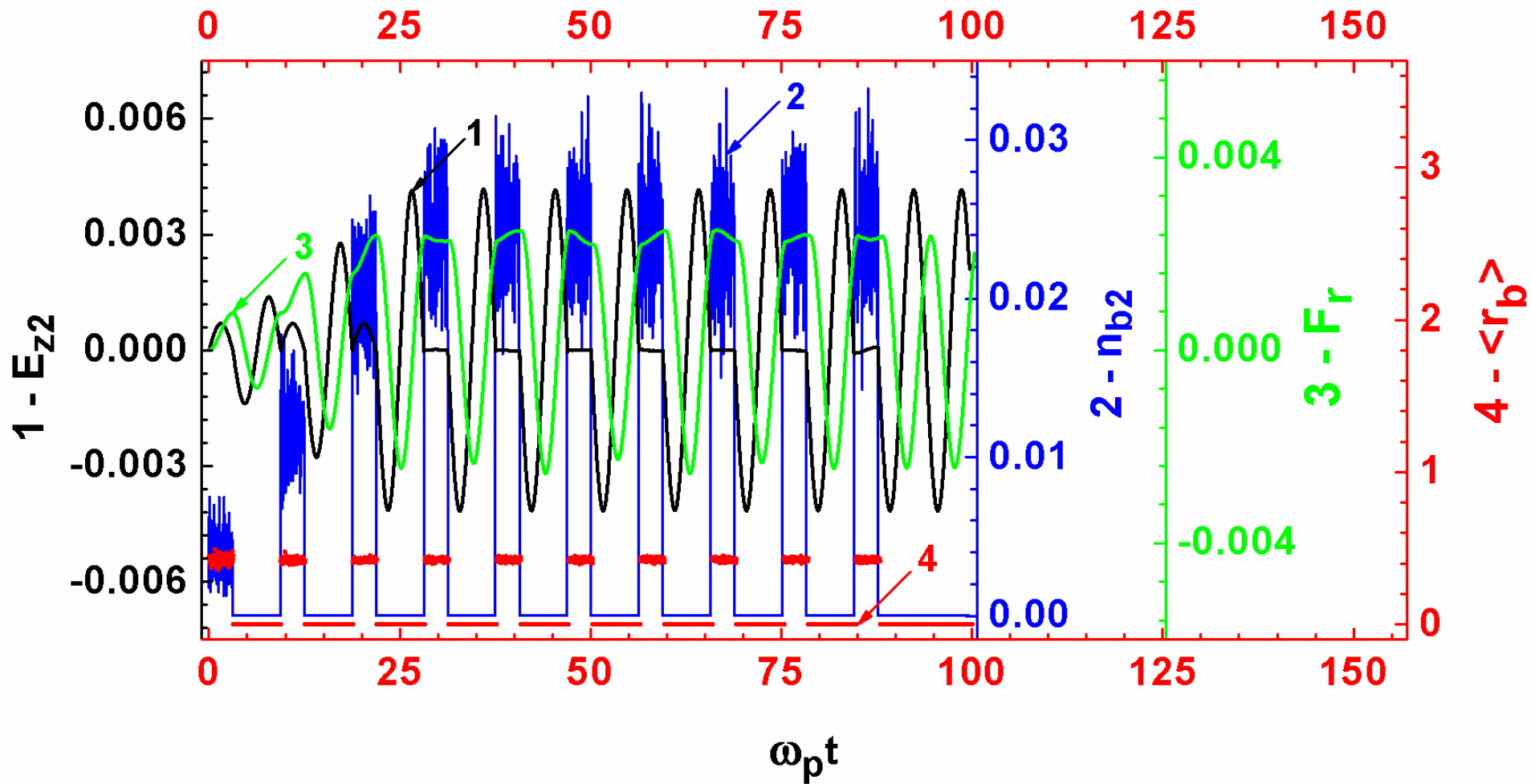


Distribution of n_b (yellow)
and coupling rate $\langle E_z \rangle$
(black)



Change of longitudinal
momentum P_z of bunches
at wakefield excitation

Coupling rates of only 1st three bunches with longitudinal wakefield do not equal zero and only 1st three bunches are decelerated.



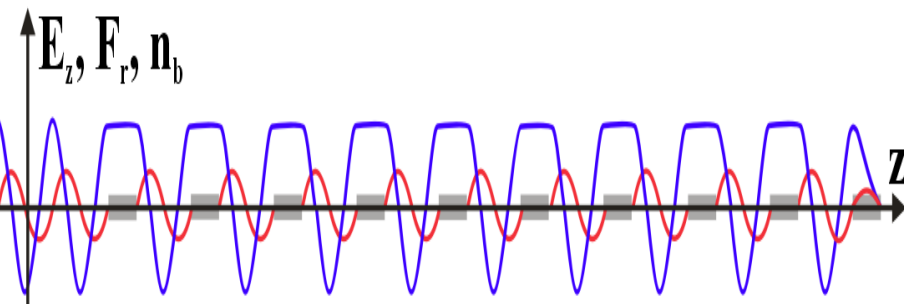
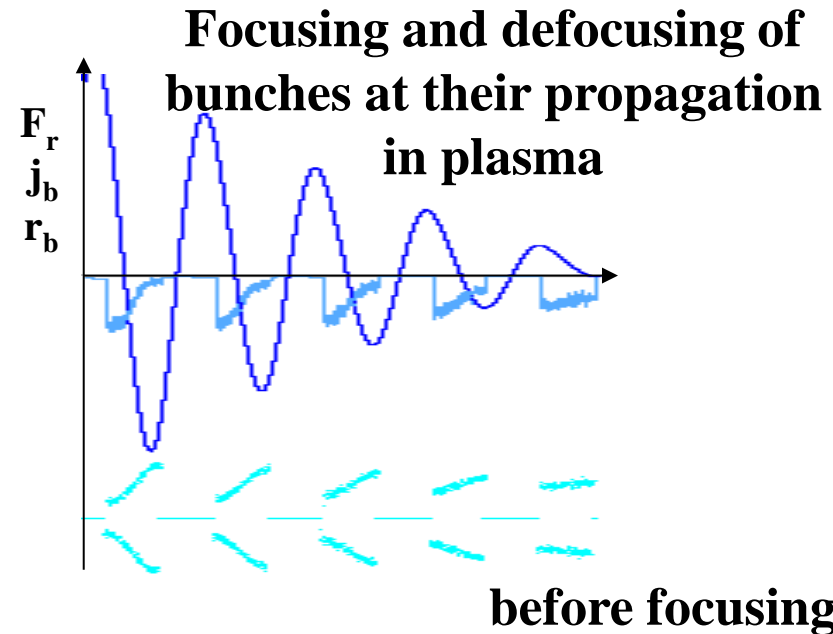
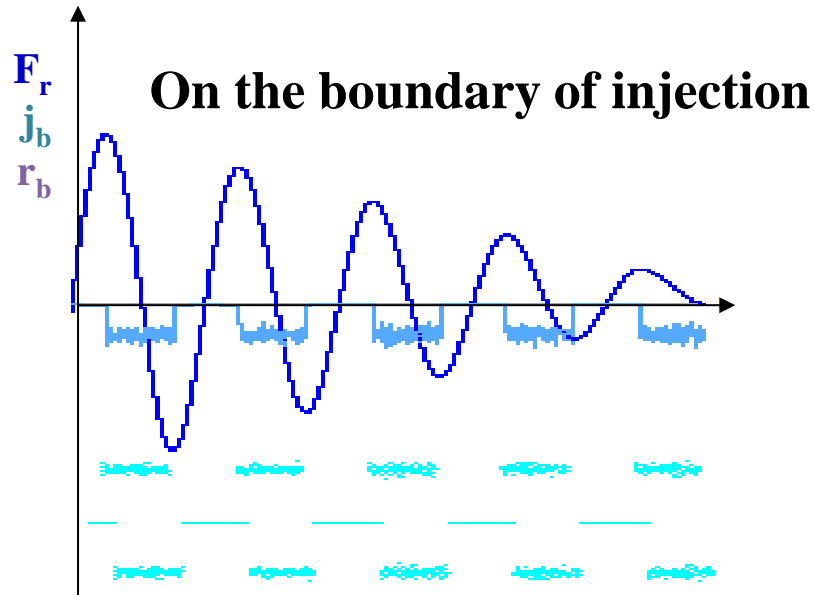
n_b (2-dark blue), $\langle r_b \rangle$ (4-red), E_z (1-black), F_r (3-green)

F_r does not depend approximately on longitudinal coordinate in regions, occupied by bunches, $F_r \approx \text{const}$, i.e. the lengthy bunches are focused identically.

Focusing of train of electron bunches by collective field in plasma

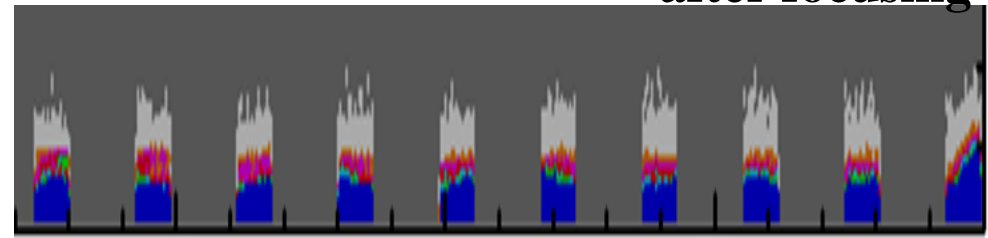
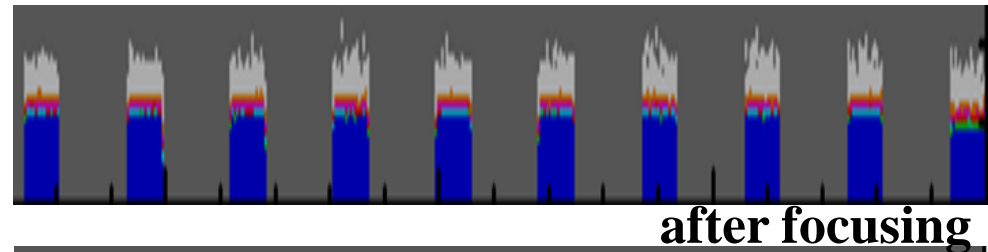
In the resonant case 1-st bunch is only focused.

1-st fronts are defocused and 2-nd fronts are focused of next bunches.



Length of bunches $\xi_{b1} = \lambda/2$, space interval between them $\delta\xi = \lambda$.

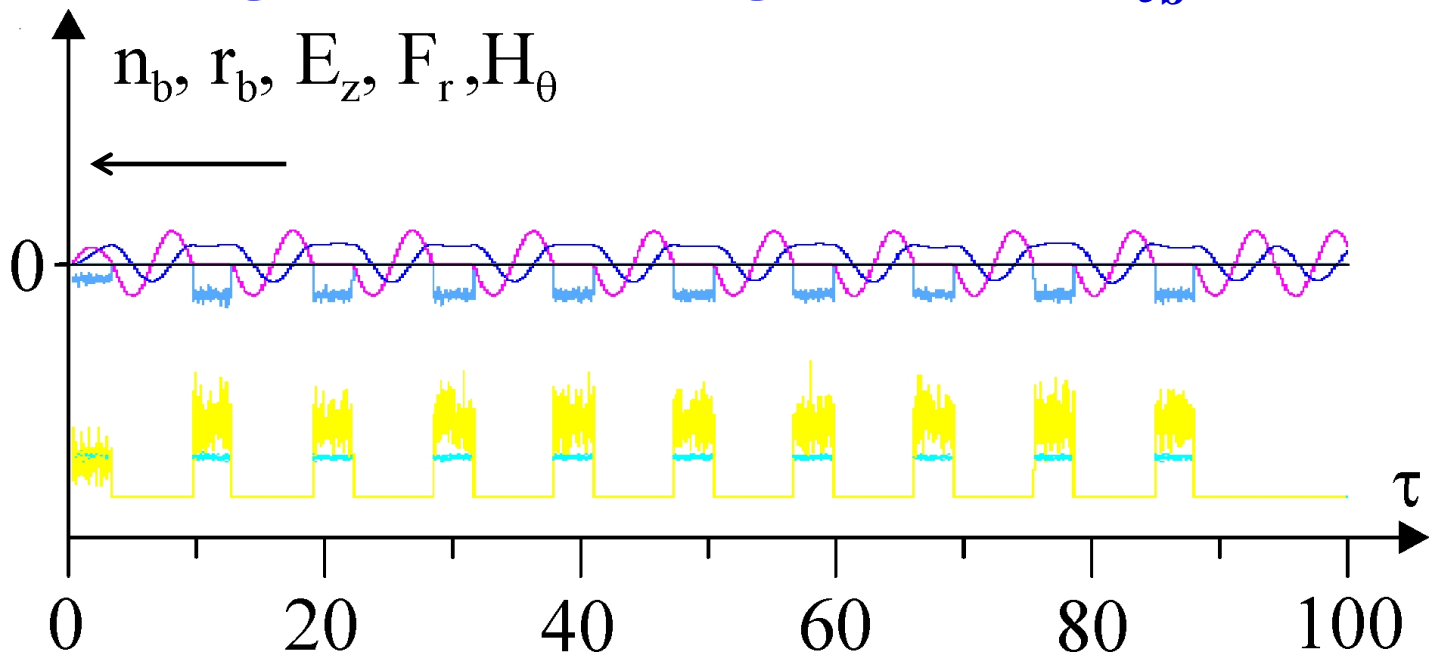
$$q_i = 2q_1, i > 1.$$



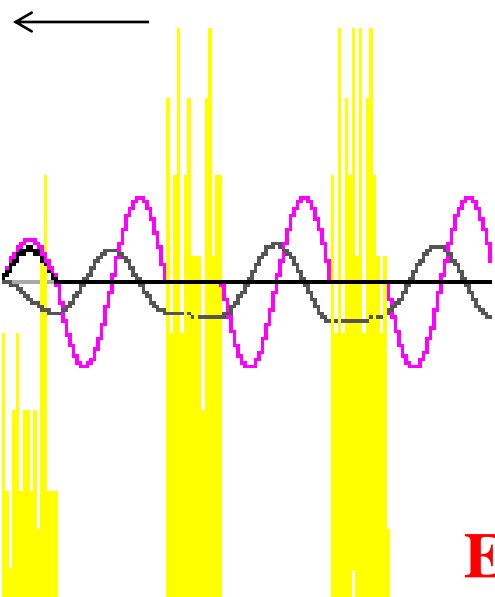
Developed plasma lens with homogeneous focusing for train of long relativistic electron bunches

For identical and uniform focusing of all relativistic electron bunches of train in wakefield plasma lens it is necessary that bunches have length $\xi_b = q(\lambda/2)$, $q=1, 2, \dots$ the charge of 1-st bunch equals half of the charges of the other bunches, the porosity between bunches equals $\delta\xi = p\lambda$, $p=1, 2, \dots$ It has been shown that only 1-st bunch is in finite longitudinal electrical wakefield $E_z \neq 0$. Other bunches are in zero $E_z = 0$. Radial wake force F_r in regions, occupied by bunches, is approximately constant along bunches.

Homogeneous focusing wakefield $\xi_b = \lambda/2$

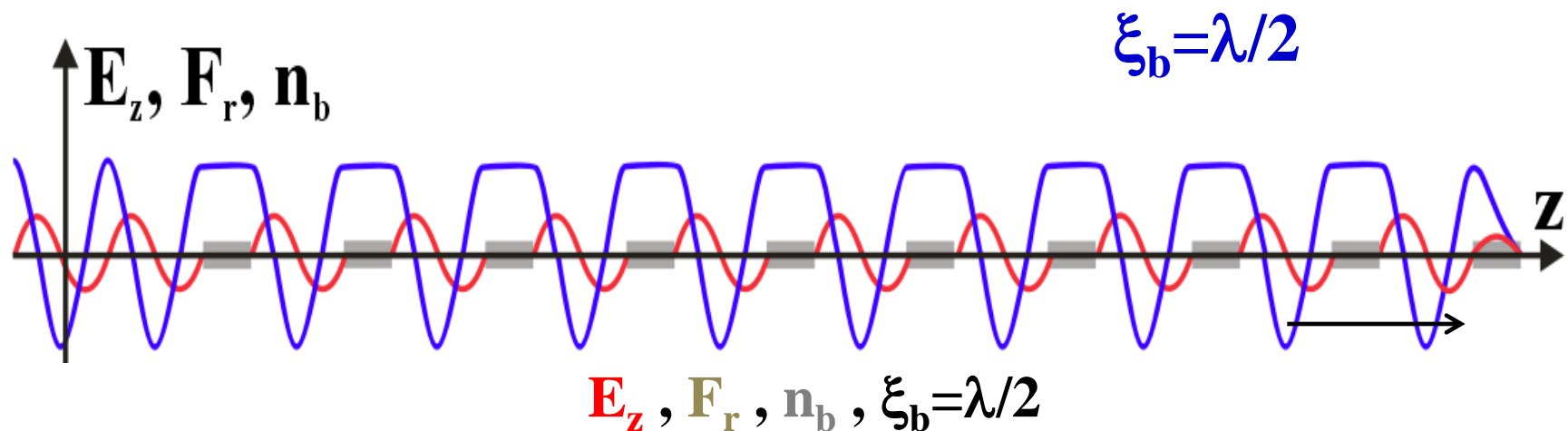


$\delta\xi = \lambda$, $Q_1 = Q_i/2$, $i=2, 3, \dots$ Fields into 2nd bunch



$$Z_{II.2}(\xi)(\xi) = (2/k)\sin(k\xi) + 2\int_0^\xi d\xi_0 \cos[k(\xi - \xi_0) + 3\pi] = 0$$

