Nuclear Physics Problems

1.1. Naturally occurring samarium includes 15.1% of the radioactive isotope ^{147}Sm, which decays by α-emission. One gram of natural Sm gives 89 ± 5 α decays per second. Calculate the half-life of the isotope ^{147}Sm, and give its uncertainty.

1.5. In order to determine the age of an ancient piece of wood, a sample of the wood is analyzed for its ^{14}C content, and gives 2.1 decays per minute. Another sample of the same size from a recently cut tree of the same type gives 5.3 decays per minute. The mean life of ^{14}C is 8267 years. *Explaining your reasoning, and any assumptions, fully,* calculate the age of the ancient piece of wood.

1.6. Explain what is meant by the *width* of an excited state, and how it is related to the state's lifetime. If a state can decay by either of two modes, with probability per unit time λ_1 of decaying by mode 1 and probability per unit time λ_2 of decaying by mode 2, what is its lifetime in terms of λ_1 and λ_2? The *partial width* of a decay mode i is equal to $\hbar\lambda_i$, where λ_i is the probability per unit time of decaying by that mode. A certain excited state of the nucleon has partial width 180 MeV for decay into a nucleon plus a pion, and partial width 120 MeV for decay into a nucleon plus two pions; there are no other decay modes. What is the lifetime of this state?

1.7. Explain what is meant by *differential scattering cross-section* and *total cross-section.* The differential scattering cross-section for Rutherford scattering of particles of charge ze and energy E off a target of charge Ze is given by

$$\frac{d\sigma}{d\Omega} = \frac{\beta^2}{16E^2\sin^4(\theta/2)}$$

where $\beta = zZe^2/4\pi\epsilon_0$ and θ is the scattering angle. Calculate the differential cross-section for scattering of 10 MeV α-particles by a gold nucleus ($Z = 79$) at an angle of 10°, in b sr^{-1} (b stands for *barn*, look up the definition).

Calculate the cross-section (in b) for the scattering of 10 MeV α-particles through an angle greater than 10°.

1.8. The cross-section for the reaction $\overline{\nu}_e + p \to n + e^+$ is about 10^{-19} b ($\overline{\nu}_e$ denotes an electron antineutrino). A typical solid contains about 10^{30} protons per cubic meter. Estimate the probability that an antineutrino will be captured on passing through a solid, per unit thickness of solid.

1.9. A beam of neutrons of kinetic energy 0.29 eV and intensity 10^5 s^{-1} traverses normally a foil of ^{235}U, of thickness 0.1 kg m^{-2}. Any neutron-nucleus collision can have one of three possible results: (a) elastic scattering of neutrons (cross section 20 mb); (b) capture of the neutron followed by the emission of a γ-ray from the nucleus (cross section 70 b); and (c) capture of the neutron followed by fission of the nucleus (cross-section 200 b).

1. Explain the units of the "thickness" of the target and of the "intensity".
2. Calculate the attenuation of the neutron beam in the foil.
3. Calculate the number of fission reactions occurring per second in the foil.

4. Assuming that the elastically scattered neutrons are scattered isotropically, calculate the flux of elastically scattered neutrons at a point 10 cm from the foil.

2.1. Explain what is meant by a *form factor*, and what its significance is. How may the form factor of a nucleus be measured?

Suppose that the charge distribution of a nucleus of radius R is *uniform*, so that the charge density is given by $\rho(\mathbf{r}) = \rho_0$, a constant, for $0 \leq r \leq R$, and $\rho(\mathbf{r}) = 0$ for $r > R$. Show that the form factor $F(q^2)$ in this case is

$$F(q^2) = \frac{3(\sin x - x\cos x)}{x^3}$$

where $x = qR/\hbar$. What does q denote here?

Expand the numerator of $F(q^2)$ about $x = 0$, and hence show that $F(0) = 1$.

The zeroes of $F(q^2)$ occur where $\tan x = x$. Show, either graphically or numerically, that the first zero occurs for x just less than $3\pi/2$.

In the scattering of 450 MeV electrons from a certain nucleus, there is a pronounced minimum (assumed to correspond to the first zero in $F(q^2)$) in the angular distribution at a scattering angle of 45°. Estimate the radius of the nucleus.

3.1. There are three possible modes of β-decay: electron emission, positron emission, and electron capture. Explain what happens in each of these processes.

Let $M_a(Z, A)$ be the *atomic* mass of an atom in its ground state which has atomic number Z and mass number A. Show that, if we neglect atomic binding energies,

1. electron emission is energetically possible for this atom if $M_a(Z, A) > M_a(Z + 1, A)$
2. electron capture is energetically possible if $M_a(Z, A) > M_a(Z - 1, A)$
3. positron emission is energetically possible if $M_a(Z, A) > M_a(Z - 1, A) + 2m_e$, where m_e is the electron mass.

3.2.

1. Explain why it is convenient to consider the *atomic* mass rather than the nuclear mass when considering the stability of a nucleus against β-decay.
2. Explain, with reference to the SEMF, why there is never more than *one* isobar with odd A that is stable against β-decay. Use the SEMF to find the stable isobar with $A = 181$. (Note: the SEMF for atomic masses is the same as that given in question 1.6, but with m_p replaced by m_H, the mass of a hydrogen atom; $m_H = 1.007825u$.)
3. Why is it possible to have more than one isobar stable against β-decay if A is even?

3.3. In the β^+-decay of ${}^{15}_{8}\mathrm{O}$ to ${}^{15}_{7}\mathrm{N}$, the maximum kinetic energy of the emitted positrons is 1.69 MeV. Use this result to estimate the radius of the nucleus ${}^{15}_{8}\mathrm{O}$. (Neutron-proton mass difference: 1.29 MeV/c^2; electron mass: 0.51 MeV/c^2)

3.4. Use the SEMF to show that the energy released when a nucleus (Z, A) emits an α-particle is approximately, for large A and Z,

$$Q = -4a_v + \frac{8a_s}{3A^{1/3}} + \frac{4a_c Z}{A^{1/3}}\left(1 - \frac{Z}{3A}\right) - \frac{4a_A(A-2Z)^2}{A^2} + B_\alpha$$

where B_α is the binding energy of the α-particle, 28.30 MeV. Use this expression to investigate the stability against α-decay of the nucleus ${}^{208}_{82}\mathrm{Pb}$ and ${}^{212}_{86}\mathrm{Rn}$.

3.5. The nucleus ${}^{244}_{96}\mathrm{Cm}$ decays to ${}^{240}_{94}\mathrm{Pu}$ by α-emission. It is found that α-particles are emitted with kinetic energies 5.902 MeV, 5.859 MeV, 5.760 MeV, and 5.608 MeV. Assuming that the most energetic α-particles are emitted in the transition to the ground state of ${}^{240}_{94}\mathrm{Pu}$, draw an energy level diagram showing all energy levels involved in the decay of ${}^{244}_{96}\mathrm{Cm}$ (neglect any nuclear recoil in the decay). Given that an excited state in ${}^{240}_{94}\mathrm{Pu}$ can only make a γ-transition to the state just below it, what are the energies of the γ-rays that will be emitted?

How might the energies of the α-particles be measured?