CP2: INTRODUCTORY ELECTROMAGNETISM

Section A

Answer **all** questions in this section.

1. A charge q with mass m moves with velocity $\mathbf{v}$ perpendicular to a uniform constant magnetic field $\mathbf{B}$, discuss its path and explain why the field does no work on the charge. [5]

2. Show that the capacitance of two conducting concentric spherical shells of radii a and b ($b > a$) is given by

$$C = \frac{4\pi\epsilon_0 ab}{(b-a)}.$$

[5]

Section B

Answer **two** of the three questions.

3. A thin conducting disc has radius a thickness b and electrical resistivity ρ. It is placed in a uniform time-dependent magnetic induction $B = B_0 \sin\omega t$ directed parallel to the axis of the disc. Assuming that ρ is large, find E at a distance $r < a$ from the axis of the disc, in the plane of the disc, and obtain an expression for the induced current density as a function of r. [9]

Derive an expression for the mean power dissipated as heat over the entire disc. [8]

Comment on how the problem changes if the resistivity ρ were not large. [3]

4. A system of charges consists of a charge $+q_2$ at the origin and charges $-q_1$ on the z axis at the two points $z = \pm a$. Using spherical polar coordinates, find the potential $V(r,\theta,\phi)$ due to these charges, taking θ to be the angle between $\mathbf{r}$ and the z axis. [4]

Expand this potential as a power series in a/r, retaining only terms up to second order. Identify the terms in your expression due to a charge, a dipole and a quadrupole. [8]

In the case that $q_2 = 2q_1$, give the form of the potential and derive expressions for the radial and angular components of the electric field due to this potential. [8]

5. A long cylindrical solenoid has radius b and n_1 turns of wire per unit length. Find the magnetic induction B inside the solenoid when a current I passes through the wire. [3]

The current varies with time. Calculate the induced e.m.f. across length l of the solenoid and hence determine its self-inductance L_1. [4]

A similar solenoid of radius $a < b$ and n_2 turns per unit length is placed inside the first one. Both solenoids have the same length. Calculate the magnetic flux through the smaller solenoid when the current in the larger is I. Hence find the e.m.f. induced in the smaller if I is changing and determine the mutual inductance of the system M. [5]

These two solenoids, which are assumed to have no resistance, act as a transformer. An AC voltage $V_1 e^{j\omega t}$ is placed across the first solenoid and a resistor R is placed across the terminals of the the second solenoid. Show that the input impedance of the first solenoid is equivalent to a circuit consisting of an inductance L_1 in parallel with a resistor R', and find the value of R'. [8]