\mathbf{E}_z , $\langle \mathbf{E}_z \rangle$, n_e , n_b



After 1-st bunch

$$\begin{aligned} E_z &\sim \int_0^{\lambda/2} d\xi_0 \cos[k(\xi - \xi_0)] = (2/k) \sin(k\xi), \\ F_r &\sim \int_0^{\lambda/2} d\xi_0 \sin[k(\xi - \xi_0)] = -(2/k) \cos(k\xi). \end{aligned}$$

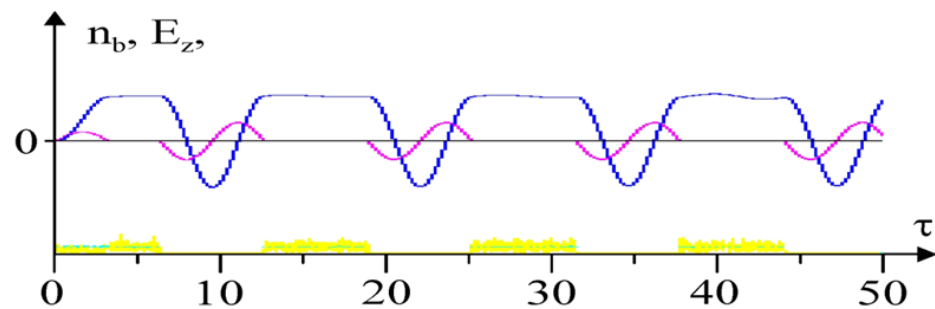
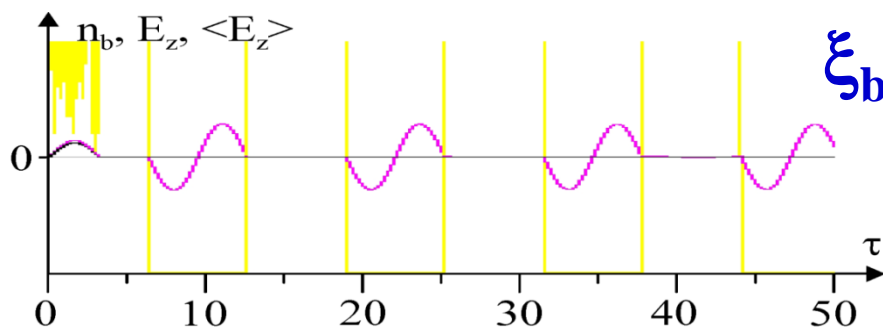
Wakefield in the middle 1-st bunch

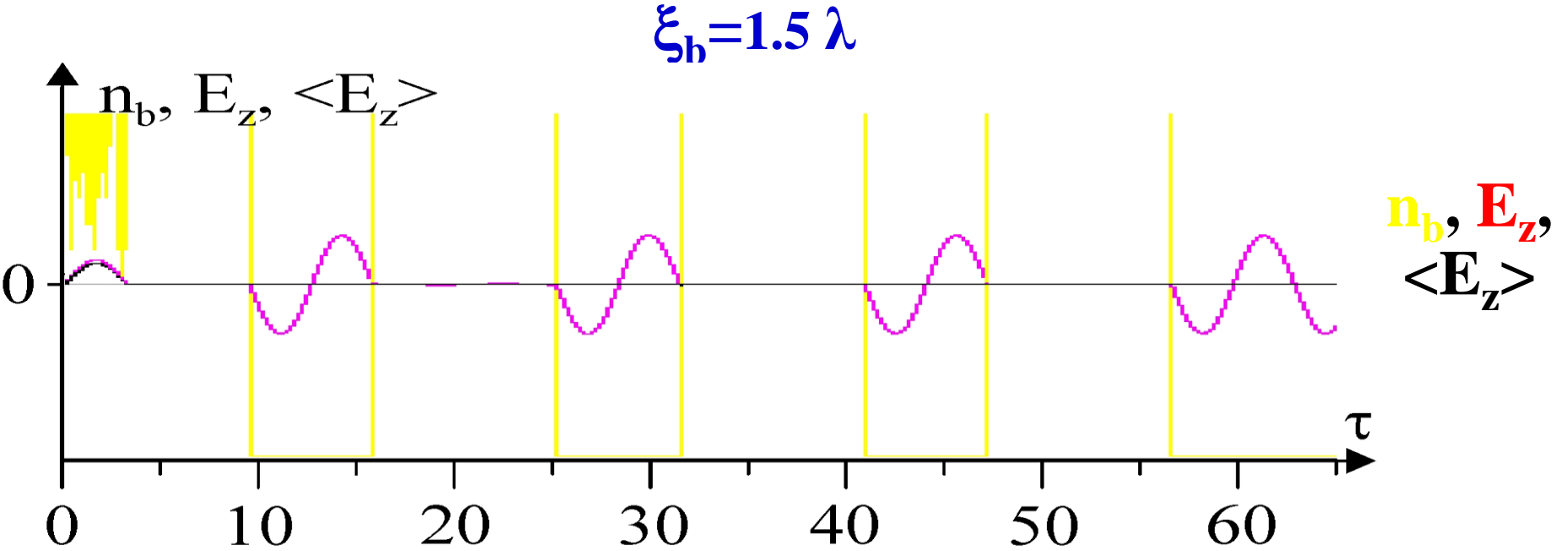
$$E_z \sim \int_0^{\lambda/4} d\xi_0 \cos[k(\xi - \xi_0)] = (1/k).$$

Wakefield inside 2-nd bunch

$$E_z \sim (2/k) \sin(k\xi) + 2 \int_0^{\xi} d\xi_0 \cos[k(\xi - \xi_0) + 3\pi] = 0.$$

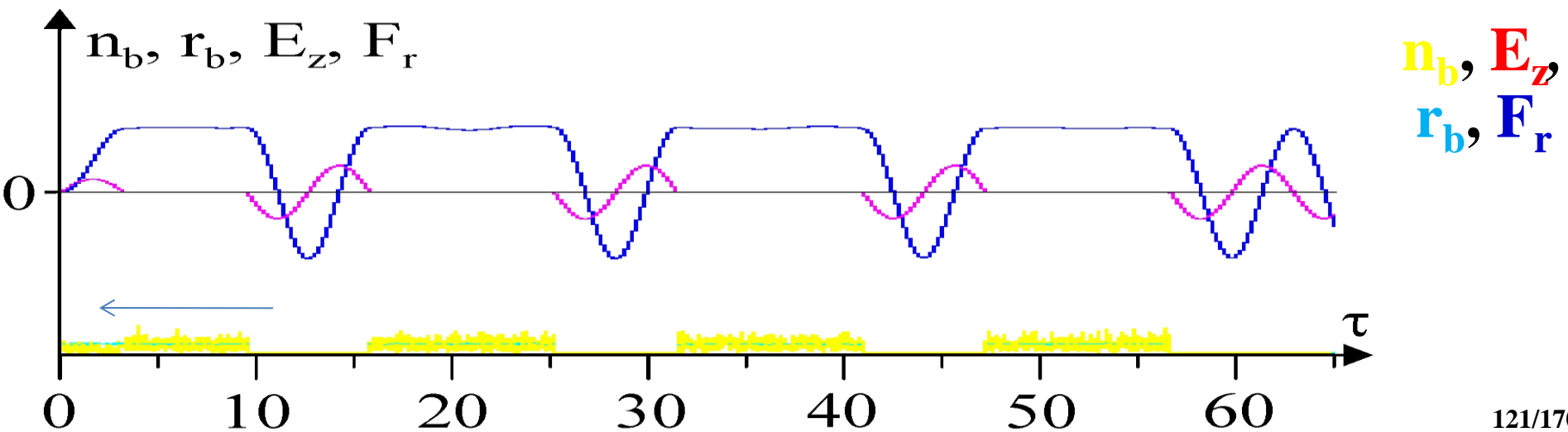
$E_z = 0$ in the regions of the location of bunches.

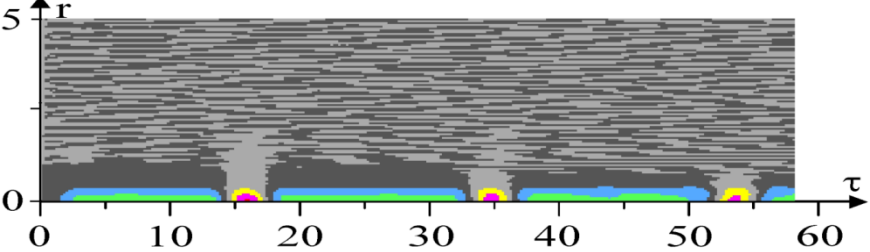




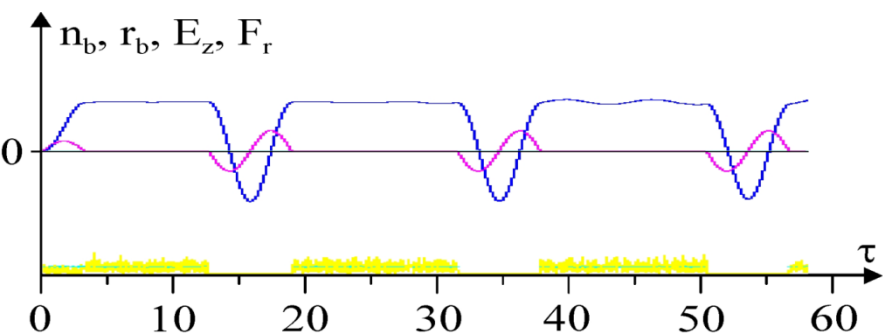
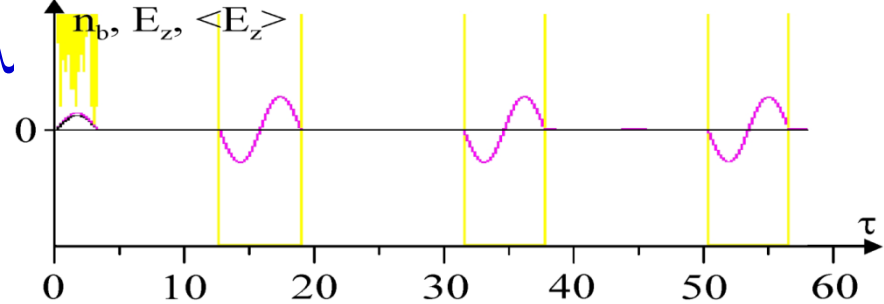
Fields into 2nd bunch

$$Z_{II.2}^{(\xi)}(\xi) = (2/k)\sin(k\xi) + 2\int_0^\xi d\xi_0 \cos[k(\xi - \xi_0) + 3\pi] = 0$$





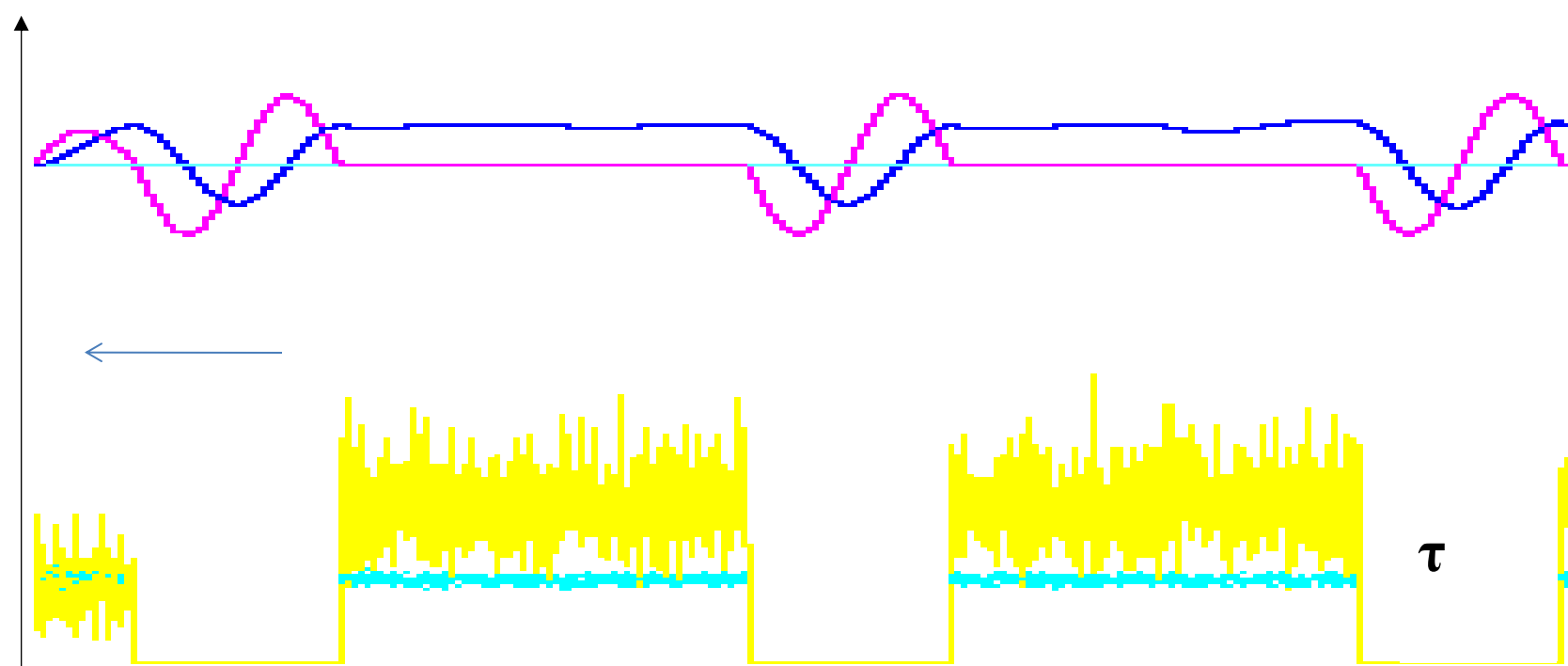
$$\xi_b = 2\lambda$$



$$\begin{matrix} \text{yellow } n_b, & \text{red } E_z, \\ \text{blue } r_b, & \text{blue } F_r \end{matrix}$$

$$\text{yellow } n_b, \text{ red } E_z, \text{ magenta } \langle E_z \rangle$$

$$n_b, r_b, E_z, F_r$$



$$\begin{matrix} \text{yellow } n_b, & \text{red } E_z, \\ \text{blue } r_b, & \text{blue } F_r \end{matrix}$$

Plasma lens with homogeneous focusing for train of short relativistic electron bunches

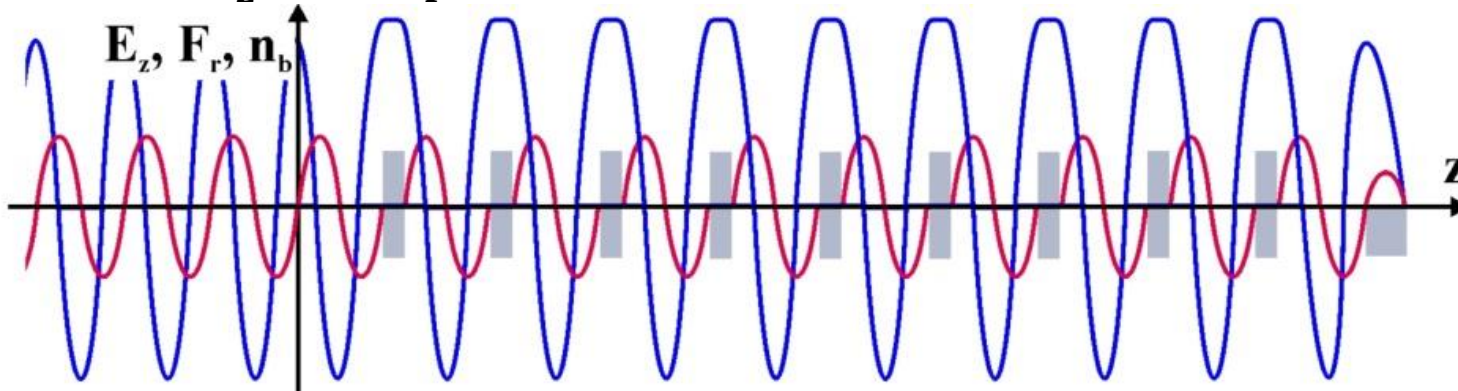
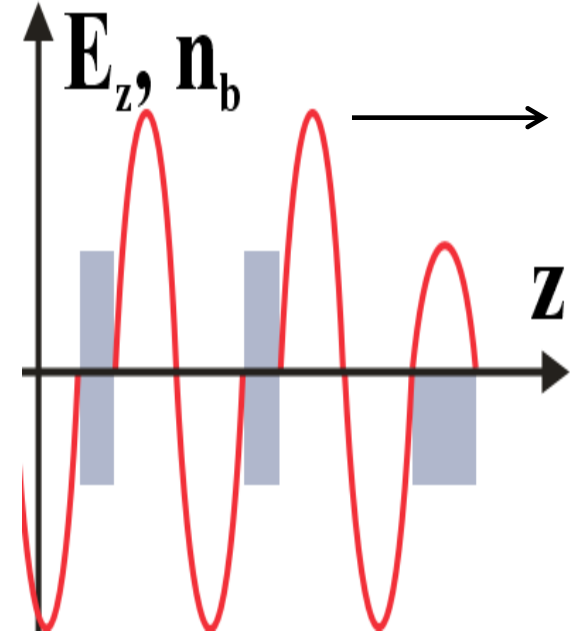
1-st lens: $\xi_{b1}=\lambda/2$, $\xi_{bi}=\lambda/4$, $n_{bi}=2n_{b1}$, $i>1$

We derive the wakefield inside the 2nd short bunch, $\xi_b=\lambda/4$, charge density of which is in 2 times more than the charge density of 1st bunch n_{b1} , the space interval between it and the first bunch is equal to $\delta\xi=\lambda$, the length of 1-st bunch $\xi_{b1}=\lambda/2$. Then inside 2-nd bunch we have

$$E_z \sim (2/k) \sin(k\xi + \pi) + 2 \int_0^\xi d\xi_0 \cos[k(\xi - \xi_0)] = 0,$$

$$F_r \sim -(2/k) \cos(k\xi + \pi) + 2 \int_0^\xi d\xi_0 \sin[k(\xi - \xi_0)] = 2/k.$$

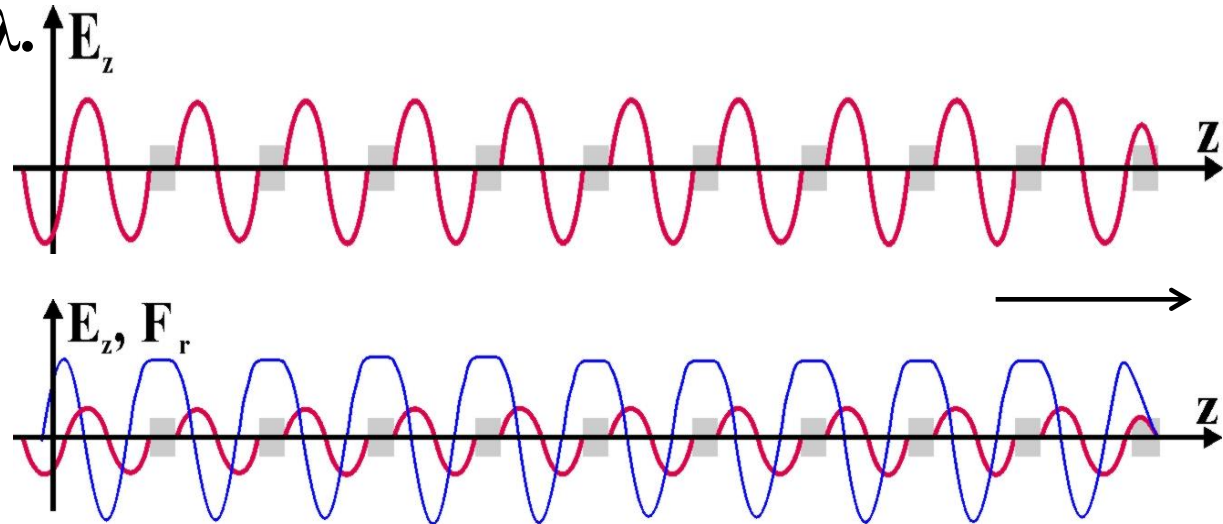
The same E_z and F_r are obtained within all next bunches.



