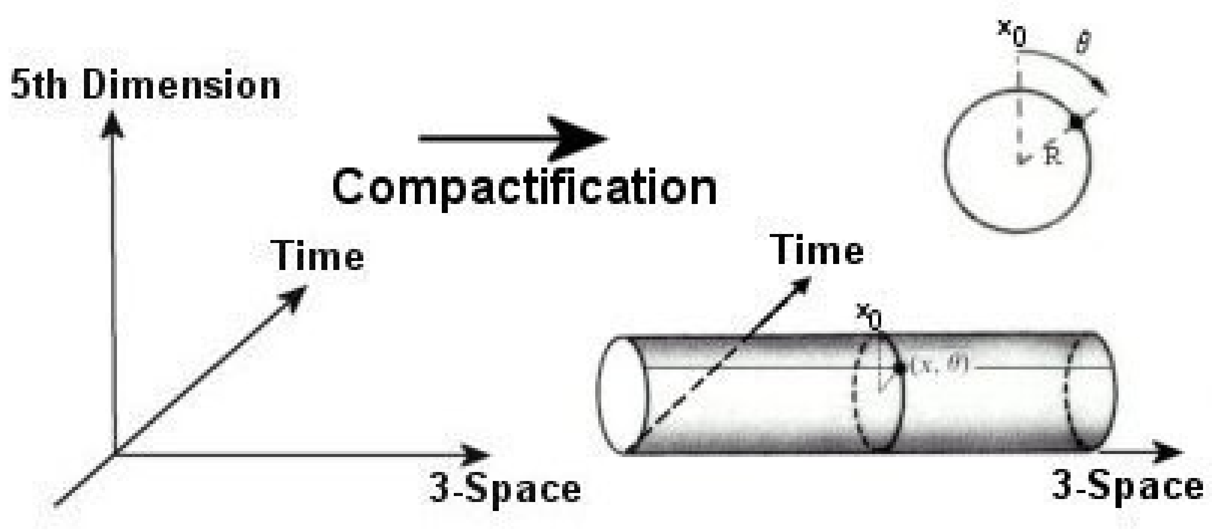
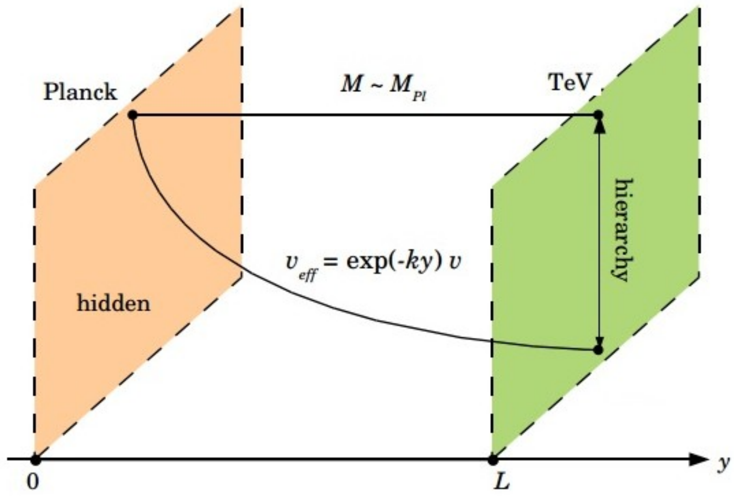
B2
Symmetry and Relativity
Lecture 4

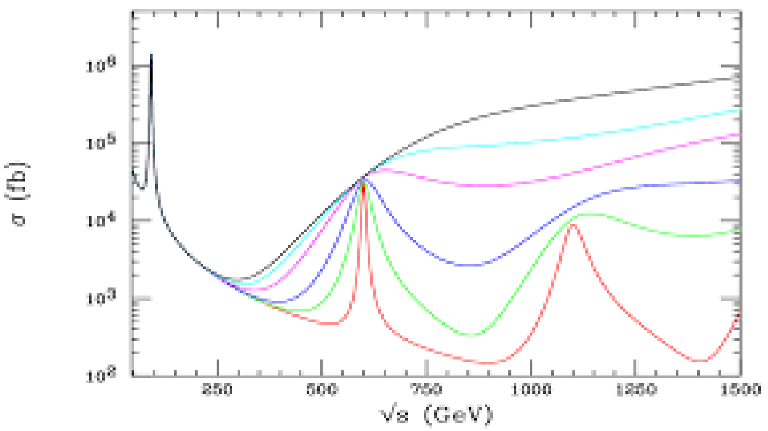
T Kaluza (1919), O Klein (1926)

- Electromagnetism + gravity in 5D
- Tightly curled up 5th dimension
- Periodic boundary condition



L Randall, R Sundrum (1999)



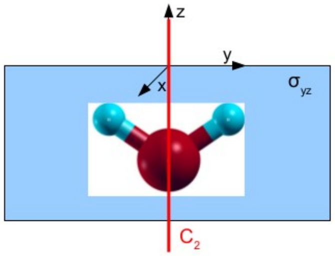


- Warped 5th dimension
  - "Such configurations are known to exist in string theory"
- Bessel root-spaced resonances
- Smoking gun?

