CP1/CP2 Collection
Classical Mechanics
Circuit Theory
HT 2008

Section CM (Classical Mechanics)

Answer 3 out of 4 questions

CM1. Write down the equations of the Lorentz transformation and use them to derive the expression for time dilation between two reference frames moving with constant relative velocity. [10]

The half life of the π meson in its own rest frame is 2.6×10^{-8} s. What fraction of a beam of π mesons would survive a journey of 100 m if their velocity relative to the laboratory were $0.98c$? [15]

CM2. A particle of mass m is projected vertically upwards from ground level with an initial speed v_0. In the absence of air resistance, and assuming the acceleration due to gravity, g, is constant, show that it reaches a height $h = v_0^2/2g$. [4]

(a) If the air resistance is proportional to the square of the instantaneous speed, show that the equation of upwards motion can be written

$$\frac{1}{2}\frac{d(v^2)}{dy} = -g - kv^2,$$

where v is the speed at height y, and k is a constant. [7]

(b) Solve this equation and show that the maximum height, h', reached in the presence of air resistance is given by [6]

$$h' = \frac{1}{2k}\ln\left(1 + \frac{kv_0^2}{g}\right).$$

(c) If $v_0 = 100\,\mathrm{m\ s^{-1}}$ and $k = 10^{-3}\,\mathrm{m^{-1}}$, calculate h and h'. [4]

(d) The particle falls back to ground level; without direct calculation, estimate the speed of arrival. [4]

CM3. The equation of motion of a damped harmonic oscillator of mass m constrained to move in the x-direction is

$$\ddot{x} + \gamma\dot{x} + \omega_0^2 x = 0,$$

where $-\gamma\dot{x}$ is the force per unit mass producing the damping and ω_0 is the angular frequency of undamped oscillations. The oscillator is lightly damped ($\gamma \ll \omega_0$).

(a) Derive an expression for the amplitude of the oscillations as a function of time. [5]

(b) Show that the angular frequency of the damped oscillations ω_f is given by [10]

$$\omega_f \approx \omega_0 \left(1 - \frac{\gamma^2}{8\omega_0^2}\right).$$

(c) Show that the energy of the oscillations drops by a factor $1/e$ in a time of approximately $1/\gamma$. [10]

CM4. A non-relativistic particle m_1 with initial velocity u along the x-axis, has a perfectly elastic collision with a stationary particle of mass m_2. After the collision the particles have equal and opposite velocities along the same axis. What is the ratio of the particle masses, m_1/m_2? What is the velocity of the centre of mass? In the centre of mass frame, what are the kinetic energies of the two particles and the total energy after the collision? [15]

A time t after the collision, the particle m_1 collides elastically with a third particle m_3, with mass equal to that of m_2, travelling with velocity u along the same axis as the first particle. Following the collision with m_3, how long will it take for m_1 to catch up with m_2? [10]