Plasma lens with homogeneous focusing for train of short relativistic electron bunches

$$\text{2-nd lens: } \xi_{bi} = \lambda/4, q_i = \sqrt{2}q_1, i > 1$$

The charge density of all bunches is $\sqrt{2}$ times more than the charge density of 1-st bunch $q_i = \sqrt{2}q_1, i > 1$, the space interval between the 1-st and 2-nd bunches is equal to $\delta\xi_{1-2} = \lambda/8$, the space interval between the other bunches is $\delta\xi = \lambda$.



One can provide such homogeneous and identical focusing for train of laser pulses. Such homogeneous and identical focusing is very important in laser wakefield acceleration.

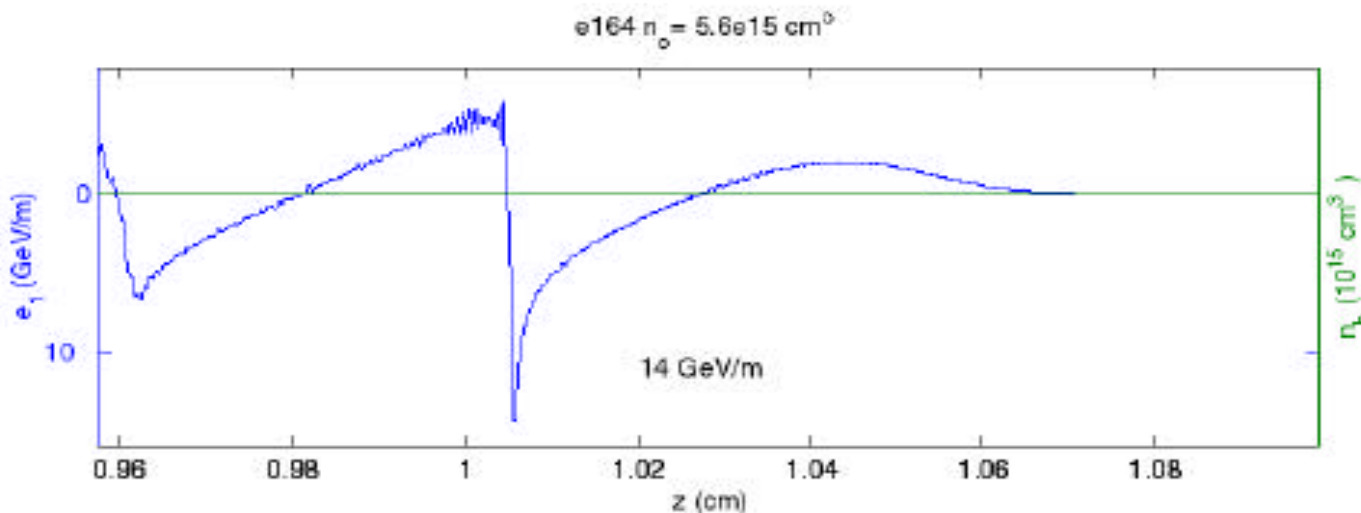
$$\text{Transformation ratio TR} = E_{\text{accel}} / E_{\text{decel}}$$

Wilson theorem: by ordinary driver the energy of witness can become not more than doubled because the transformation ratio for ordinary driver

$$\text{TR} = E_{\text{acc}} / E_{\text{dec}} \leq 2.$$

It is necessary to derive

$$\text{TR} \gg 1.$$



Wakefield excitation in plasma by shaped train of electron bunches with linear growth of charge

By numerical simulation it has been shown that E_z in the areas of bunches - drivers location does not depend on z along every bunch and along the train of bunches at certain lengths of bunches and interbunch gaps if the charge is shaped along each bunch and along train according to linear law. The large transformation ratio

$$TR \approx 2\pi N$$

is achieved.

Introduction

At electron acceleration by wakefield the transformation ratio is important. It is determined as

$$\text{TR}_\varepsilon = \Delta\varepsilon_w / \Delta\varepsilon_{\text{dr}}$$

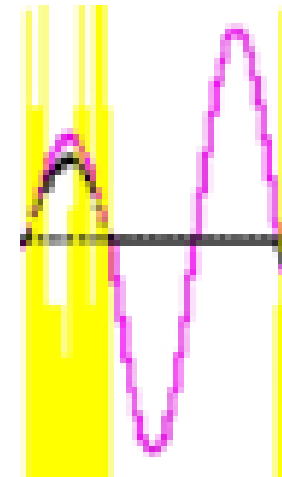
ratio of the energy, received by witness, to energy, lost by driver. The transformation ratio can be approximately defined as a ratio

$$\text{TR} = E_{\text{acel}} / E_{\text{decel}}$$

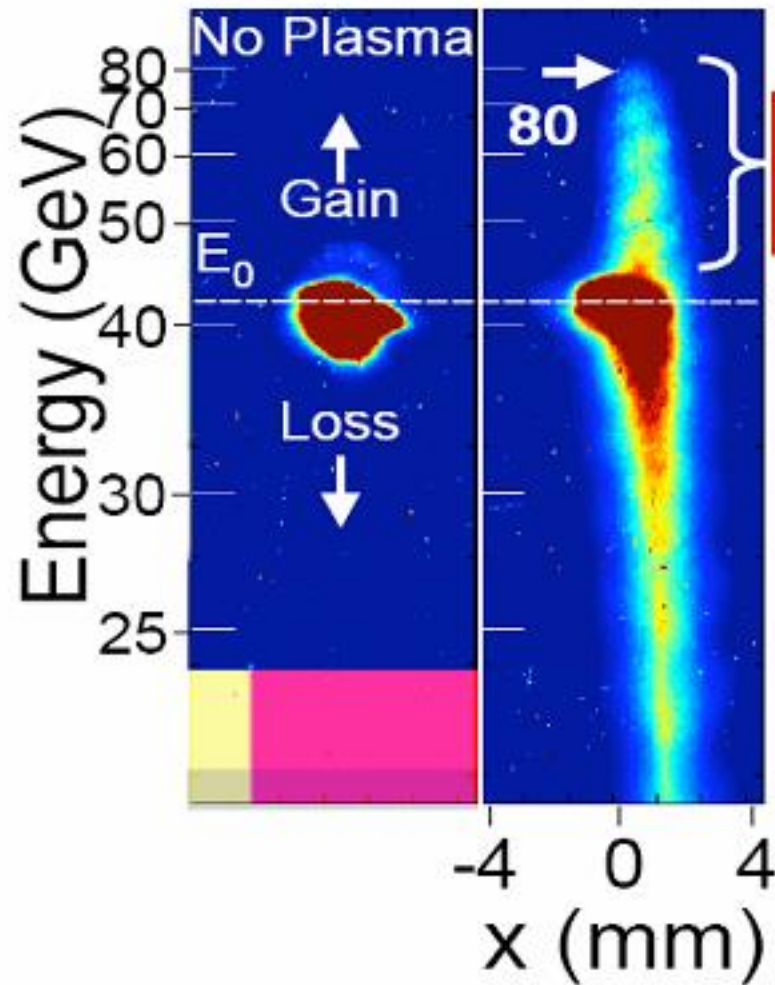
of the wakefield, which are excited in plasma and accelerating electrons E_{acel} to the field in which an electron bunch is decelerated E_{decel} .

$$\text{TR} \leq 2.$$

P. B. Wilson, 1985



Experiment E-167 in SLAC

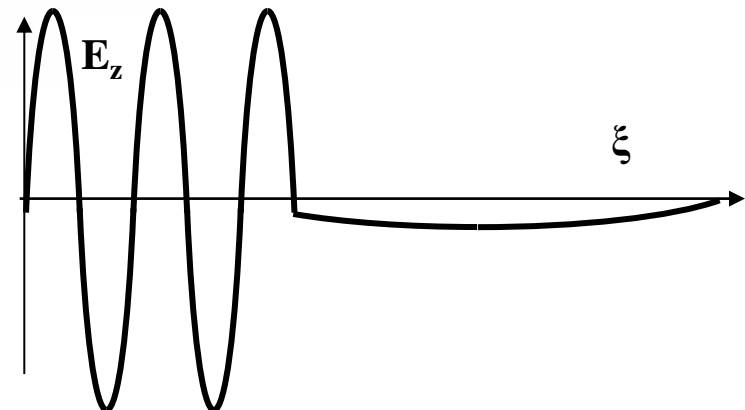


$\approx 9.6 \times 10^8 e^-$
 $\approx 154 \text{ pC}$

$E_0 = 42 \text{ GeV}$,
 $N = 1.75 \times 10^{10} e^-$,
 $n_e = 2.6 \times 10^{17} \text{ cm}^{-3}$,
 $L_p = 90 \text{ cm}$

Energy doubling

They only doubled, and if use shaped bunch, one can accelerate much stronger.



**Using train of bunches for wakefield excitation one can increase T_E .
In the case of bunches of finite dimensions the methods of T_E
increase has been investigated**

R.D.Ruth, A.W.Chao, P.L.Morton, P.B.Wilson. 1985.

P.Chen, J.M.Dawson, R.W.Huff, T.C.Katsouleas. 1985.

Bane, K. L. F., P. Chen, P. B. Wilson. 1985.

Chen, P. et al. 1986.

T. Katsouleas. 1986.

Laziev, E., V. Tsakanov and S. Vahanyan. 1988.

K.Nakajima. 1990.

V.A.Balakirev, G.V.Sotnikov, Ya.B.Fainberg. 1996.

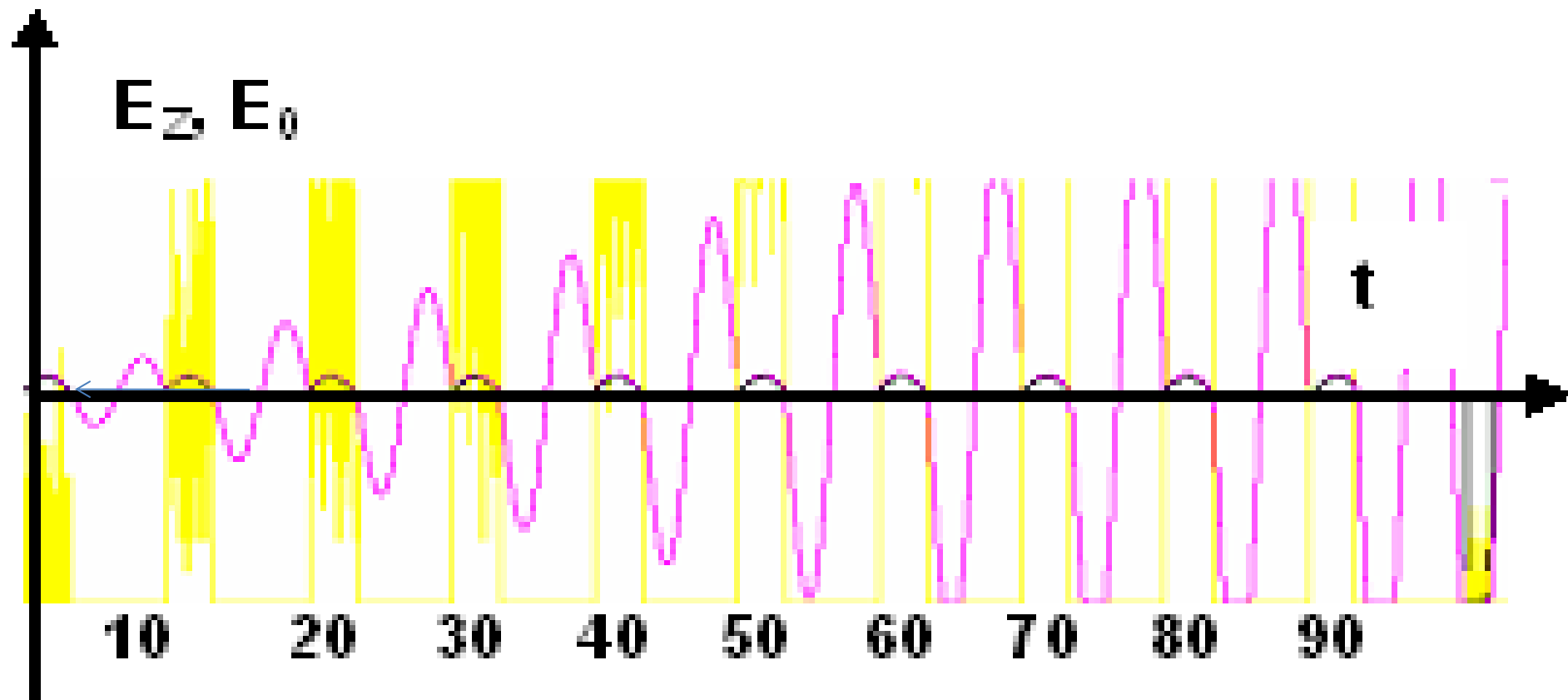
**C. Jing, A. Kanareykin, J. G. Power, M. Conde, Z. Yusof, P.
Schoessow, W. Gai. 2007.**

B. Jiang, C. Jing, P. Schoessow, J. Power, W. Gai. 2012.

E. Kallos, T. Katsouleas, P. Muggli et al. 2007.

K.V.Lotov, V.I.Maslov, I.N.Onishchenko, I.P.Yarovaya. 2011.

T_E increases at not very large amplitudes $TR=2N$. N – number of bunches in train. At bunch length $\xi_b=\lambda/2$ and spatial interval between bunches λ .



Thus all bunches are decelerated identically, but in inhomogeneous forces. I.e. the electrons of bunches are not decelerated fully.