Other applications

- B2 emphasizes continuous (Lie) groups
  - Rotations
  - Lorentz group
- Discrete groups important in CMP, chemistry
  - Symmetries → molecular vibration modes
  - Degenerate perturbation theory
- Rubik's Cube

Symmetry group: C_{2v} (water)



Symmetry elements:

E: identity
C_2: rotation by π
σ_{yz}: mirror plane (yz)
σ_{xz}: mirror plane (xz)

Multiplication table C_{2v}:

C_{2v}	E	C_2	σ_{yz}	σ_{xz}
E	E	C_2	σ_{yz}	σ_{xz}
C_2	C_2	E	σ_{xz}	σ_{yz}
σ_{yz}	σ_{yz}	σ_{xz}	E	C_2
σ_{xz}	σ_{xz}	σ_{yz}	C_2	E

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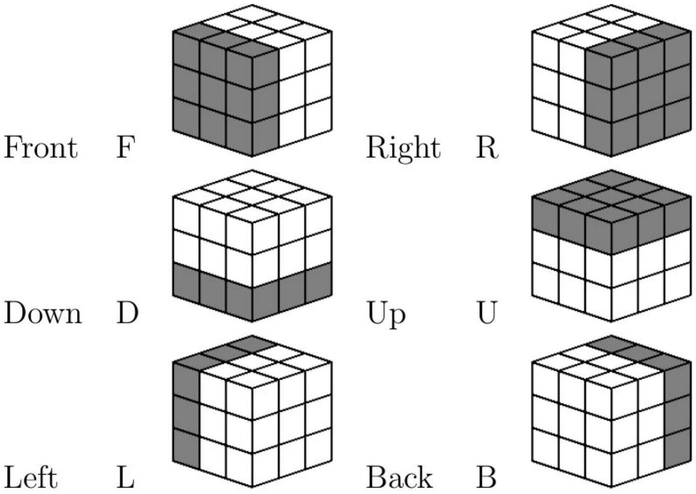

Generators	Size	Factorization
U	4	2^2
U, RR	14400	$2^6 \cdot 3^2 \cdot 5^2$
U, R	73483200	$2^6 \cdot 3^8 \cdot 5^2$
RRLL, UUDD, FFBB	8	2^3
Rl, Ud, Fb	768	$2^8 \cdot 3$
RL, UD, FB	6144	$2^{11} \cdot 3$
FF, RR	12	$2 \cdot 3^2$
FF, RR, LL	96	$2^5 \cdot 3$
FF, BB, RR, LL, UU	663552	$2^{13} \cdot 3^4$
LLUU	6	$2 \cdot 3$
LLUU, RRUU	48	$2^4 \cdot 3$
LLUU, FFUU, RRUU	82944	$2^{10} \cdot 3^4$
LLUU, FFUU, RRUU, BBUU	331776	$2^{12} \cdot 3^4$
LUlu, RUru	486	$2 \cdot 3^5$

Permutation group

	e	231	312	213	132	321
e	e	231	312	213	132	321
231	231	312	e	321	213	132
312	312	e	231	132	321	213
213	213	132	321	e	231	312
132	132	321	213	312	e	231
321	321	213	132	231	312	e

- Multiplication table: specifies where positions 123 end up
- Identity element: e takes 123 to 123
- Not commutative ("non-Abelian")

Permutation group

	e	231	312	213	132	321
e	e	231	312	213	132	321
231	231	312	e	321	213	132
312	312	e	231	132	321	213
213	213	132	321	e	231	312
132	132	321	213	312	e	231
321	321	213	132	231	312	e

- Subgroup: e, 231, 312 multiply among selves
  - Cyclic permutations

Permutation group

	e	231	312	213	132	321
e	e	231	312	213	132	321
231	231	312	e	321	213	132
312	312	e	231	132	321	213
213	213	132	321	e	231	312
132	132	321	213	312	e	231
321	321	213	132	231	312	e

- Off-diagonal quadrants self-contained
  - 213, 132, 321 swap two positions, not cyclic
  - Not subgroups: no identity element

Permutation group

	e	231	312	213	132	321
e	e	231	312	213	132	321
231	231	312	e	321	213	132
312	312	e	231	132	321	213
213	213	132	321	e	231	312
132	132	321	213	312	e	231
321	321	213	132	231	312	e

- Don’t need all elements to traverse group
  - Only need identity, permutation (231), swap (213)
  - “Generators”
  - Elements e, p, pp, s, sp, spp
  - Inverses: ppp = ss = e