Problem Set 3
Nuclear Physics 2005, by Armin Reichold

This draws mainly on lectures 7 and 8 discussing alpha decay barriers and Fermi theory of β-decays

Q1 [Tunneling in α-decays]

1. Why are naturally occurring nuclei with A close to 60 not unstable against alpha decay?
2. If you neglect the existence of nuclear spins and considering coulomb barrier (in alpha decays) which are large compared to the alpha particles kinetic energy, you can approximate the dependence of the mean life time of an alpha unstable nucleus on the charge Z and the alpha particle velocity v by this formula $\tau \approx \exp(\frac{4\pi Ze^2}{4\pi\epsilon_0\hbar v})$ Discuss the basic assumptions made in the derivation of this expression.
3. In the decay of $^{238}_{92}U \rightarrow ^{234}_{90} Th + \alpha$, the emission of alpha particles of energy 4.195, 4.147 and 4.038 MeV is observed. Use the above formula to estimate the relative strengths of these three lines.
4. State briefly why consideration of the spins of the states of $^{234}_{90}Th$ might change your estimates of the relative line strengths.
   Hint: One of the assumptions you made in deriving the formula for the decay time might not be valid for some of the decays if you have to assume different spins for the various final states.

Q2. [kinematics, Mössbauer effect, finals 1997, Q5]
By considering the conservation of energy and momentum show that a free electron cannot absorb a photon. Why can absorption occur if the electron is initially bound? *(5 points)*

A certain narrow nuclear γ-transition is examined in both emission and absorption. It is found that the peak absorption energy E_a differs slightly from the peak emission energy E_e. Find an approximate expression for the difference ΔE. *(6 points)*

Discuss whether the effect would be observable for the γ-transition to the ground state of ^{198}Hg from the 0.411 MeV state, which has a lifetime of 2.2×10^{-11} s. *(6 points)*

Discuss briefly why it is sometimes possible for transitions to occur for which $\Delta E \approx 0$. *(8 points)*

Q3. [Fermi theory of β-decay, neutrino reactions, finals 1998, Q2]
State briefly the basic assumptions of the Fermi theory of nuclear β decay. *(5 points)*

When this simple theory is applied to the reaction $\nu_e + n \rightarrow e^- + p$ the predicted cross section is given by $\sigma = \frac{2\pi}{\hbar}\,\frac{1}{c}\,g^2\,\frac{4\pi(pc)^2}{(hc)^3}$, where p is the momentum in the final state. Discuss the origin of the factors in the above expression. *(5 points)*

The principal source of energy produced in the Sun is nuclear fusion via the pp chain:

$$
\begin{aligned}
p + p &\rightarrow {}^2_1H + e^+ + \nu + 0.42\,\mathrm{MeV} &(1)\\
p + {}^2_1H &\rightarrow {}^3_2He + \gamma + 5.49\,\mathrm{MeV} &(2)\\
{}^3_2He + {}^3_2He &\rightarrow {}^4_2He + p + p + 12.86\,\mathrm{MeV} &(3)
\end{aligned}
$$

On average the neutrinos each take away 0.26 MeV. The flux of radiant energy at the distance of the Earth from the Sun is $1.37\,\mathrm{kW\,m^{-2}}$. Calculate the flux of solar neutrinos at the Earth. *(5 points)*

Higher energy neutrinos can also be produced by reaction chains that involve the production of ^{8}B which undergoes β decay. The end point of the neutrino energy spectrum is 14 MeV. According to the Standard Solar Model, the flux of these neutrinos is 10^{-4} of the neutrinos produced by reaction (1).

A neutrino detector contains 10^6 kg of heavy water. Neutrinos may undergo the reaction $\nu_e + D \rightarrow p + p + e^-$ The threshold for this reaction is 1.4 MeV. Estimate the approximate rate of neutrino interactions of this type due to the neutrinos from ^{8}B. How might such neutrino reactions in the detector be observed in practice? *(10 points)*

[$g = 1.4 \times 10^{-62}\,\mathrm{J\,m^3}$.]