Wakefield excitation in plasma by bunches of train with shaped charge according to linear law

In

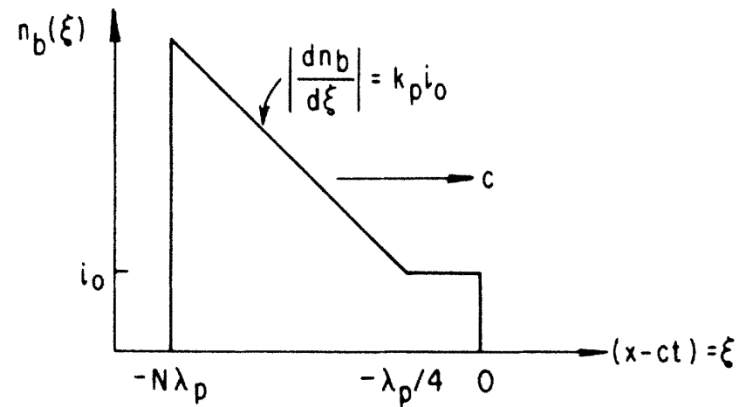
Bane, K. L. F., P. Chen, P. B. Wilson, 1985,
Chen, P. *et al.*, 1986.

in the case of one long bunch the charge of which grows along bunch, the larger transformation ratio T_E can be obtained

$$TR=2\pi N$$

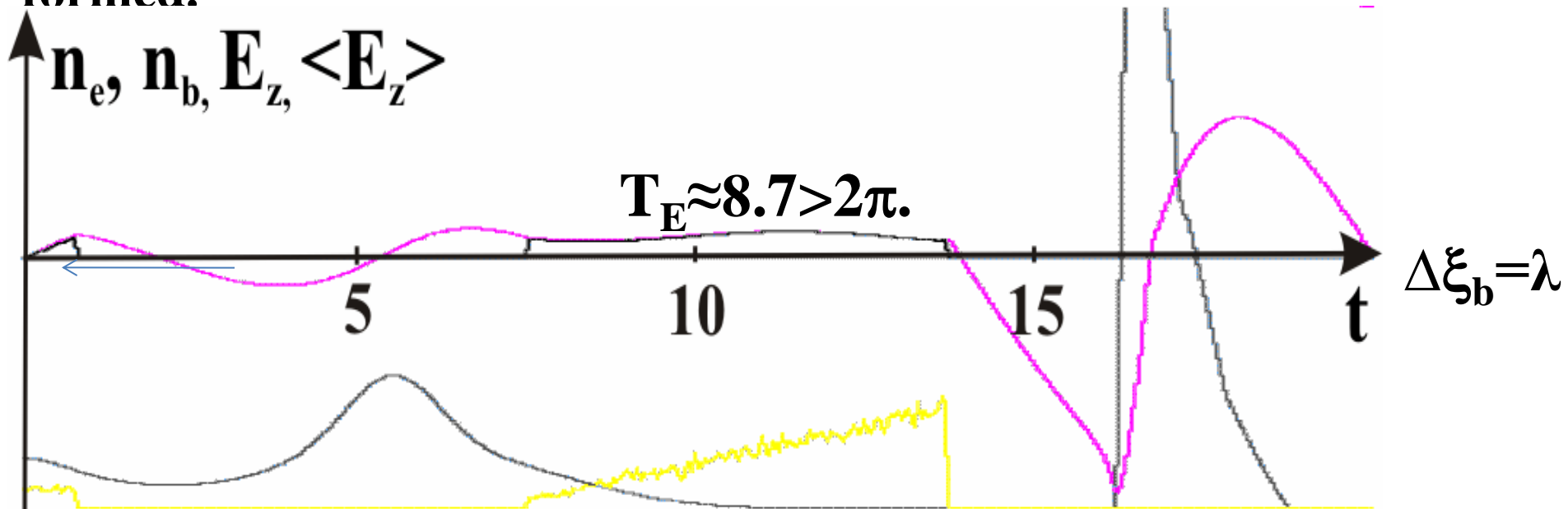
$N=L_b/\lambda$ is the number of wavelengths λ , distributed along bunch L_b .

We consider this charge shaping of train of bunches along z according to linear law. $\xi_b=\lambda$. The interbunch gap also equals $\delta\xi=\lambda$. Before this sequence at some distance a rectangular bunch of length $\lambda/4$ has been placed. Then $T_E>2\pi N$ can be derived in nonlinear case, N is the number of bunches. It has been shown that large TR can be achieved also for $\xi_b=\lambda, 2\lambda, ..$ and for $\delta\xi=0, \lambda, 2\lambda, ...$



Wakefield excitation in plasma by bunch with shaped charge according to linear law

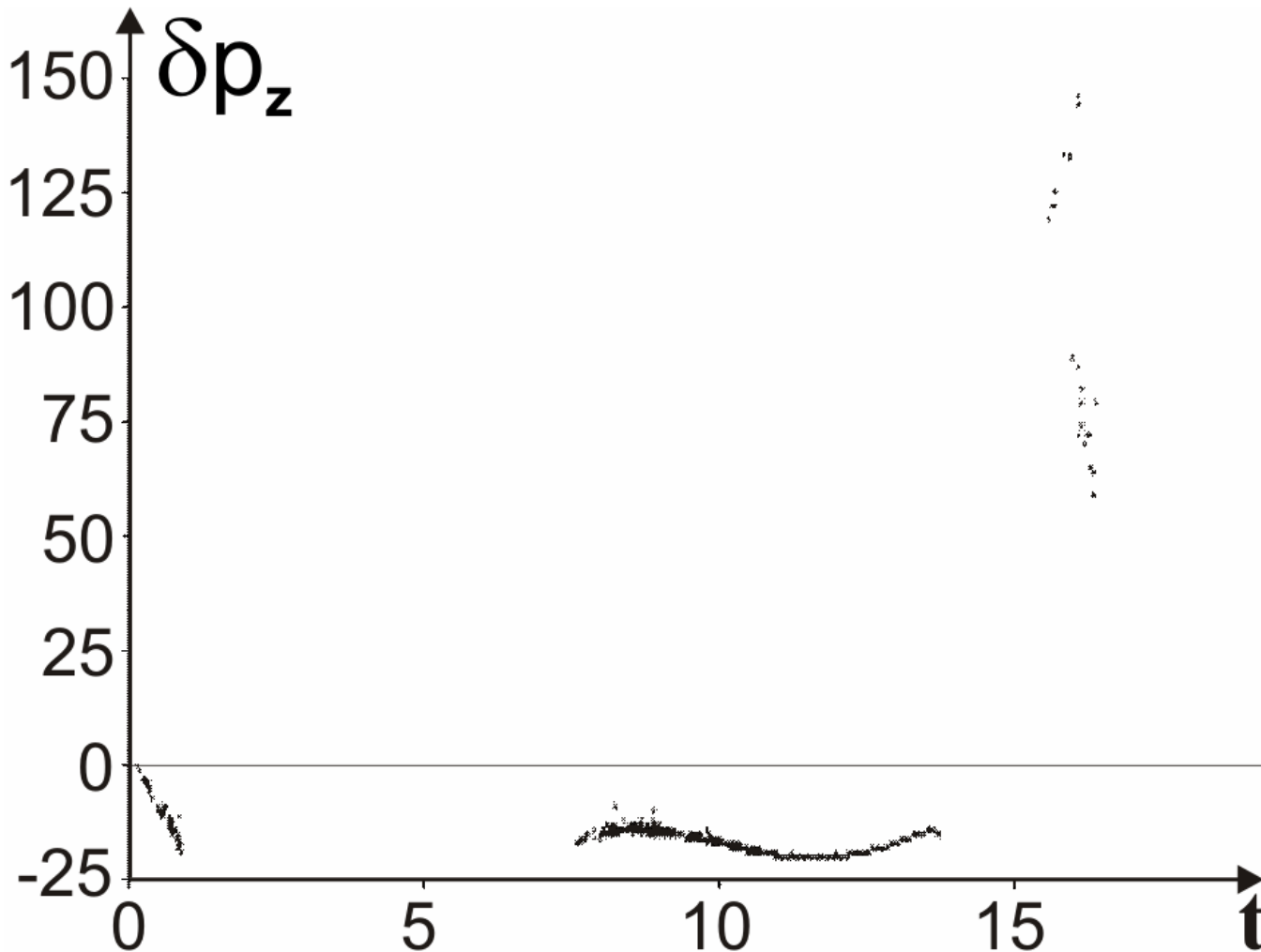
Before bunch a rectangular bunch of length $\lambda/4$ is placed. Bunch of large charge is used that after it the bubble and E_z steepening are formed.



E_z , coupling of bunch electrons with E_z (black), n_b , n_e (серая)

All electrons of all bunches are decelerated approximately in identical E_z . In other words, decelerating E_z approximately does not depend on z along bunch.

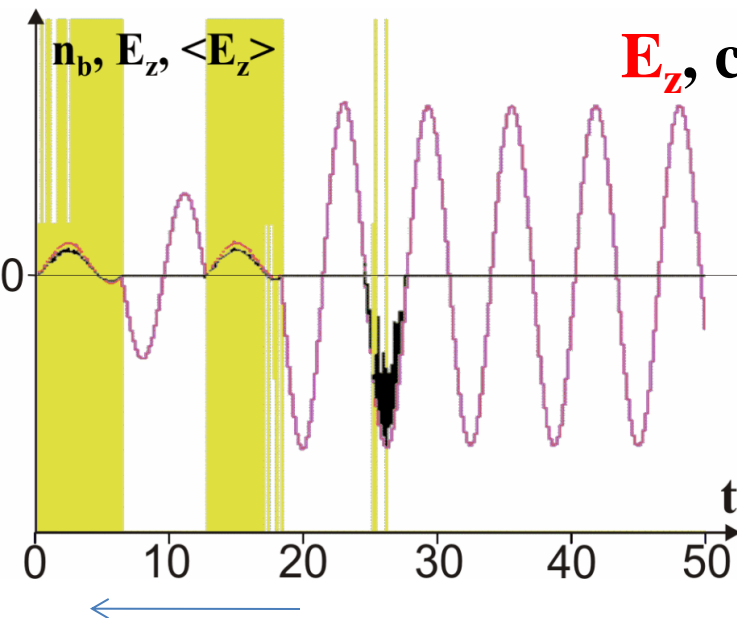
Wakefield excitation in plasma by bunch with shaped charge according to linear law



Change of longitudinal momentum of bunches δp_z at wakefield excitation

$TR_\epsilon \approx 7.73$, i.e. $2\pi < TR_\epsilon < TR$.

Wakefield excitation in plasma by two bunches with shaped charge according to linear law , $\xi_b=\lambda$, $\delta\xi=\lambda$

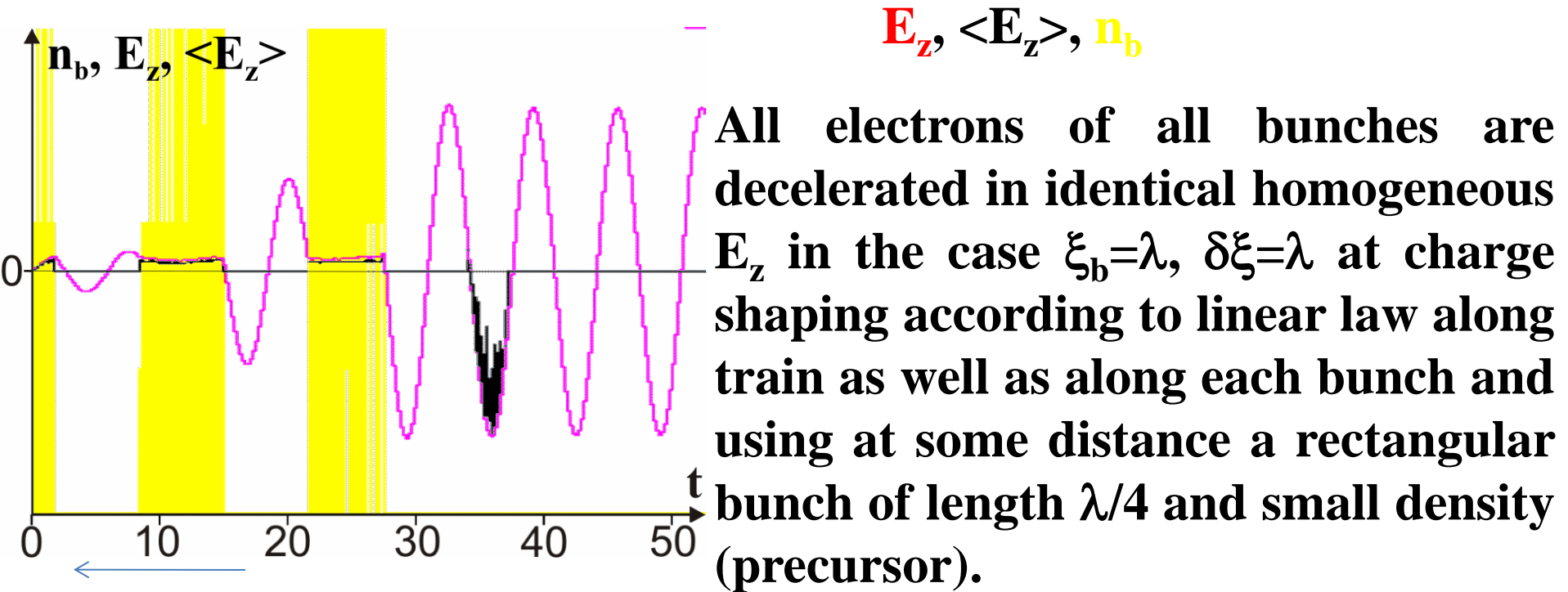


E_z , coupling of bunch with E_z (black), n_b

In this case it is impossible to obtain the full deceleration of all electrons of bunches, because decelerating wakefield is strongly inhomogeneous along bunch.

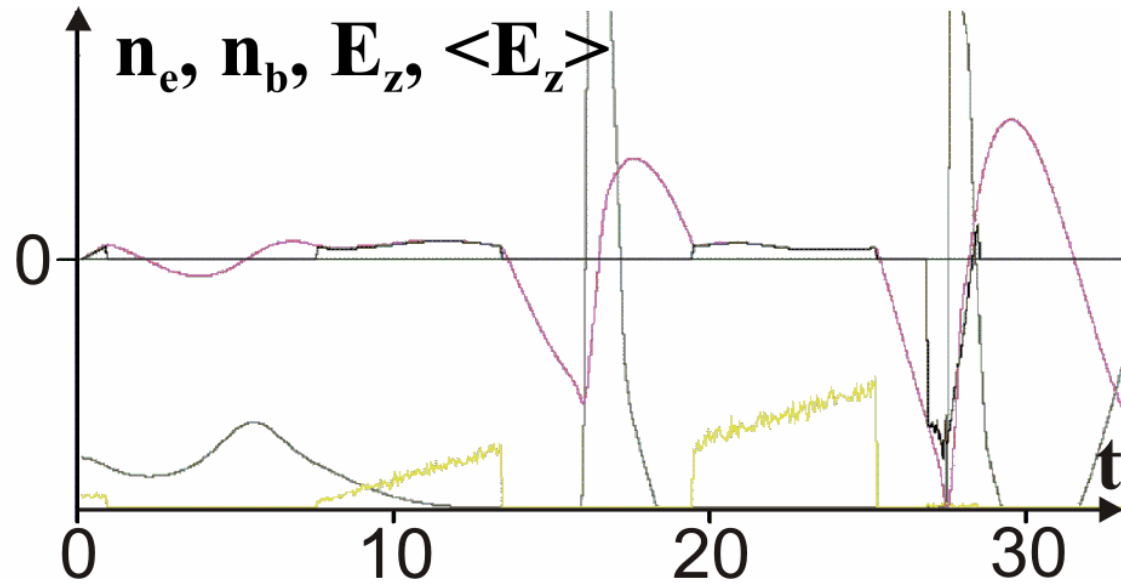
Also TR does not equal to maximal one. Namely, $TR \approx 3$ after 1st bunch and $TR \approx 6$ after 2nd bunch. I.e. approximately $TR \approx 3N$, N is the number of bunches.

Wakefield excitation in plasma by two bunches with shaped charge according to linear law with precursor



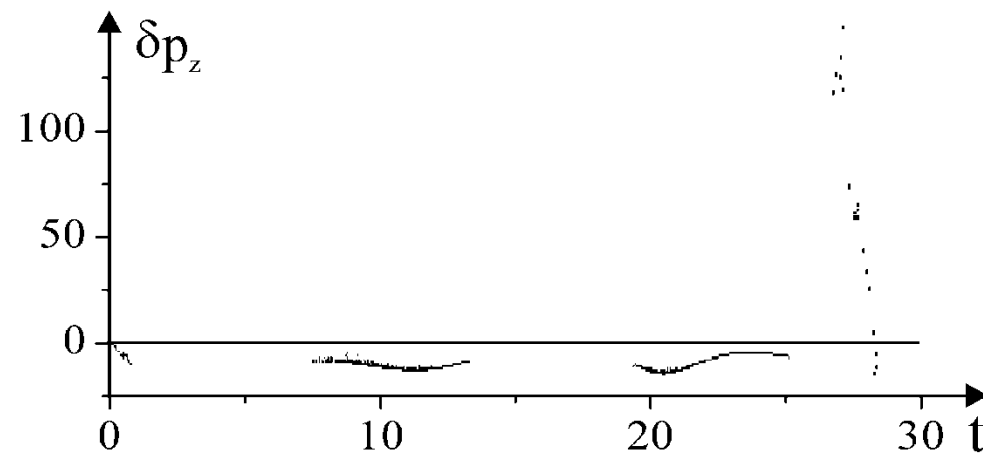
One can obtain maximal TR and complete deceleration of all bunches – drivers.

Nonlinear wakefield excitation in plasma by two bunches with shaped charge according to linear law with precursor



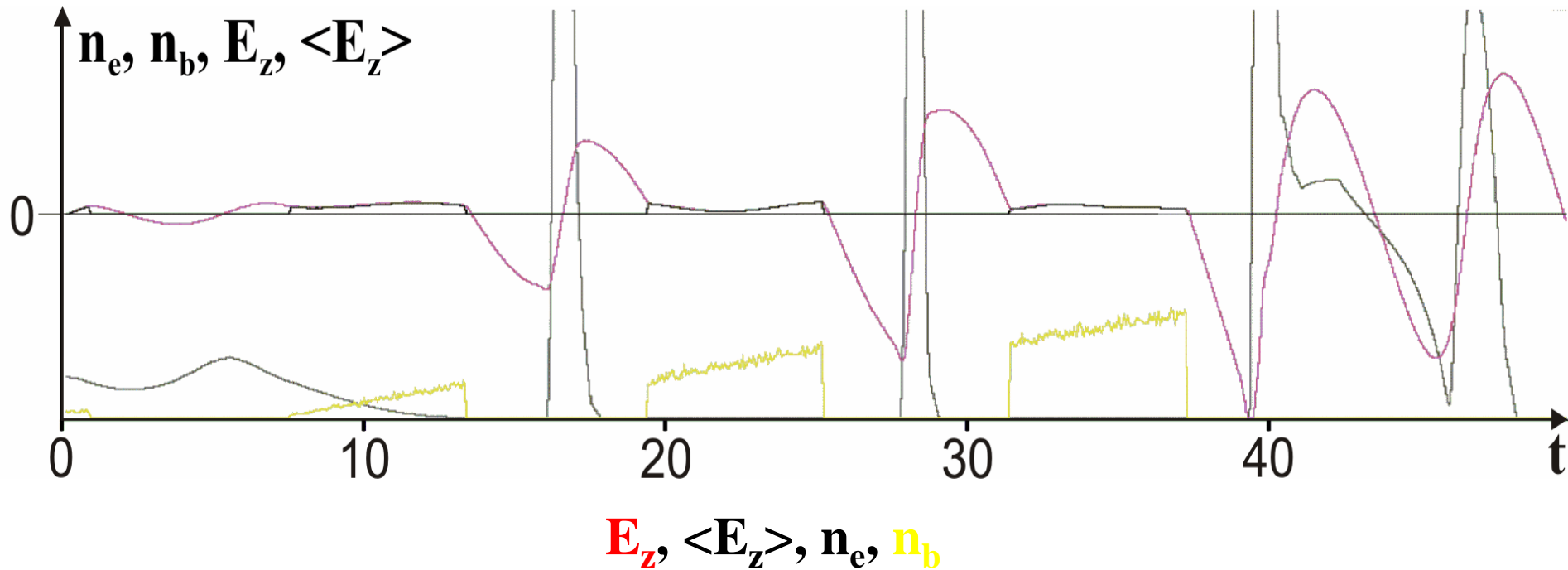
$E_z, \langle E_z \rangle, n_e, n_b$

In nonlinear case also all electrons are decelerated approximately in identical E_z .



