Atomic Physics
Review of hydrogen and helium
from D.N. Stacey

1. Certain quantities which appear in the theory of hydrogen have wider application in atomic physics: the Bohr radius a_0, the Rydberg constant R, the fine structure constant α, and the Bohr magneton μ_B. Give a defining expression for each, and write a sentence or two on each explaining their significance.

2. (a) A muon has mass 206 times the electronic mass, and charge equal to that of an electron. The particle may be captured by an atom and the radiation which is emitted as the muon cascades through the energy levels can be measured. Assuming the muon-nucleus system can be treated as hydrogen-like, find the energy in MeV of the photon emitted as the muon goes from a state with principal quantum number 2 to the ground state in an atom of lead. Neglect the reduced mass correction.

(b) The experimental value is around 5 MeV. This is likely to be significantly different from your theoretical value. What effect(s) might be responsible for the discrepancy? [Hint: find the order of magnitude of the size of the region occupied by the muon in its ground state.]

3. [A non-mathematical quesiton, which invites you to think about properties of the hydrogenic wave-functions.] Neglecting relativistic and other small effects, the energies of the s, p, d,... states in hydrogen for a given value of the principal quantum number n are the same. They have the same potential energy, and the same kinetic energy. However, a student argues as follows: "Compared with high angular momentum states of the same n, the low angular momentum states have a higher probability of being near the nucleus, so their potential energy must be more negative. Their low angular momentum means they have lower kinetic energy also. So why don't the low angular momentum states lie deeper in energy?"

Explain to the student what is wrong with his argument.

4. Which of the following levels do not exist in hydrogen, and why? $3f\,^2F_{3/2}$, $3p\,^2P_{3/2}$, $3s\,^2S_{3/2}$, $3d\,^3D_2$, $2p\,^2D_{3/2}$.

5. For this question, you need to have to hand a derivation of the expression for the shifts in the positions of the energy levels of hydrogen-like atoms when spin-orbit interaction is taken into account. The result can be written

$$\Delta E(n,l,s,j) = \frac{\mu_0}{4\pi} Z^4 g_s \mu_B^2 \frac{1}{n^3 a_0^3 l(l+1/2)(l+1)} \times \frac{j(j+1) - l(l+1) - s(s+1)}{2}$$

We are concerned with the physics behind this expression and its derivation.

(a) In simple "vector-model" terms, what is the direction of the B-field experienced by the electron, relative to its orbital angular momentum? What is the direction of the intrinsic magnetic moment of the electron relative to its spin angular momentum?

(b) In the light of (a), and given that the interaction is of the form $-\mu \cdot \mathbf{B}$, should the energy be higher or lower for $j = l + 1/2$ as compared with $j = l - 1/2$? Check that your conclusion is correct by inspecting the expression given above.

(c) Explain in physical terms where the Z^4 dependence comes from.

6. Show that the splitting due to spin-orbit interaction between the $J = 3/2$ and $J = 1/2$ levels of the $2p$ configuration in hydrogen is 11GHz. Draw a diagram showing the positions of the split levels with reference to the unperturbed level, and label the levels with the appropriate quantum numbers. Using the known n-dependence, find the splitting also of the $3p$ configuration.

7. A way of studying the small shifts caused by quantum electrodynamic effects is to use high-Z atoms, with all the electrons stripped off except one. One method is to take a tunable laser operating in the visible region (say around 600nm) and excite transitions within the fine structure of the $n = 2$ manifold. Using your expression for the spin-orbit splitting as a function of Z, suggest a suitable element for study.

8. (a) Explain qualitatively why the energy required to remove one electron from the ground state of the helium atom is much larger than that in hydrogen, while for other states the hydrogenic value is quite a good approximation, *e.g.*, the $1s2s$ levels in helium have energies very similar to the $2s$ energy in hydrogen.

(b) The $1s2s$ configuration in helium is split into two levels. Explain briefly the physical origin of the splitting, and how it comes about that these different energies should be associated with different relative orientations of the intrinsic spins of the two electrons (so that one of the levels is called a "singlet", and the other a "triplet").