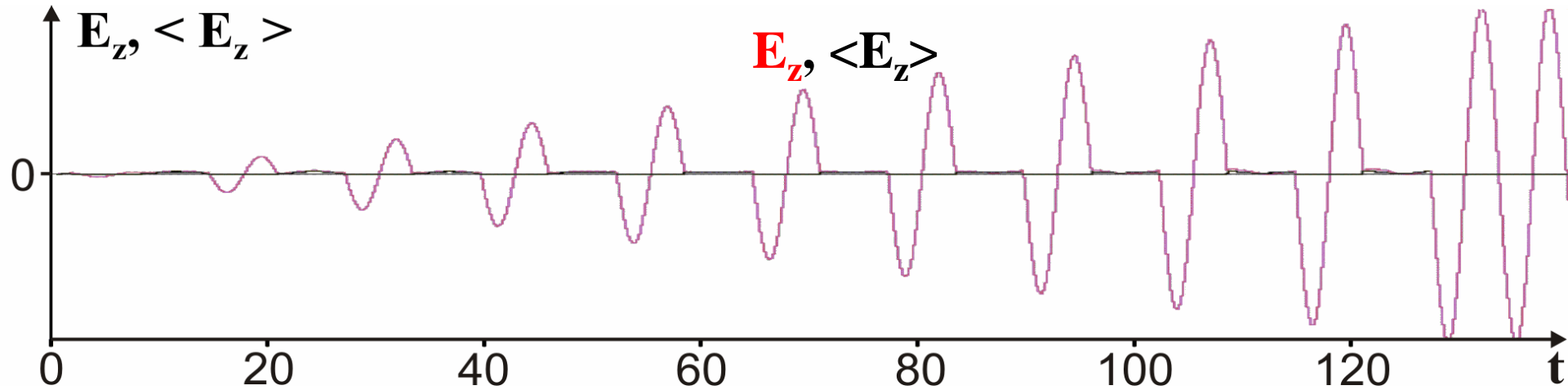
δp_z of bunches

Wakefield excitation in plasma by three bunches with shaped charge according to linear law with precursor, $\xi_b = \lambda, \delta\xi = \lambda$

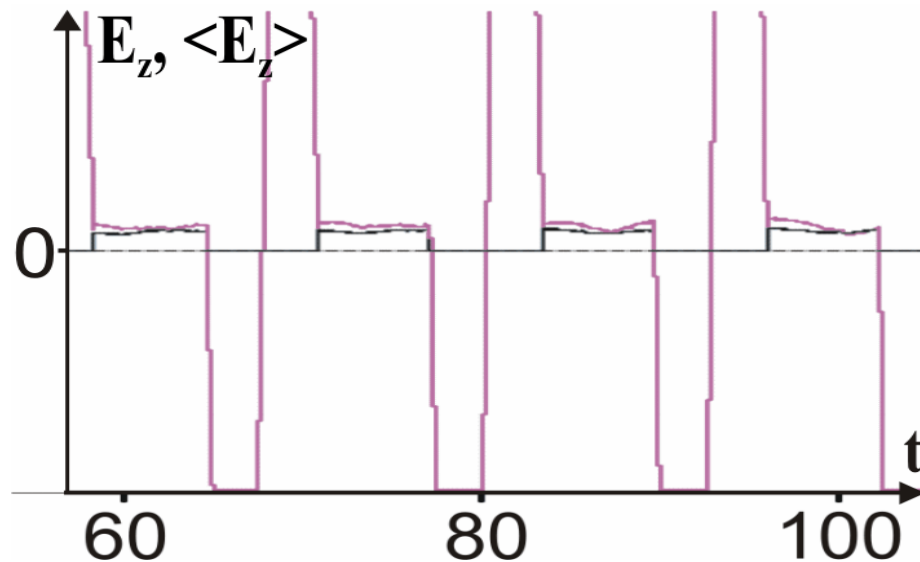


All electrons are decelerated approximately in identical E_z . After 1st bunch $T_E \approx 7.4$, after 2nd bunch $TR \approx 14.1$, after 3rd bunch $TR \approx 19.7$. I.e. $TR > 2\pi N$. N is the number of bunches.

Wakefield excitation in plasma by 10 bunches with shaped charge according to linear law, $\xi_b = \lambda$, $\delta\xi = \lambda$



Part of Fig. is shown in detail in:



Wakefield excitation with high transformation ratio in plasma by infinite train of shaped bunches-drivers and bunches-witness acceleration

There are several reasons for infinite train use :

- 1) large number of accelerated electrons is needed;
- 2) large TR is needed.

However there appeared difficulties:

- 1) it is impossible to extend strongly the train, shaped on the linear law, because a maximal charge is limited;
- 2) with train lengthening, shaped according to the linear law, Fr grows along it, the latter can destroy drivers.



As a result the infinite sequence of electron bunches - drivers is derived. The bunches-drivers of asymptotic part of sequence are identical each other and are represented rectangular trapezoid. These bunches - drivers are alternated with bunches-witnesses.

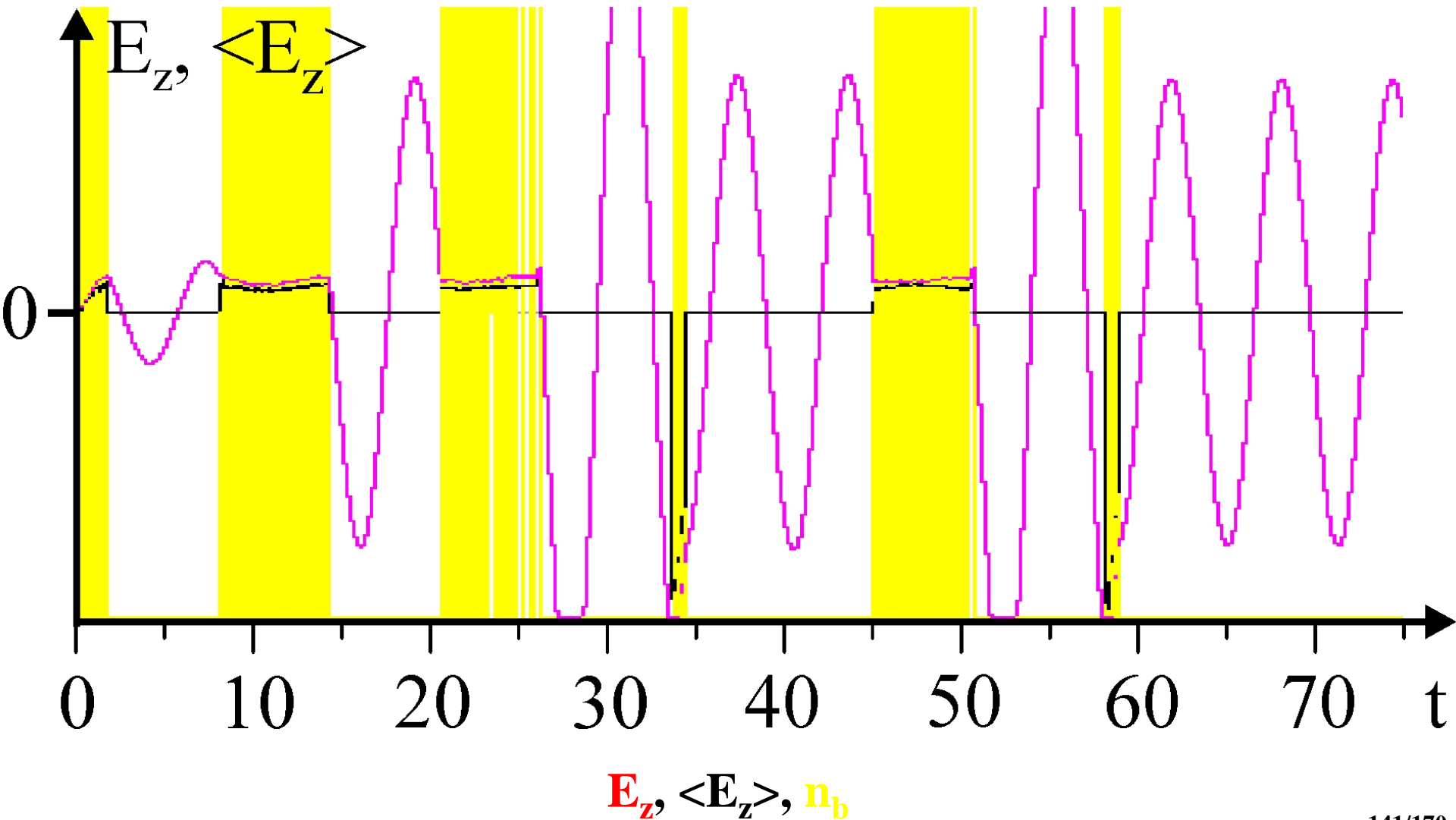
Each short train (period) of bunch – drivers restore the wakefield amplitude after each bunch – witness. The charge of every bunch is shaped along bunch according to linear law.

The advantages of this train are following:

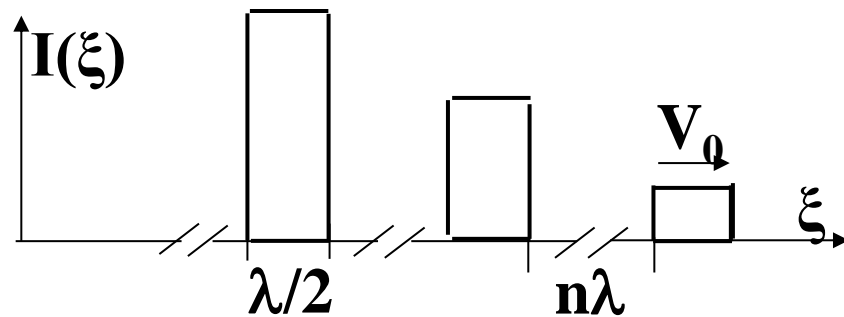
- 1) large transformation ratio $TR=2\pi N_{fr}$;**
- 2) homogeneous E_z for bunches-drivers along every bunch and along the train;**
- 3) in nonlinear case bunches - drivers are focused by identical focusing forces;**
- 4) a long train of bunches - witnesses.**

In the head of train of bunches a bunch – precursor is placed. Then short train of electron bunches, a charge in which grows according to the linear law both along every bunch and along the train, follows. After this short train an asymptotic long (infinite) periodic train of electron bunches - drivers and bunches – witnesses follows. Charges of these bunches - drivers are identical, but they are distributed according to the linear law along every bunch. These bunches - drivers are alternated with bunches–witnesses.

Wakefield excitation with high transformation ratio in plasma by infinite train of shaped bunches-drivers and bunch-witness acceleration



Transformation Ratio at Wakefield Excitation in Dielectric Cavity by Shaped Train of Homogeneous Electron Bunches with Linear Growth of Current



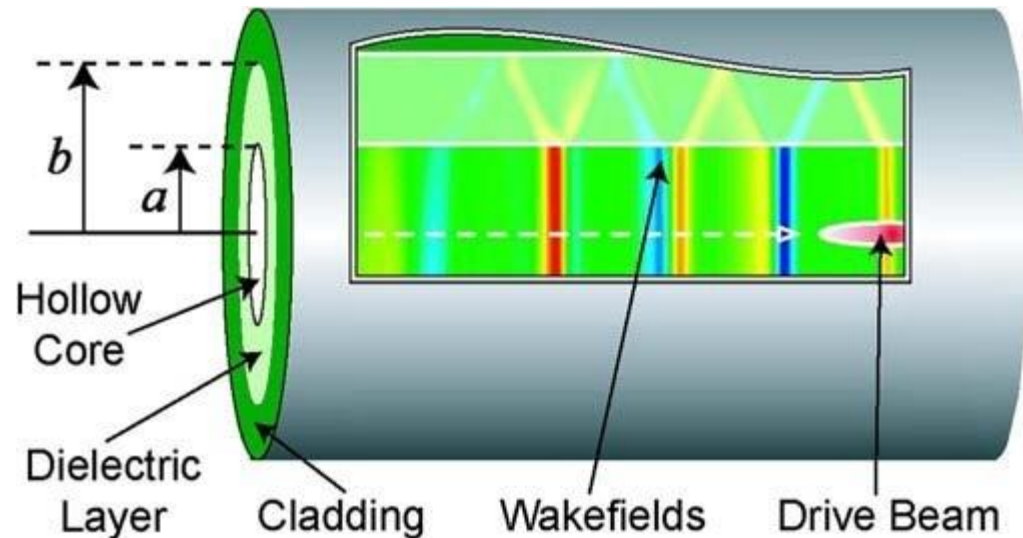
$$E_{\text{metal}} \ll E_{\text{dielectric}} \ll E_{\text{plasma}}$$

$$E_z \approx 10 \text{ GeV/m}$$

Measurements

M. C. Thompson et al.

of the breakdown threshold **in a dielectric** for wakefield produced by short 28.5 GeV electron bunches have shown $E=13.8 \text{ GV/m}$.



Introduction

TR is important in the method of particle acceleration by wakefield E_z , excited in dielectric cavity by train of electron bunches. TR is defined as ratio

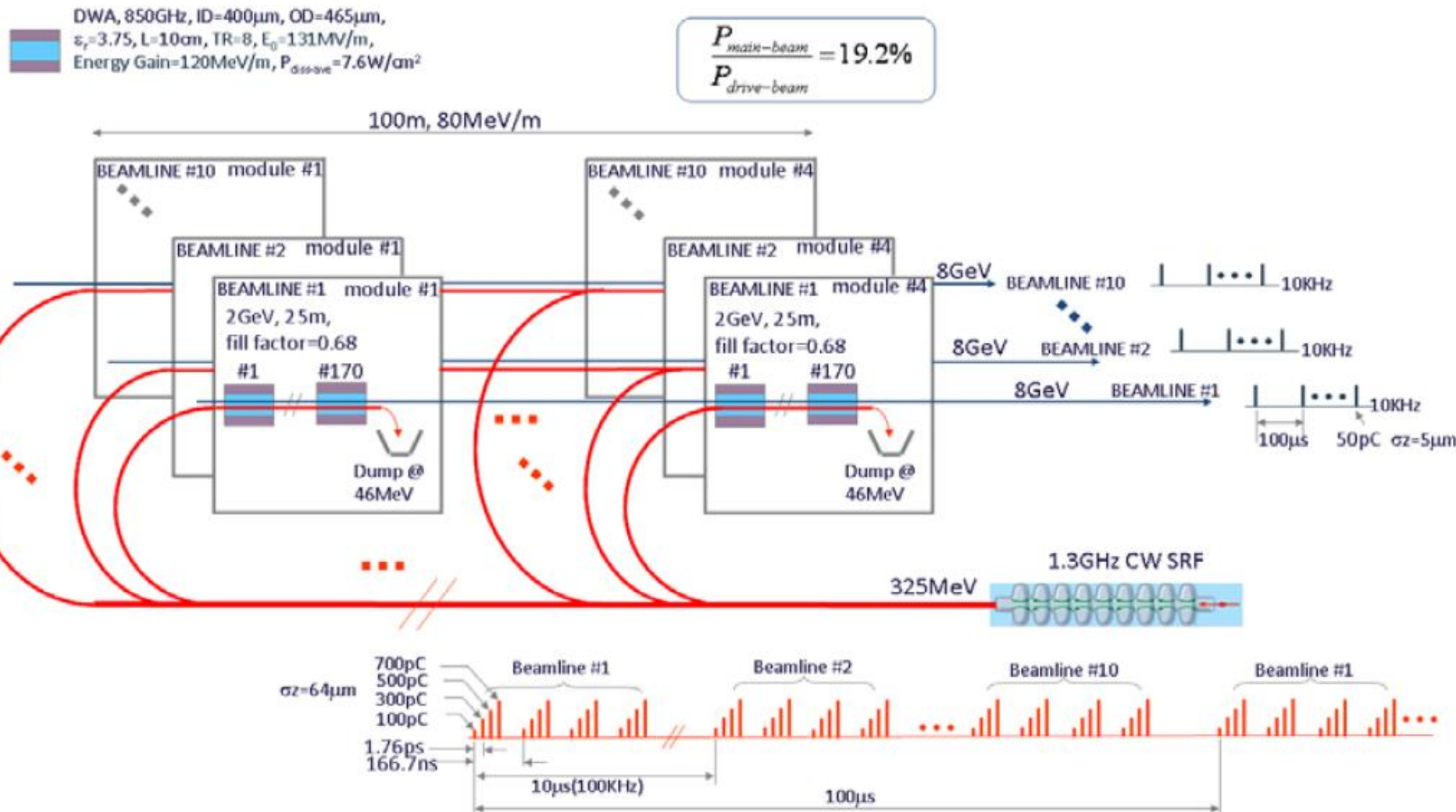
$$TR = \frac{E_{z\max}^+}{E_{z\max}^-}$$

of the wakefield $E_{z\max}^+$ which is excited by sequence of the electron bunches, to the field $E_{z\max}^-$ in which an electron bunch is decelerated.

TR determines energy, to which witness bunch can be accelerated at fixed energy of driver bunch. In typical conditions $TR \leq 2$.

Many investigations on TR increase at wakefield excitation and on their application for particle acceleration :

- R.D. Ruth, A.W. Chao, P.L. Morton, P.B. Wilson. 1985;
K.Nakajima. 1990; S.S. Vahanyan, E.M. Laziev, V.M. Tsakanov . 1990;
V.A. Balakirev, I.N.Onishchenko, G.V. Sotnikov, Ya. B. Fainberg. 1996;
C. Jing, A. Kanareykin, J. G. Power, M. Conde, Z. Yusof, P. Schoessow, and
W. Gai. 2007; E. Kallos et al. 2007;
K.V. Lotov, V.I.Maslov, I.N.Onishchenko. 2010;
B. Jiang, C. Jing, P. Schoessow, J. Power, W. Gai. 2012;
V.I.Maslov, I.N.Onishchenko, I.P.Yarovaya. 2012.

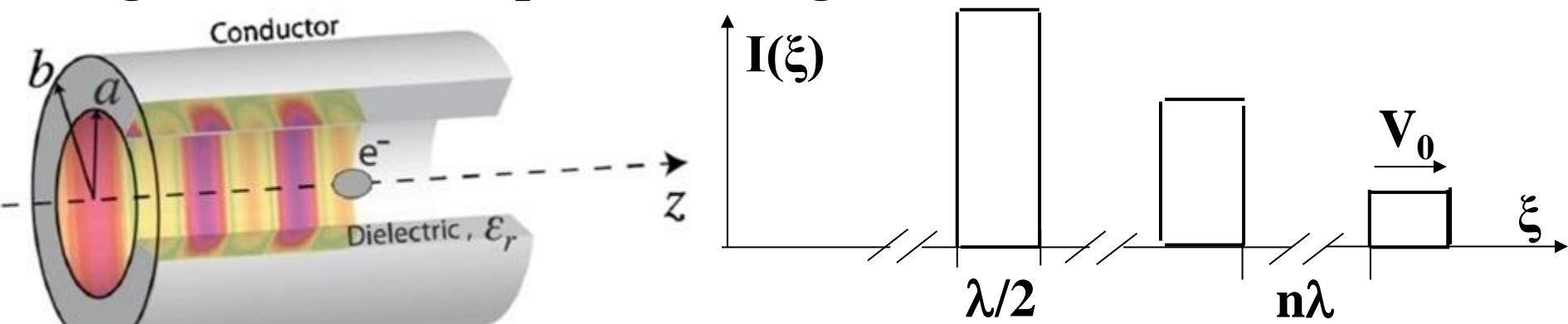


Dielectric wakefield waveguide accelerator, using a ramped bunch train technique to enhance transformer ratio

Transformation ratio at wakefield excitation in dielectric cavity by shaped train of homogeneous electron bunches with linear growth of current

Cavity has advantage in comparison with waveguide.

We consider possibility of TR increase at wakefield excitation in dielectric cavity by train of homogeneous electron bunches, the charge of which is shaped according to linear law.



We consider injection of driver bunches of length

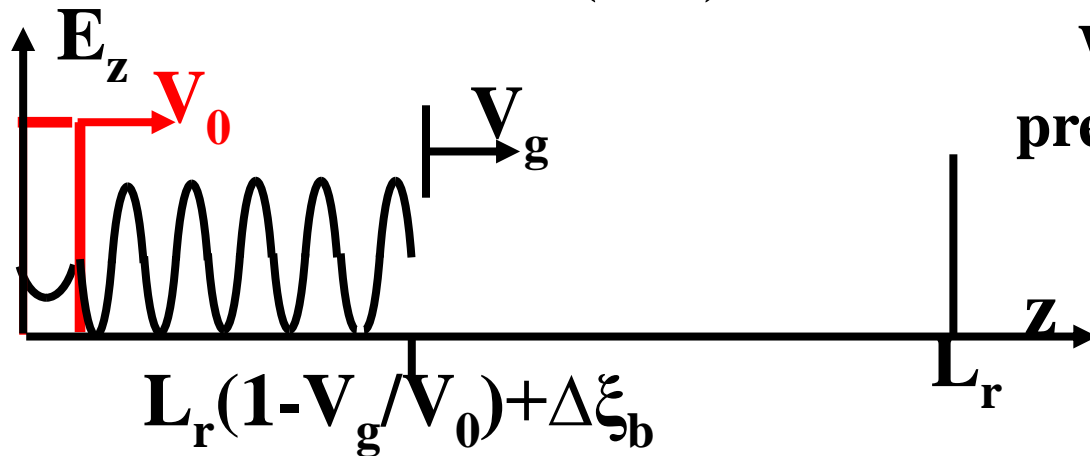
$$\Delta\xi_b = \lambda/2$$

The choice of such length of bunches is determined by the necessity to provide large TR and E_z .

$$n_b(z, t) = n_{b0} (2N - 1), \quad N \geq 1, \quad T(N-1) < t < T(N-1) + \frac{(L + \Delta\xi_b)}{V_0}, \quad 0 < V_0 (t - T(N-1)) - z < \Delta\xi_b$$

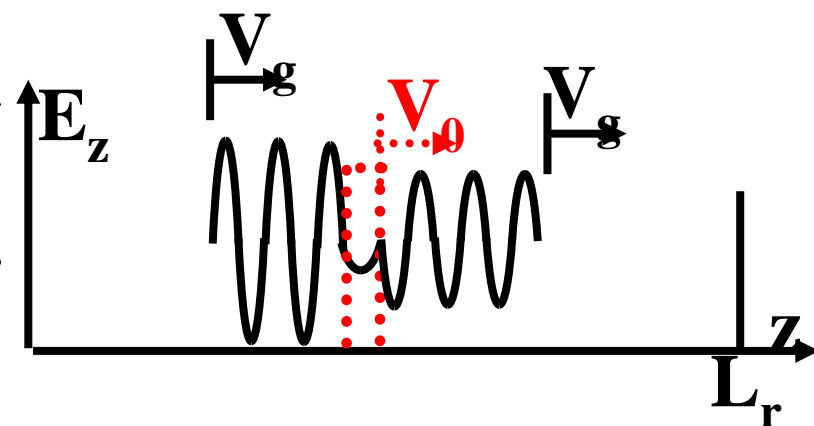
A next $N+1$ -th bunch is injected in the cavity, when the back wavefront of wakefield pulse, excited by previous N bunches, is on the injection boundary ($z=0$). At this moment the leading edge of the wakefield pulse, located at the distance from the injection boundary, equal to

$L_r(1-V_g/V_0)+\Delta\xi_b$ is located at the distance $L_r(V_g/V_0)-\Delta\xi_b$ from the end of the resonator ($z=L$).

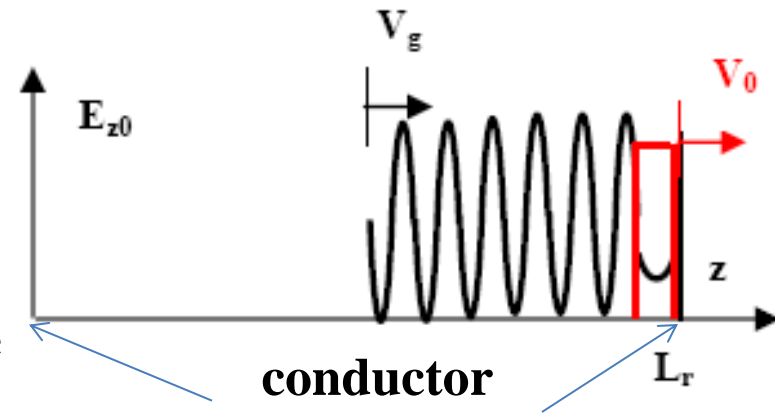


Wakefield pulse, excited by previous N bunches, when $N+1$ -th bunch is injected in the cavity

Wakefield pulse, excited by previous N bunches and excited by $N+1$ -th bunch, when $N+1$ -th bunch is in the middle of the cavity



Wakefield pulse, excited by N+1 bunches, when N+1-th bunch leaves cavity



E_z is small and identical for all bunches but non-uniform along them. Then one can provide a TR.

Because next N+1-th bunch is injected in the cavity, when the back wavefront of wakefield pulse, excited by previous N bunches, is on the injection boundary and N+1-th bunch reaches the end of the cavity together with the leading edge of the wakefield pulse, created by the previous N bunches, wakefield pulses, excited by all consistently injected bunches, are coherently added. In other words, coherent accumulation of wakefield is realized.

For achieving a large TR several conditions should be correct. Namely, we choose the length of the cavity L, the group velocity V_g , the bunch repetition frequency ω_d and the wave frequency ω_0 , which satisfy the following equalities

$$T = \frac{2L_r}{V_g} = \frac{2\pi}{\omega_m} = \frac{\pi q}{\omega_0}, \quad q = 1, 3, \dots \quad \frac{V_g}{V_0} = \frac{4L_r}{q\lambda}$$

At wakefield pulse excitation by 1-st bunch the wakefield in the whole cavity within the time

$0 < t < \frac{L_r + \Delta\xi_b}{V_0}$ is proportional to

$$Z_{\parallel}(z, t) = \left(\frac{1}{k}\right) \left[\theta(V_0 t - z) - \theta(V_0 t - \Delta\xi_b - z) \right] \sin[k(V_0 t - z)] + \\ + \left(\frac{2}{k}\right) \left[\theta(V_0 t - \Delta\xi_b - z) - \theta(V_g t - z) \right] \sin[k(V_0 t - z)]$$

1-st term is the field inside of 1-st bunch, 2nd term is the wakefield after 1st bunch. Thus, after 1-st bunch $TR=2$.

Inside 2-nd bunch

$$0 < \xi = V_0(t - T) - z < \Delta\xi_b$$

the wakefield on the times $T < t < T + \frac{L_r + \Delta\xi_b}{V_0}$ is proportional

$$Z_{\parallel}(z, t) = \left[\theta(V_0(t - T) - z) - \theta(V_0(t - T) - \Delta\xi_b - z) \right] k^{-1} \sin[k(V_0(t - T) - z)]$$

The decelerating field into 2-nd bunch equals to decelerating field into 1-st bunch.

After 2-nd bunch $\xi = V_0 (t-T) - z > \Delta \xi_b$ **on the times**
 $T < t < T + (L_r + \Delta \xi_b) / V_0$ **wakefield is proportional to**
 $Z_{||}(z, t) = \left[\theta(V_0 (t-T) - \Delta \xi_b - z) - \theta(V_g (t-T) - z) \right] \times$
 $\times 4k^{-1} \sin \left[k (V_0 (t-T) - z) \right]$ **Thus, after 2-nd bunch TR=4 .**

At pulse excitation by N-th bunch within time

$$T(N-1) \leq t \leq T(N-1) + \frac{(L + \Delta \xi_b)}{V_0} \quad T = \frac{2L}{V_g}$$

the wakefield is proportional to

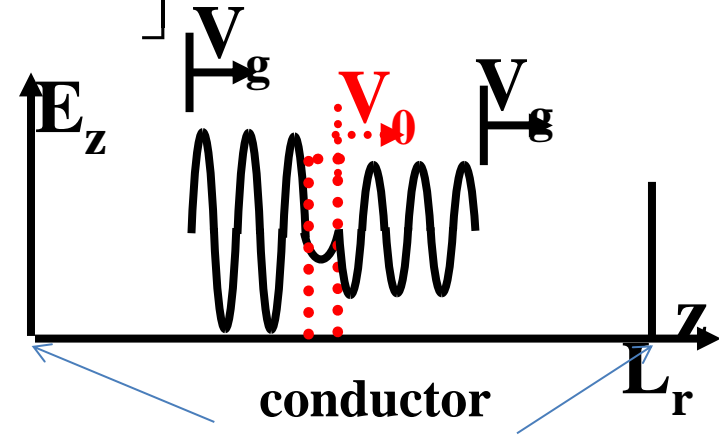
$$Z_{||}(z, t) = \left(\frac{1}{k} \right) \left[\theta(V_0 (t-T(N-1)) - z) - \theta(V_0 (t-T(N-1)) - \Delta \xi_b - z) \right] \sin(k\xi) +$$

$$+ \left[\theta(V_0 (t-T(N-1)) - \Delta \xi_b - z) - \theta(V_g (t-T(N-1)) - z) \right] \left(\frac{2N}{k} \right) \sin(k\xi) +$$

$$+ \left[\theta(V_g (t-T(N-1)) + L_r \left(1 - \frac{V_g}{V_0} \right) - z) - \theta(V_0 (t-T(N-1)) - z) \right] \times$$

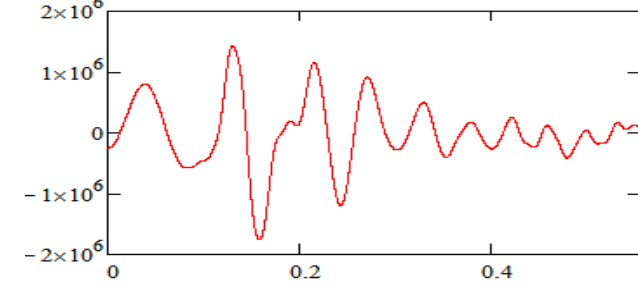
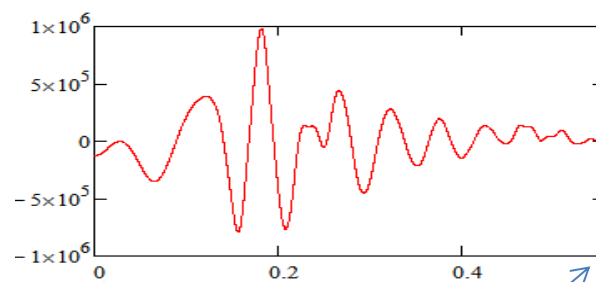
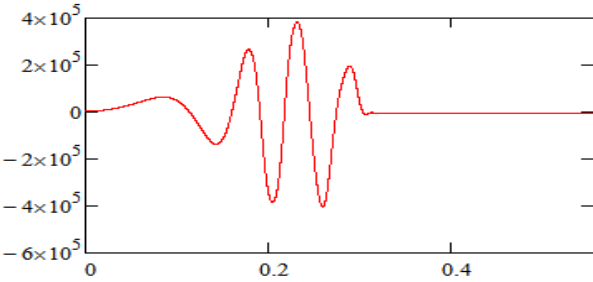
$$\times \left(\frac{2}{k} \right) (N-1) \sin \left[k (V_0 t - z) \right]$$

$$\begin{aligned}
Z_{\parallel}(z, t) = & \left(\frac{1}{k} \right) \left[\theta(V_0(t-T(N-1))-z) - \theta(V_0(t-T(N-1))-\Delta\xi_b - z) \right] \sin(k\xi) + \\
& + \left[\theta(V_0(t-T(N-1))-\Delta\xi_b - z) - \theta(V_g(t-T(N-1))-z) \right] \left(\frac{2N}{k} \right) \sin(k\xi) + \\
& + \left[\theta(V_g(t-T(N-1))+L_r \left(1 - \frac{V_g}{V_0} \right) - z) - \theta(V_0(t-T(N-1))-z) \right] \times \\
& \times \left(\frac{2}{k} \right) (N-1) \sin[k(V_0 t - z)] \quad \xi \equiv V_0 t - z
\end{aligned}$$



1-st term is the decelerating field inside N-th bunch, 2-nd term is wakefield after N-th bunch, 3-rd term is field before N-th bunch, excited by N-1 bunches. Decelerating field inside N-th bunch is equal to decelerating field inside 1-st bunch.

Thus, after N-th bunch $R=2N$.



conductor

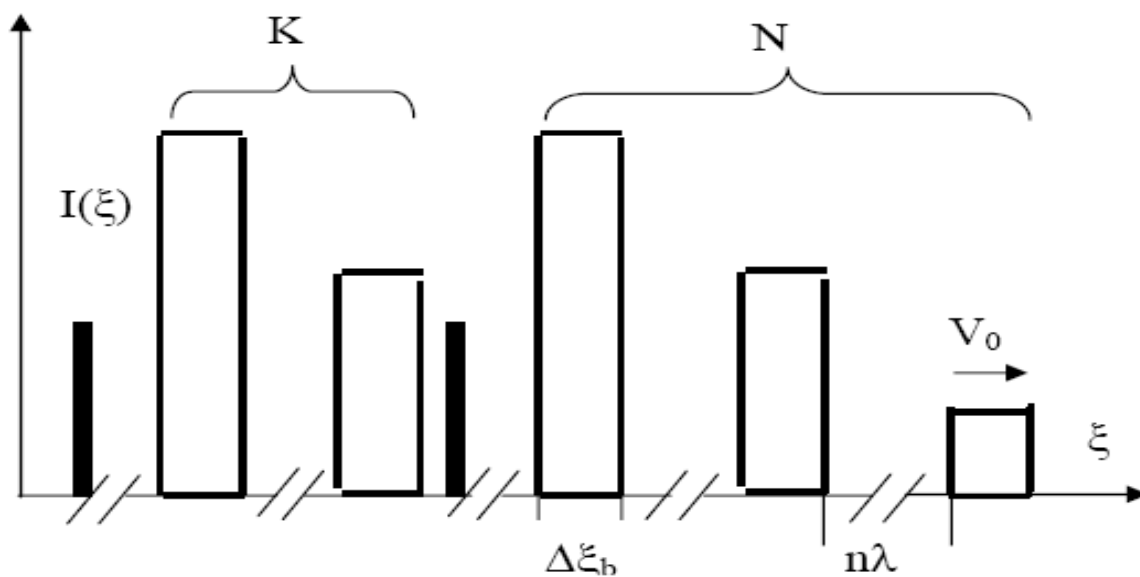
**TR after 1-st bunch
equals $TR_{01} \approx 1.9$ and
close to $TR_{01 \text{ theor}} = 2$**

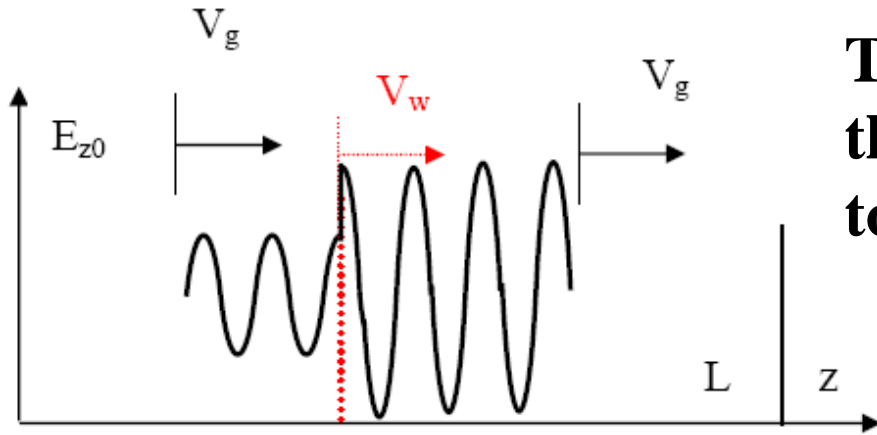
**TR after 2-nd bunch
equals $TR_{02} \approx 3.4$ and
smaller than
 $TR_{02 \text{ theor}} = 4$**

**TR after 3-rd bunch
equals $TR_{03} \approx 5$ and
smaller than
 $TR_{03 \text{ theor}} = 6$**

Infinite periodical train of short trains of shaped drivers, interchanged by witnesses

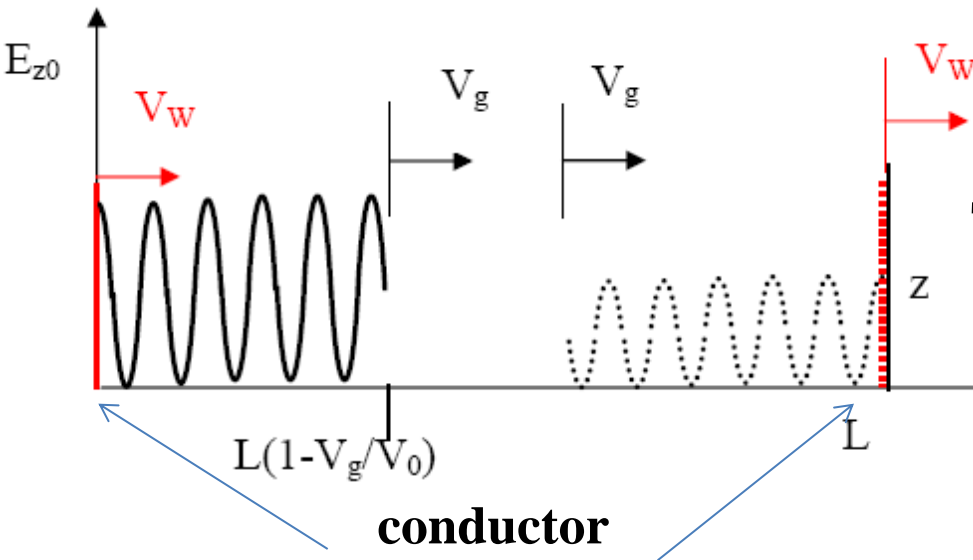
For the increase of number of accelerated electrons we consider the case, when after N shaped bunches the train continues as periodical infinite train of interchanging short (K bunches-drivers) trains of the shaped drivers and separate witnesses. Then after every K -th driver a witness follows, so that it can take away considerable energy.





Thus after witness the amplitude of the wakefield decreases from $E_{z0}=NE_{11}$ to $\chi E_{z0}=(N-K)E_{11}$, $\chi=\frac{(N-K)}{N}<1$

$K=N(1-\chi)$. E_{11} is wakefield after 1-st bunch.



TR equals

$$TR \approx \frac{[NE_{11}+(N-K)E_{11}]}{2E_{sl}} = (N-\frac{K}{2}) \frac{E_{11}}{E_{sl}}$$

E_{sl} is maximum decelerating wakefield in region of drives.

We use that $TR=2N$ known at $K=0$. Then $\frac{E_{11}}{E_{sl}}=2$

We obtain the coupling of TR with χ and with number of bunches N of the train, after which the periodic wakefield is set,

$$TR=N(1+\chi).$$

From balance of energies one can derive

$$q_W TR = q_W (2N - K) = (2/\pi) \sum_{i=1}^K q_{dr i} = (2/\pi) q_1 K (2N - K).$$

$q_{dr i}$ is charge of i -th driver bunch of train.

Then the ratio of charge of witness q_W to charge of 1-st driver q_1 equals

$$q_W/q_1 = (2/\pi) K.$$

One can see that $q_W \geq q_1$, however for $q_K = (2K-1)q_1$,

$$q_W/q_K = 1/\pi(1-1/2K),$$

and for $q_N = (2N-1)q_1$,

$$q_W/q_N = K/\pi(N-1/2).$$

The maximal ratio q_W/q_N equals $q_W/q_N = 1/\pi(1-1/2N)$ at $K=N$, i.e. at $\chi=0$. But

$$q_W/q_N \geq 1/\pi(N-1/2),$$

because $K \geq 1$.

Thus minimal TR equals $TR=N$ for infinite train. From here one can derive coupling of decrease rate χ of wakefield (from E_{z0} to χE_{z0}) after witness bunch with ratio of witness charge to driver charge q_W/q_{dr}

$$q_W/q_1 = (2/\pi) N(1-\chi).$$

TR equals to

$$TR = 2N - (\pi/2)(q_W/q_1).$$

The maximal TR equals to

$$TR = 2N - 1.$$

for infinite sequence.

Transformation ratio at wakefield excitation in a dielectric cavity at charge ramping of bunch train by linear law

Here TR is investigated theoretically. In many cases

$$\mathbf{TR=E_2/E_1}$$

We consider injection of bunches with length $\Delta\xi_b = \lambda$ the charge of which is ramped according to linear law, both along the train of bunches and along each bunch (in each bunch the charge is distributed accordingly to rectangular trapezoid), in the dielectric cavity of length L. The choice of such length of bunches is determined by the necessity to provide large TR and E_z .

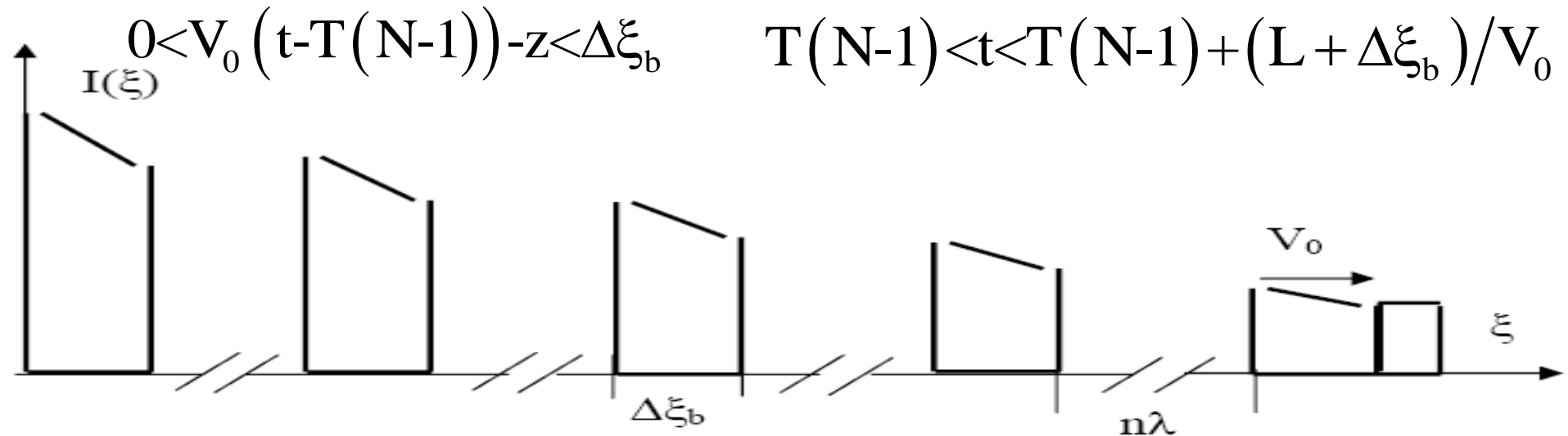
Charge density of short rectangular bunch – precursor and of train of rectangular trapezoid bunches is distributed according to

$$n_b(z, t) = n_{b0} \quad 0 < V_0 t - z < \Delta \xi_0 \quad 0 < t < (L + \Delta \xi_0) / V_0$$

$$n_b(z, t) = n_{b0} \left[1 - \pi/2 + (V_0 t - z) 2\pi/\lambda \right] \quad N = 1$$

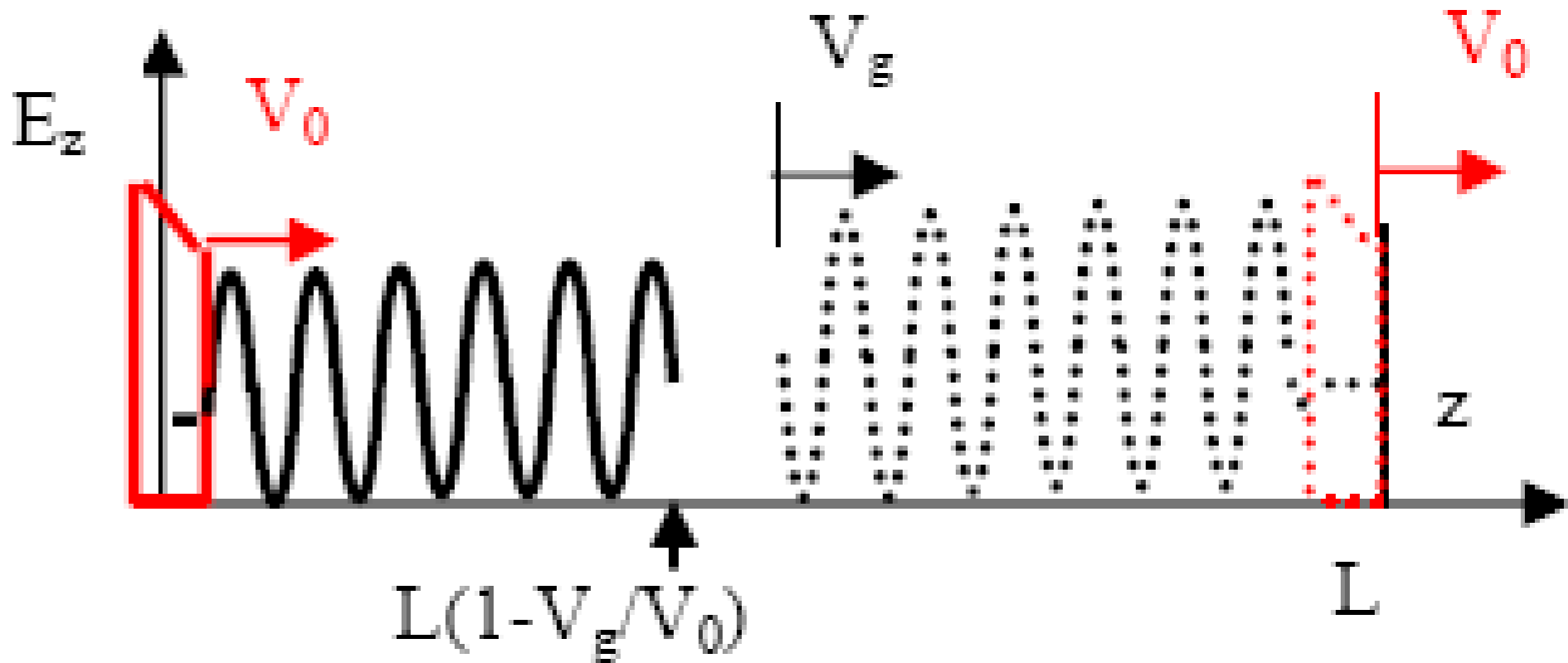
$$\Delta \xi_0 < V_0 t - z < \Delta \xi_0 + \Delta \xi_b \quad \Delta \xi_0 / V_0 < t < (L + \Delta \xi_0 + \Delta \xi_b) / V_0$$

$$n_b(z, t) = n_{b0} \left[1 + (N - 1) 2\pi + \left[V_0 (t - T(N - 1)) - z \right] 2\pi/\lambda \right] \quad N > 1$$

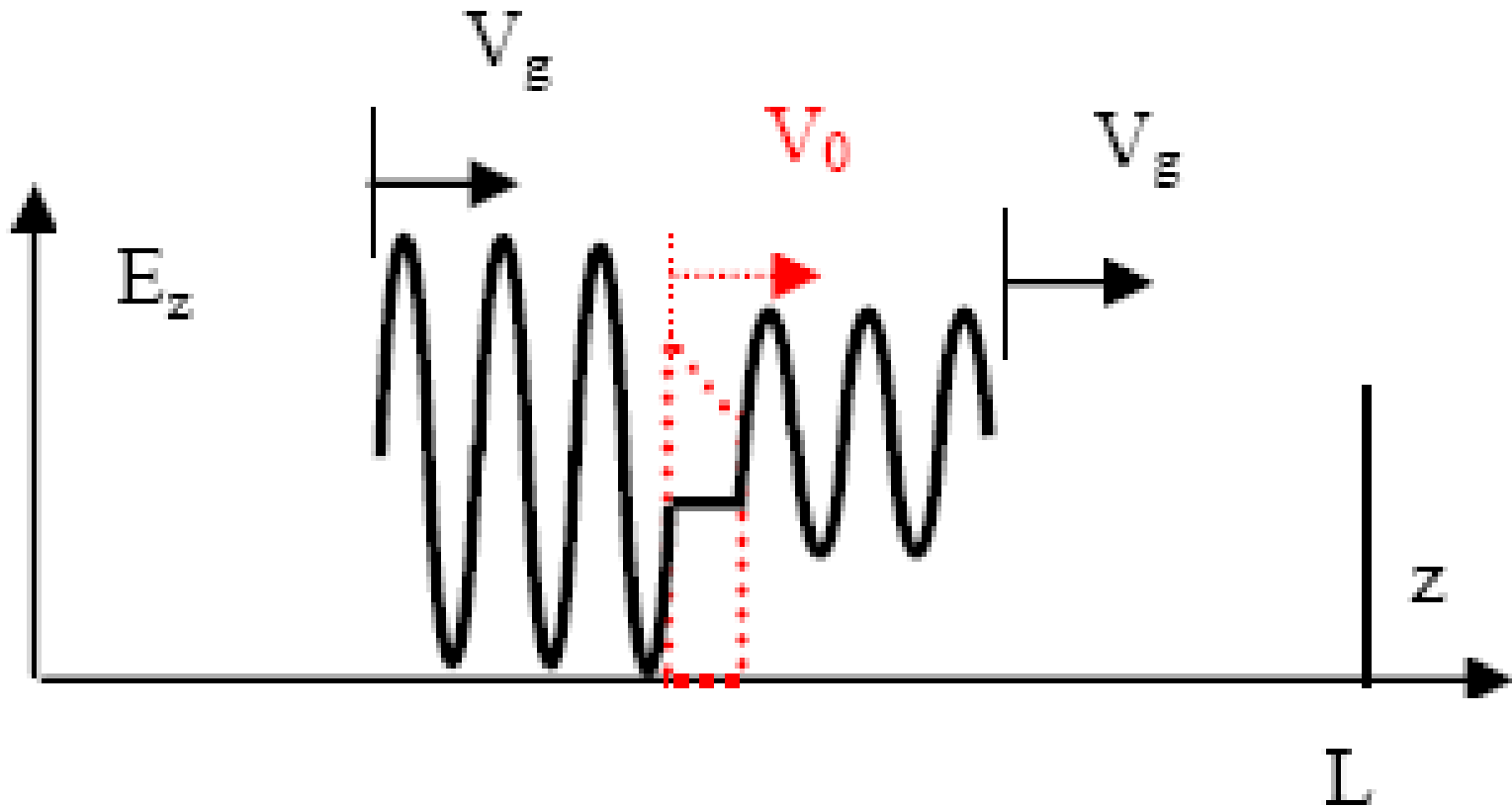


The charge distribution of short rectangular bunch - precursor and of train of rectangular trapezoid bunches

A next bunch leaves the cavity, when 1st wavefront of wakefield pulse, excited by previous bunches, is on the end of the cavity ($z=L$).



The wakefield pulse (dotted), excited by three bunches, when 3-rd bunch leaves the cavity



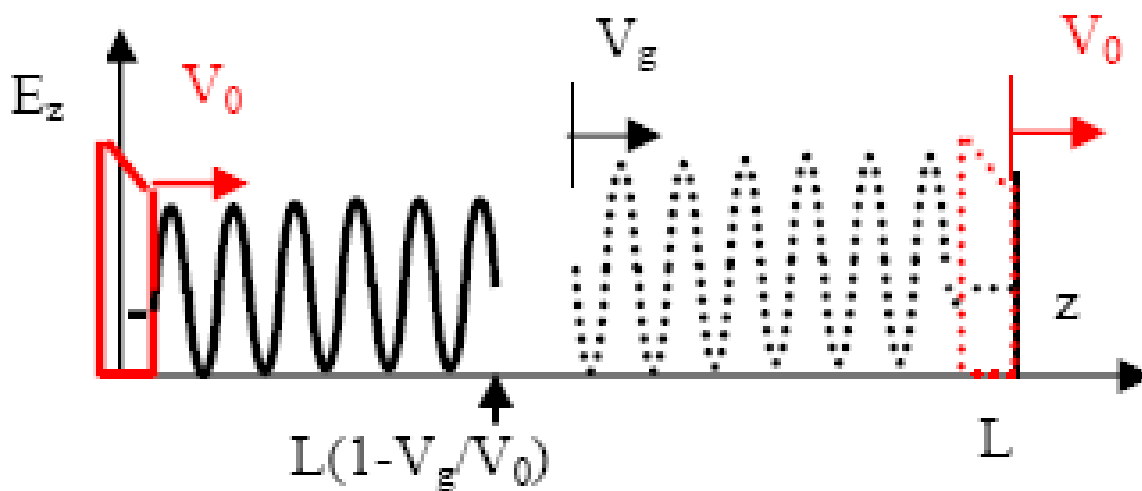
An approximate view of the wakefield pulse, excited by previous two bunches and excited by 3-rd bunch, when 3-rd bunch is in the middle of the cavity

Excited longitudinal wakefield E_z is non-uniform along the bunch - precursor $\xi \leq \Delta\xi_0$, and for all major bunches the decelerating wakefield E_z is homogeneous (in other words the same) and small. Then one can provide a large transformation ratio TR. But several conditions should be correct for this purpose. Namely, we choose the length of the cavity L , the group velocity V_g and the frequency of bunch repetition rate ω_m and the wave frequency ω_0 , which satisfy the following equalities

$$T = 2L/V_g = 2\pi/\omega_m = 2\pi n/\omega_0 \quad n = 1, 2, \dots$$

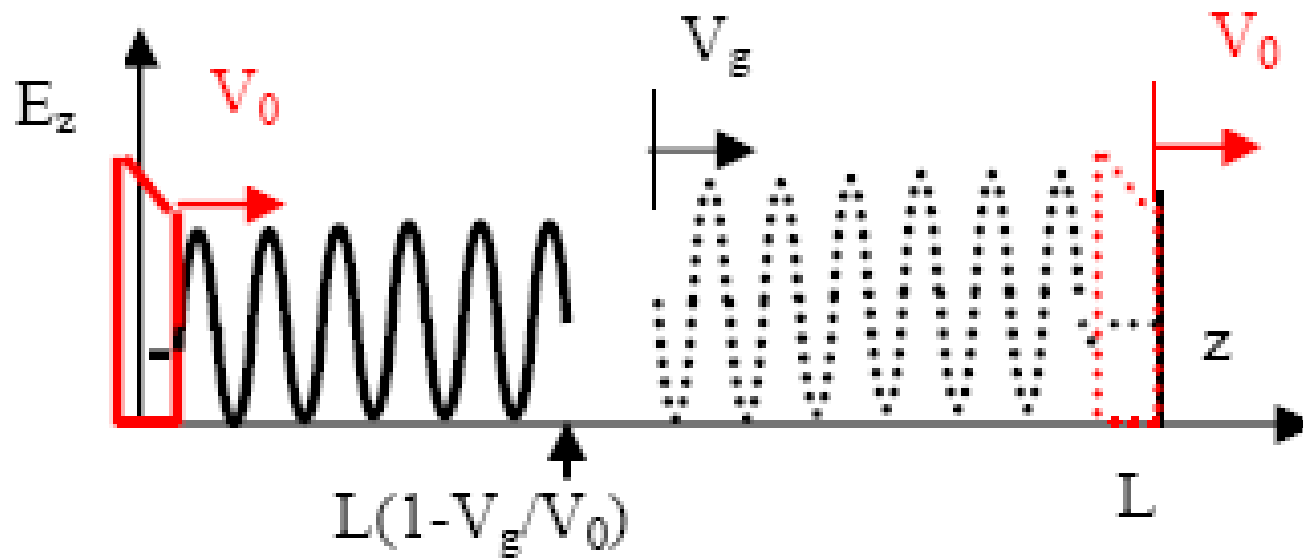
For selected length of the cavity L and n , equal $L/\lambda=4$ and $n=10$ group velocity should be equal $V_g/V_0=0.8$. V_0 is the beam velocity. For $L/\lambda=5$ and $n=16$ group velocity should be equal $V_g/V_0=0.625$.

Thus, all the next bunches after 1st one begin to be injected in the cavity (on the boundary $z=0$), when the trailing edge of the wakefield pulse, created by the previous bunches, is located at the point $z=0$. At this time the leading edge of the wakefield pulse, located at the distance from the injection boundary, equal to $L(1-V_g/V_0)$ (see Fig.), is located at the distance $L(V_g/V_0)$ from the end of the cavity ($z=L$).



**Wakefield pulse
(continuous), excited by
previous two bunches,
when 3-rd bunch is
injected in the cavity**

Again injected bunch reaches the end of the cavity together with the leading edge of the wakefield pulse, created by the previous bunches. Then wakefield pulses, excited by all consistently injected bunches, are coherently added. In other words, coherent accumulation of wakefield is realized.



The wakefield pulse (dotted), excited by three bunches, when 3-rd bunch leaves the cavity

At wakefield pulse excitation by the 1-st bunch the wakefield in the whole cavity within the time

$$0 < t < (L + \Delta\xi_0 + \Delta\xi_b) / V_0$$

equals to

$$\begin{aligned} E_z(z, t) \sim I_0 \{ & \theta(V_0 t - z) \theta(z - V_0 t + \Delta\xi_0) \sin(k\xi) + \\ & + \theta(V_0 t - \Delta\xi_0 - z) \theta(z - V_0 t + \Delta\xi_0 + \Delta\xi_1) + \\ & + \left[(1 + 2\pi) \cos(k\xi) + \sin(k\xi) \right] \theta(V_0 t - \Delta\xi_1 - \Delta\xi_0 - z) \theta(z - V_g t) \} \end{aligned}$$

1-st term is the field inside of the bunch - precursor, the 2nd term is the field inside the 1-st bunch, the 3-rd term is the wakefield after the 1st bunch. The field is uniform inside the 1-st bunch.

$$\xi = V_0 t - z$$

At wakefield excitation by the N-th bunch the wakefield in whole cavity within the time

$$T(N-1) \leq t \leq T(N-1) + (L + \Delta\xi_b)/V_0 \quad T = 2L/V_g$$

equals to

$$\begin{aligned} E_z(z, t) \sim I_0 \{ & \theta[V_0(t-T(N-1))-z]\theta[z+\lambda-V_0(t-T(N-1))]+ \\ & + [(1+2\pi(N-1))\cos(k\xi)+\sin(k\xi)] \times \\ & \times \{ \theta[V_g(t-T(N-1))+L(1-V_g/V_0)-z]\theta[z-V_g(t-T(N-1))]- \\ & - \theta[V_0(t-T(N-1))-z]\theta[z+\lambda-V_0(t-T(N-1))] \} + \\ & + 2\pi\cos(k\xi)\theta[V_0(t-T(N-1))-\lambda-z]\theta[z-V_g(t-T(N-1))] \}. \end{aligned}$$

Here the last term is the wakefield, excited by the N-th bunch, the first term is the slowing down field in the N-th bunch (uniform, small and the same for all bunches) and the second term is the wakefield, excited by N-1 bunches.

Thus, if the lengths of all bunches are equal $\Delta\xi_b = \lambda$, after the N-th bunch the transformation ratio is equal to

$$R = \left[1 + (1 + 2\pi N)^2 \right]^{1/2} \approx 2\pi N$$

Bane, K. L. F., P. Chen, P. B. Wilson. 1985.

Chen, P. et al. 1986.

Laziev, E., V. Tsakanov, S. Vahanyan. 1988.

B. Jiang, C. Jing, P. Schoessow, J. Power, W. Gai. 2012.

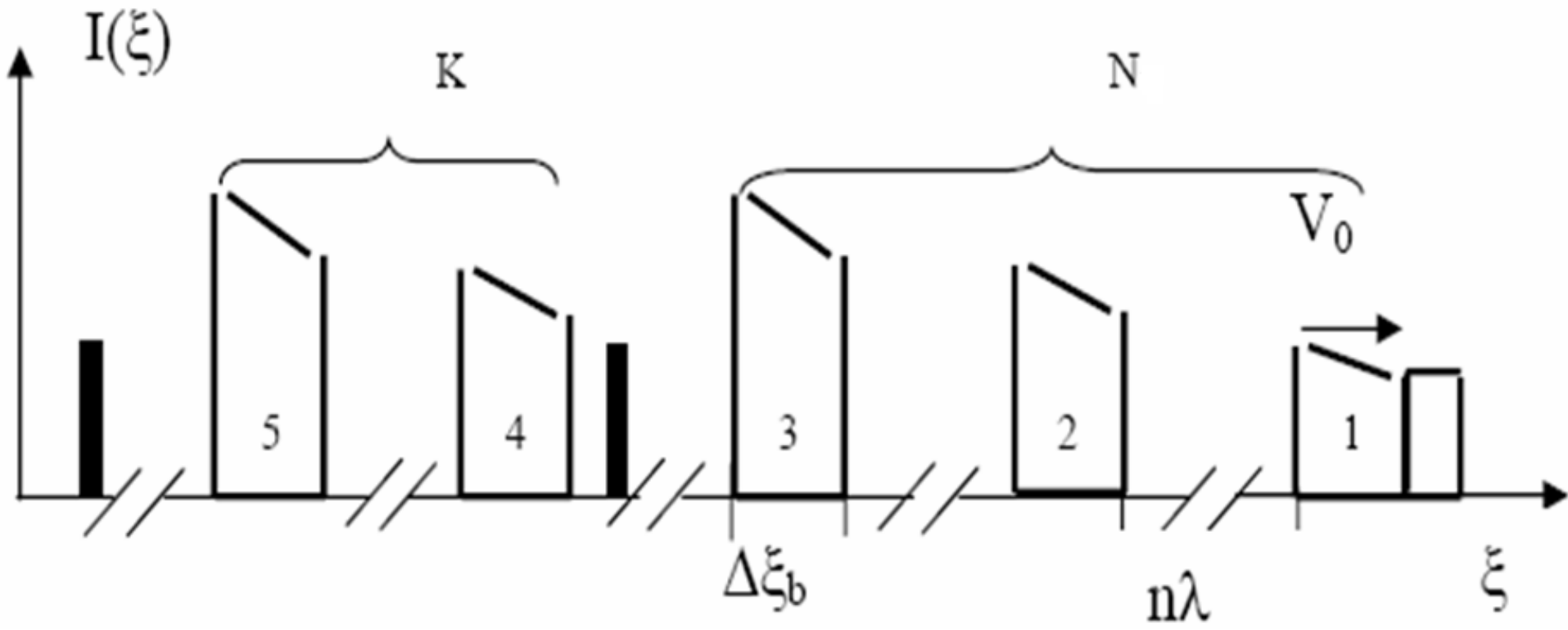
Thus the choice of such ramped sequence of bunches provides not only large R, but also large amplitude of excited wakefield

$$E_{0N} \approx N E_{01}$$

Thus the total charge Q_Σ of train is proportional $Q_\Sigma \approx N^2 Q_1$, i.e. it is the much larger in comparison with nonramped sequence, for which $Q_\Sigma = N Q_1$.

Asymptotic infinite train of short trains of drivers- rectangular trapezoids, interchanged by accelerated «high-current» bunches

We consider the case, when after N bunches- trapezoids the train continues as asymptotic infinite train of interchanging short (K bunches-trapezoids) trains of the shaped drivers and separate "high-current" witnesses. Then after every K -th bunch-driver a "high-current" witness follows, so that it takes away considerable energy.



Thus after witness the amplitude of the wakefield decreases from $E_{z0}=NE_1$ to $\chi E_{z0}=(N-K)E_1$, $\chi=(N-K)/N<1$, $K=N(1-\chi)$. In this case R equals

$$\mathbf{R}\approx[NE_1+(N-K)E_1]/2E_{sl}=(N-K/2)(E_1/E_{sl})$$

We use that the transformation ratio $R=2\pi N$ known at $K=0$. Then $E_1/E_{sl}=2\pi$ and we derive the connection of R with χ and with number of bunches N of the train, after which the periodic quasi-stationary asymptotic wakefield is set

$$\mathbf{R}\approx\pi N(1+\chi).$$

Here E_{sl} is the decelerating wakefield in the region of being of bunch-trapezoid.

We specify that the words "high-current" witness mean.

From balance of energies one can derive

$$\mathbf{R}q_{\mathbf{w}} = \mathbf{K}q_{\mathbf{N}} \left[1 + \pi(2\mathbf{N} - \mathbf{K}) \right] / \left[1 + \pi(2\mathbf{N} - 1) \right]$$

$$q_{\mathbf{N}} = \mathbf{I}_0 \lambda \left[1 + \pi(2\mathbf{N} - 1) \right]$$

$q_{\mathbf{N}}$ is the charge of \mathbf{N} -th bunch.

Then the ratio of witness charge to driver charge equals

$$q_{\mathbf{w}} / q_{\mathbf{N}} = \mathbf{K} / \pi(2\mathbf{N} - 1) = (1 - \chi) / \pi(2 - 1/\mathbf{N})$$

One can see that the maximal ratio $q_{\mathbf{w}}/q_{\mathbf{n}}$ equals $q_{\mathbf{w}}/q_{\mathbf{n}}=1/\pi(2-1/\mathbf{N})$ at $\mathbf{K}=\mathbf{N}$, i.e. at $\chi=0$. Thus $\mathbf{R}=\pi\mathbf{N}$ for infinite train.

As $\mathbf{K} \geq 1$, then

$$q_{\mathbf{w}} / q_{\mathbf{N}} \geq 1 / \pi(2\mathbf{N} - 1)$$

The transformation ratio, equal to

$$R=1+\pi(2N-1)$$

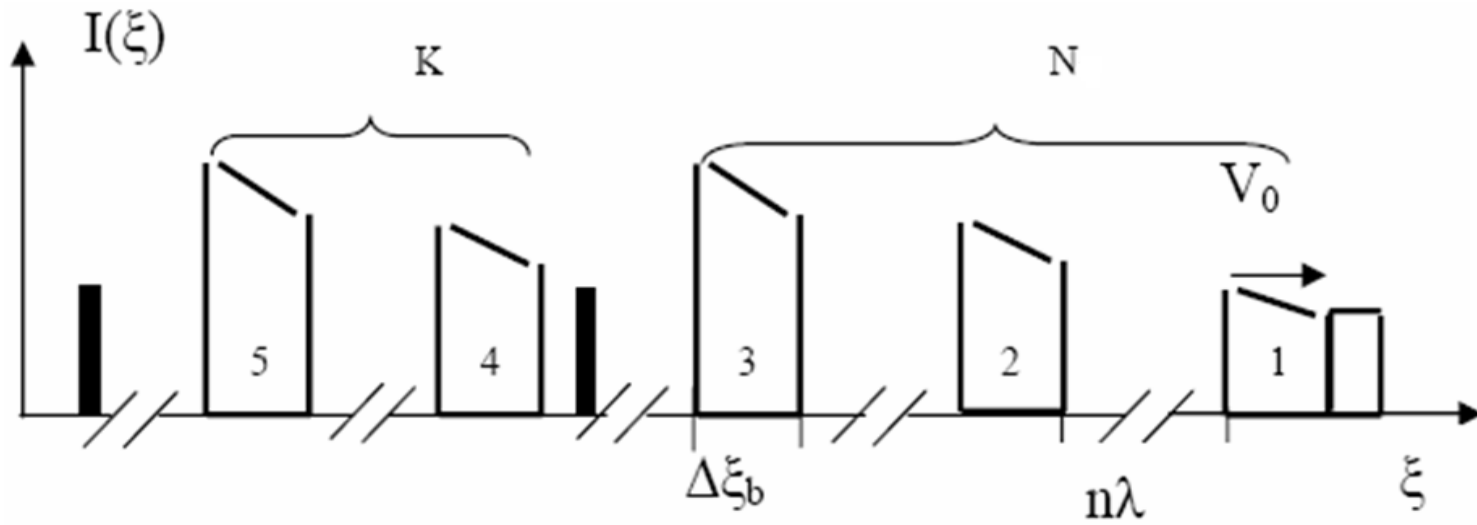
is achieved at $q_w \ll q_n$.

From previous expressions one can derive the connection of R with N and with q_w/q_n

$$R = 2N\pi \left[1 - \left(1 - 1/2N \right) \pi q_w / q_n \right]$$

Thus $\pi N \leq R \leq 1 + \pi(2N - 1)$, because

$$1/\pi(2N-1) \leq q_w/q_n \leq 1/2\pi(1-1/2N)$$



Thus, using decelerating wakefield, equal
 $E_{\text{dec}}=80\text{MV/m}$, then at

$$q_{\text{W}}/q_{\text{N0}} = 2/5\pi, \quad N = 3$$

(in this case K equals $K=2$) R equals $R \approx 4\pi$ and accelerating wakefield equals $E_{\text{ac}}=1\text{GV/m}$. Thus, at accelerator length, equal 250 m, driver bunches with energy 20 GeV are fully decelerated (or on each from 10 decelerating sections, each of length 25 m, driver bunches with energy 2 GeV are fully decelerated). Thus witness bunch are accelerated up to energy 250 GeV.

Thank you!

Сократить диэлектрик

Добавить:

-гауссовские сгустки [Deleue]

- (стр. 24, 65) профилирование лазерных сгустков по

интенсивности согласно:

= 1:3:5: ... (через 1.5λ) $TR=2N$

= 1:2:3: ... (через λ) $TR \approx \pi N$

=1:5:9: ... (через 1.5λ)

= 1:3:4:4: ... (через 1.5λ)

=1:5:7:7: ... [Deleue]